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Solution search performance of a multi-agent solution solving method

Kenta Kohinata† Takuya Shindo Kenya Jinno

†Department of Electrical and Electronics Engineering, Nippon Institute of Technology
‡Gakuendai 4-1, Miyashiro-Saitama, Saitama, 345-8501, Japan
Email: e1112228@estu.ac.jp, t-shindo@nit.ac.jp, jinno@nit.ac.jp

Abstract—In this article, we propose a novel deterministic multi-agent solution solving method that is based on the analysis result of our proposed canonical deterministic particle swarm optimization. The proposed method improves the local search ability by the controlling the search range. The proposed multi-agent solution solving method is similar to meta-heuristics. Comparing with other meta-heuristics, the solution search performance of the proposed multi-agent solution solving method is worse since the system is a deterministic system. However, the deterministic system is easy to implement because the system does not contain the stochastic factors. Thus, if the optimum solution search ability of the deterministic system is improved, the deterministic system is very useful. In this article, we propose the novel method to improve the solution search performance.

1. INTRODUCTION

Particle Swarm Optimization (abbr. PSO) which was proposed by J. Kennedy and R. Eberhart, is an optimum solution searching algorithm [1] [2]. Although the PSO is a very simple conception, the PSO exhibits a superior performance to search the optimum solution of the given objective function. Each particle of the PSO flies the solution space based on its excellent location information and the best location information in the swarm. In general, the rigorous theoretical analysis of the dynamics of the particle of the PSO is quite difficult since the PSO contains stochastic factors. To analyze the dynamics theoretically, we proposed a canonical deterministic PSO (abbr. CD-PSO) [4] that does not contain the stochastic factors. [3]. The analysis results indicate that the fundamental motion of the particles exhibits a spiral motion around the current best position in the phase space. The spiral motion is described by two parameters; a rotation angle and a damping factor. The solution search performance of CD-PSO depends on these two parameters. Our analysis results clarified that CD-PSO exhibits superior performance if the rotation angle is set as goldenangle. Also, we clarified that the damping factor relates to the convergence property. In general, the search performance of metaheuristics is caused by the stochastic factors. In metaheuristics, the diversity of individuals is the most important because plural individuals search an optimal location. The stochastic factors generate the diversity. Also, the escaping ability from the local minimum is caused by the stochastic factors. Therefore, the search performance of the deterministic system is inferior to the stochastic system in metaheuristics. On the other hand, the deterministic system is easy to implement because the system does not contain the stochastic factor. Thus, if the optimum solution search ability of the deterministic system can be improved, we can say the deterministic system is very useful. In order to improve the search ability of the CD-PSO, we proposed the modification CD-PSO [5]. The most important point of the modification CD-PSO is its rotation radius in the phase space becomes a constant. This structure is useful to generate the diversity, and the particles can escape the local minimum [5]. However, the local search ability of the proposed method is worse, because the system expects to search exhaustively around the excellent solution. To overcome this problem, we propose a novel deterministic multi-agent solution solving method which is based on the analysis result of the CD-PSO. The proposed method will improve the local search ability by the controlling the search range.

2. CD-PSO

The novel deterministic multi-agent solution search method is based on our proposed CD-PSO. Therefore, we introduce our proposed CD-PSO at first.

The conventional PSO is described by the following equations.

$$\begin{cases} \boldsymbol{v}_j^{t+1} = w\boldsymbol{v}_j^t + c_1 r_1(\boldsymbol{pbest}_j^t - \boldsymbol{x}_j^t) + c_2 r_2(\boldsymbol{gbest}^t - \boldsymbol{x}_j^t) \\ \boldsymbol{x}_j^{t+1} = \boldsymbol{x}_j^t + \boldsymbol{v}_j^{t+1} \end{cases} \tag{1}$$

where, $\boldsymbol{v}_j^t$ and $\boldsymbol{x}_j^t$ denote the velocity vector and the location vector of the j-th particle on the t-th iteration, respectively. $\boldsymbol{pbest}_j^t$ means the location that gives the best value of the evaluation function of the j-th particle until the t-th iteration. $\boldsymbol{gbest}^t$ means the location which gives the best value of the evaluation function on the t-th iteration in the swarm. $w \geq 0$ is an inertia weight coefficient, $c_1 \geq 0$, and $c_2 \geq 0$ are acceleration coefficients, and $r_1 \in [0, 1]$ and $r_2 \in [0, 1]$ are two separately generated uniformly distributed random numbers.

