2008 International Symposium on Nonlinear Theory and its Applications
NOLTA'08, Budapest, Hungary, September 7-10, 2008

The Mixed Time-Frequency Steady-State Analysis Method for Nonlinear Circuits Driven by Multitone Signals

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Abstract—This paper presents the mixed time-frequency steady-state analysis method for efficient simulation of circuits whose excitation frequencies are widely separated. These circuits can be written by multitime partial differential equations. In this paper, an axis of the slow time-scale is formulated in the time domain and another axis of the fast time-scale is formulated in the frequency domain.

We show that computational cost, however, is not dependent on the interval of frequencies, whereas for the harmonic balance or transient analysis, it increases exponentially as the interval of frequencies increases.

1. Introduction

For some modulation circuits used in digital communications, time-scales of an information signal and a carrier are widely separated. If such circuits are simulated by using the transient analysis of the time domain simulator, such as SPICE, computational cost increases, because it is necessary to use small time steps of numerical integration for high frequency and to calculate over long time for low frequency. Harmonic balance[1][2] is available as a nonlinear circuit analysis method in the frequency domain. In the case of harmonic balance for multitone, multidimensional Fourier transform[3] or almost-periodic Fourier transform[4] is used. However, if frequencies are separated, the computational cost will increase. Furthermore, the size of a matrix that should be solved for many harmonics becomes large.

In this paper, a mixed time-frequency steady-state analysis method for a multitime partial differential equation[5] to simulate these circuits efficiently is presented. This method is based on the envelope method[6][7]. It formulates one axis of the fast time-scale in the frequency domain and formulates another axis of the slow time-scale in the time domain. The time-axis is discretized in time and differential terms are integrated numerically. At each discretized time, the nonlinear elements expressed with Fourier series are linearized by iterations of Newton's method. Therefore, computational cost is not dependent on the interval of frequencies. So, it is effective for circuits whose frequencies of an informational signal and a carrier are widely separated.

The algorithm of the mixed time-frequency steady-state analysis method is described in Section 2. In Section 3, formulation for some circuit elements is presented. In Section 4, the simulation costs of the mixed time-frequency method, harmonic balance and transient analysis are compared by referring to an example circuit.

2. Mixed Time-Frequency Steady-State Analysis Method

2.1. Multitime Partial Differential Equation

The following differential algebraic equation form for describing a circuit is used.

$$\frac{\partial q(v(t))}{\partial t} + i(v(t)) + u(t) = 0. \quad (1)$$

We consider the circuit driven by two tones where frequencies are widely separated as shown in Fig.1. It can be shown that if $v(t_i, t_c)$ and $u(t_i, t_c)$ denote the bivariate forms of the circuit unknowns and excitations[5], then the following multitime partial differential equation is the correct generalization of (1) for the bivariate case:

$$\frac{\partial q(v(t_i, t_c))}{\partial t_i} + \frac{\partial q(v(t_i, t_c))}{\partial t_c} + i(v(t_i, t_c)) + u(t_i, t_c) = 0,$$

where t_i denotes slow time-scale for low frequency and t_c denotes fast time-scale for high frequency.

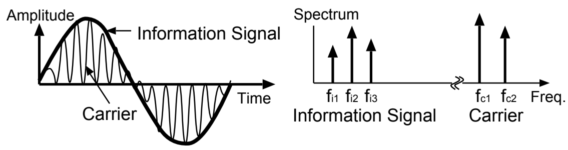

Figure 1: Information signal and carrier

2.2. Algorithm of Mixed Time-Frequency Method

In this method, t_i-axis of (1) is handled in the time domain and t_c-axis of (1) is transformed into the frequency domain as shown in Fig.2. Then, t_i-axis is discretized in time and differential terms are integrated numerically. For t_c-axis, it is expanded into Fourier series as follows:

$$v(t_i, t_c) = \sum_{k=-K}^{K} V_k(t_i, \omega_c) e^{jk\omega_c t_c}, \quad (2)$$

where ω_c is an angular frequency of a carrier. The nonlinear elements expressed with Fourier series are linearized by Newton's iterations at each discretized time. When the envelopes of the spectra, which change with time for every harmonics, reached a steady state, these envelopes for one period of low frequency are transformed into the frequency domain. Finally, we can get steady-state responses summing up all spectra shifted to each fundamental frequency as shown in Fig.3.

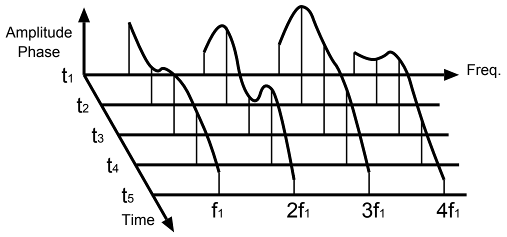

Figure 2: Two axes of time and frequency

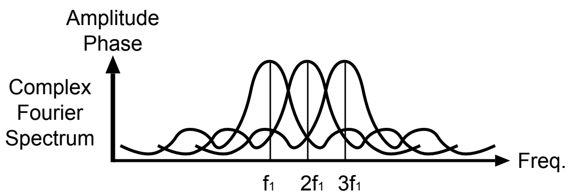

Figure 3: Summation of shifted spectra

A flowchart of the mixed time-frequency steady-state analysis method is shown in Fig.4.

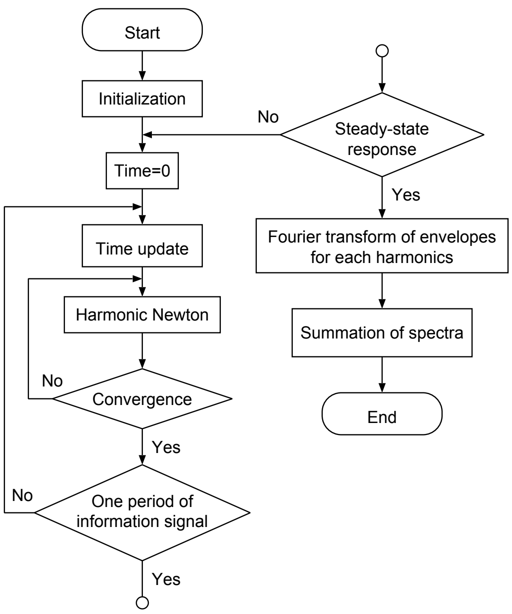

Figure 4: A flowchart of the mixed time-frequency steady-state analysis method

3. Formulation for Each Element

3.1. Dynamic Element (Capacitor)

The relation of the current $i(t)$ and the voltage $v(t)$ of the linear capacitor C is given by

$$i(t) = C\frac{dv(t)}{dt}. \tag{3}$$

It is assumed that $v(t)$ can denote bivariate form $v(t_i, t_c)$. Substituting (2) into (3), we have

$$i(t_i, t_c) = \sum_{k=-K}^{K} I_k(t_i, \omega_c)e^{jk\omega_c t_c},$$

where

$$I_k(t_i, \omega_c) = jk\omega_c C V_k(t_i, \omega_c) + C\frac{dV_k(t_i, \omega_c)}{dt_i}. \tag{4}$$

Discretizing in time of (4), the trapezoid formula is applied to the differential term. We obtain

$$I_k(t_{i,n+1}) = G_C V_k(t_{i,n+1}) + I_{C,n},$$

where

$$\begin{cases} G_C &= jk\omega_c C + \frac{2C}{h} \\ I_{C,n} &= -\frac{2C}{h}V_k(t_{i,n}) - C\frac{dV_k(t_{i,n})}{dt_i} \end{cases}, \tag{5}$$

and n denotes a subscript of discrete time and $h = t_{n+1} - t_n$ is a time step of numerical integration. Then, an equivalent circuit consists of a conductance G_c and current source I_c in parallel.

3.2. Nonlinear Conductance (Diode)

We assume the nonlinear characteristic of diode is given by $i_D = i(v)$. When harmonic Newton is applied to the diode, we can consider that the equivalent circuit at m-th Newton iteration consists of current vector $I_D^{(m)}$ and conductance matrix $G_D^{(m)}$ in parallel. Vector $I_D^{(m)}$ and matrix $G_D^{(m)}$ are shown as follows:

$$I_D^{(m)} = I^{(m)} - \frac{dI^{(m)}}{dV^{(m)}}V^{(m)}$$

$$G_D^{(m)} = \frac{dI^{(m)}}{dV^{(m)}} = \begin{bmatrix} G^{(m)}(0) & G^{(m)}(-1) & \cdots & G^{(m)}(-2K) \\ G^{(m)}(1) & G^{(m)}(0) & \cdots & G^{(m)}(-2K+1) \\ \vdots & \vdots & \ddots & \vdots \\ G^{(m)}(2K) & G^{(m)}(2K-1) & \cdots & G^{(m)}(0) \end{bmatrix},$$

where K is the number of harmonics. Each element of $G_D^{(m)}$ can be expressed using discrete Fourier transform as follows.

$$G^{(m)}(k-l) = \frac{\partial I^{(m)}(k)}{\partial V^{(m)}(l)} = \frac{1}{S}\sum_{s=0}^{S} g^{(m)}(s)e^{-j\frac{2\pi(k-l)s}{S}},$$

where S is the number of time points, k, l denote harmonics and

$$g^{(m)}(s) = \frac{\partial i^{(m)}(v(s))}{\partial v^{(m)}(s)}.$$

In the case of multicarrier whose angular frequencies are ω_{c_1} and ω_{c_2}, conductance matrix $G_{D'}^{(m)}$ is calculated by follows[8].

$$\boldsymbol{G}_{D'}^{(m)} = \begin{bmatrix} G^{(m)}(\Omega_{-K} - \Omega_{-K}) & \cdots & G^{(m)}(\Omega_{-K} - \Omega_{K}) \\ G^{(m)}(\Omega_{-K+1} - \Omega_{-K}) & \cdots & G^{(m)}(\Omega_{-K+1} - \Omega_{K}) \\ \vdots & \ddots & \vdots \\ G^{(m)}(\Omega_{K} - \Omega_{-K}) & \cdots & G^{(m)}(\Omega_{K} - \Omega_{K}) \end{bmatrix},$$

where

$$G^{(m)}(\Omega_k - \Omega_l) = \frac{\partial I^{(m)}(\Omega_k)}{\partial V^{(m)}(\Omega_l)} = \frac{1}{S}\sum_{s=0}^{S} g^{(m)}(s) e^{-j(\Omega_k - \Omega_l)t_s},$$

$$t_s = \frac{sT}{S}, \qquad T = \mathrm{GCD}\left(\frac{2\pi}{\omega_{c_1}}, \frac{2\pi}{\omega_{c_2}}\right),$$

where GCD(·) denotes greatest common divisor and $\Omega_k, \Omega_l = m_{k_1}\omega_{c_1} + m_{k_2}\omega_{c_2}$ for m_{k_1}, m_{k_2} are both integers.

3.3. Information Signal

When we consider an information signal $v_{in}(t_i) = A\sin(\omega_i t_i)$, $A\sin(\omega_i t_{i,n})$ should be added to the equivalent circuit at each discrete time t_n, as a DC source.

4. Example Circuit

An example circuit and its equivalent circuit at n-th discrete time and m-th Newton iteration is shown in Fig.5. v_{in} of excitation is used as the following signal:

$$v_{in}(t_i, t_c) = A\sin(\omega_1 t_i) + B\sin(\omega_2 t_c).$$

The solved equation is shown by (6). For confined space, the number of harmonics used is $K = 1$ here.

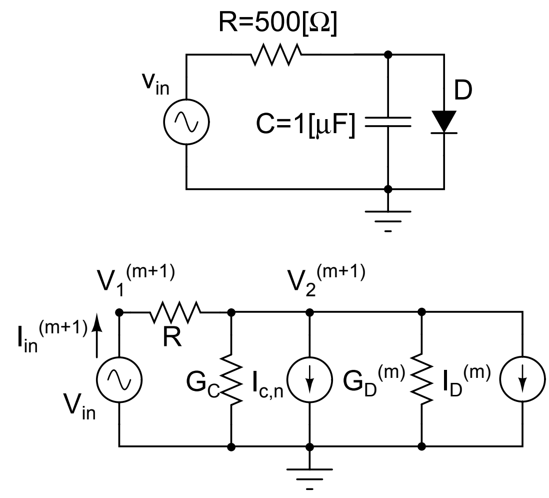

Figure 5: An example circuit and its equivalent circuit

5. Result

Steady-state response of the example circuit in Fig.5 is calculated by the mixed time-frequency steady-state analysis method, harmonic balance and transient analysis. The numbers of harmonics around high frequency used are $K = 5, 10$, and 64 discretized time points are used in one period of low frequency for the mixed method. The number of harmonics used is $K_1 = K_2 = 5$ in harmonic balance with multitone. The number of discretized time points used is 50 in one period of high frequency in the transient analysis. Computational time of these three methods is shown in Fig.6. The horizontal axis is a ratio of ω_1 and ω_2 and the vertical axis is a computational time. In the harmonic balance and transient analysis, the computational time increases exponentially. However, in the mixed time-frequency steady-state analysis method, it is not dependent on the ratio of two excitation frequencies. If frequency scales are separated more than two or three figures, the mixed method is found to be more effective than other methods.

$$\begin{bmatrix} \frac{1}{R} & 0 & 0 & -\frac{1}{R} & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{R} & 0 & 0 & -\frac{1}{R} & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{R} & 0 & 0 & -\frac{1}{R} & 0 & 0 & 1 \\ -\frac{1}{R} & 0 & 0 & \frac{1}{R} + G_C(-\omega_2) + G_D^{(m)}(0) & G_D^{(m)}(-\omega_2) & G_D^{(m)}(-2\omega_2) & 0 & 0 & 0 \\ 0 & -\frac{1}{R} & 0 & G_D^{(m)}(\omega_2) & \frac{1}{R} + G_C(0) + G_D^{(m)}(0) & G_D^{(m)}(-\omega_2) & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R} & G_D^{(m)}(2\omega_2) & G_D^{(m)}(\omega_2) & \frac{1}{R} + G_C(\omega_2) + G_D^{(m)}(0) & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{1,n+1}^{(m+1)}(-\omega_2) \\ V_{1,n+1}^{(m+1)}(0) \\ V_{1,n+1}^{(m+1)}(\omega_2) \\ V_{2,n+1}^{(m+1)}(-\omega_2) \\ V_{2,n+1}^{(m+1)}(0) \\ V_{2,n+1}^{(m+1)}(\omega_2) \\ I_{in,n+1}^{(m+1)}(-\omega_2) \\ I_{in,n+1}^{(m+1)}(0) \\ I_{in,n+1}^{(m+1)}(\omega_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_{C,n}(-\omega_2) - I_D^{(m)}(-\omega_2) \\ I_{C,n}(0) - I_D^{(m)}(0) \\ I_{C,n}(\omega_2) - I_D^{(m)}(\omega_2) \\ jB/2 \\ A\sin(\omega_1 t_{i,n+1}) \\ -jB/2 \end{bmatrix} \quad (6)$$

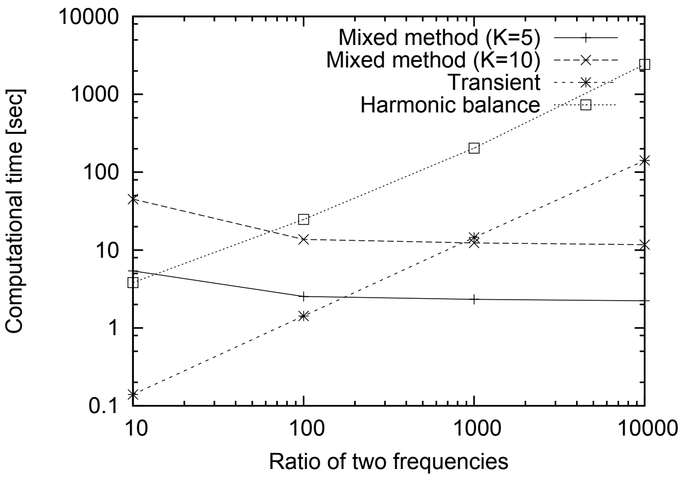

Figure 6: Comparison of computational time between three methods

6. Conclusions

The mixed time-frequency steady-state analysis method is described. An axis of the slow time-scale is formulated in the time domain and another axis of the fast time-scales is formulated in the frequency domain. It was shown that computational cost is not dependent on the interval of frequencies, whereas for other methods it increases exponentially as the interval of frequencies increases. Therefore, the mixed time-frequency method is calculated efficiently, if the time-scales are widely separated.

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