To analyze the dynamics of the conventional PSO, the random coefficients have been omitted from the conventional PSO. We rewrite the best location information as follows.

$$\begin{cases} \boldsymbol{p}_j^t = \dfrac{c_1 \boldsymbol{pbest}_j^t + c_2 \boldsymbol{gbest}^t}{c} \\ c = c_1 + c_2 \end{cases} \quad (2)$$

We normalize the location information by $\boldsymbol{p}_j^t$. Without loss of generality, we consider one-dimensional case. In this case, Eq. (1) is transformed into the following matrix form:

$$\begin{bmatrix} y_j^{t+1} \\ v_j^{t+1} \end{bmatrix} = \begin{bmatrix} w & -c \\ w & 1-c \end{bmatrix} \begin{bmatrix} y_j^t \\ v_j^t \end{bmatrix}, \quad (3)$$

where, $y_j^t = x_j^t - p_j^t$.

The behavior of the particle is governed by the eigenvalues of the matrix in Eq. (3). The eigenvalue λ is derived as follows.

$$\lambda = \frac{(1+w-c) \pm \sqrt{(1+w-c)^2 - 4w}}{2} \quad (4)$$

This system is a discrete-time system. Therefore, if the eigenvalues exist within the unit circle on the complex plane, the system is said to be stable. When the eigenvalues are complex conjugate numbers, the behavior of the trajectory of Eq. (3) in the phase space $v_j - y_j$ becomes a spiral motion. We have clarified that the system exhibits an excellent solution search performance when the particle exhibits the spiral motion in the phase space [4] [5]. In this case, the damping factor Δ and the rotation angle θ are given as follows.

$$\Delta = \sqrt{\mathrm{Im}(\lambda)^2 + \mathrm{Re}(\lambda)^2} = \sqrt{w}, \quad (5)$$

$$\theta = \arctan \frac{\mathrm{Im}(\lambda)}{\mathrm{Re}(\lambda)} = \arctan \frac{\sqrt{4w - (1+w-c)^2}}{(1+w-c)}. \quad (6)$$

To clarify the effect of the eigenvalues, we derive a canonical deterministic PSO (abbr. CD-PSO) [4] [5].

$$\begin{bmatrix} y_j^{t+1} \\ v_j^{t+1} \end{bmatrix} = \begin{bmatrix} \delta & -\omega \\ \omega & \delta \end{bmatrix} \begin{bmatrix} y_j^t \\ v_j^t \end{bmatrix} \quad (7)$$

The damping factor Δ and the rotation angle θ of CD-PSO are derived as

$$\Delta = \sqrt{\delta^2 + \omega^2}, \quad (8)$$

$$\theta = \arctan \frac{\omega}{\delta}. \quad (9)$$

By using these parameters, Eq. (7) is rewritten as follows.

$$\begin{bmatrix} y_j^{t+1} \\ v_j^{t+1} \end{bmatrix} = \Delta \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} y_j^t \\ v_j^t \end{bmatrix} \quad (10)$$

If the rotational angle is set by the golden angle, the search position becomes not overlapped [6] [7]. The golden angle ϕ is defined as

$$\phi = 180(3 - \sqrt{5})[\mathrm{deg}]. \quad (11)$$

An example of the particle behavior in the phase space is shown in Fig. 1. In this case, the rotation angle θ is set as the golden angle ϕ, and the damping factor is set as $\Delta = 1.0$. Figure 1 indicates the system keeps to oscillate since the damping factor Δ equals to 1.0. If the absolute value of the damping factor is less than 1, the particle converges to the origin in the phase space which corresponds to a fixed point. However, the particle of CD-PSO cannot escape the local minimum if the particle traps the local minimum [6] [7].

To improve the global search ability, we proposed a control method that set a lower limit of the rotation radius on the phase space [5]. When the rotation radius is under a criterion value R, the proposed method resets the velocity of the particle as follows.

$$\begin{bmatrix} x_j^t \\ v_j^t \end{bmatrix} = \begin{bmatrix} p_j^t + R\cos\theta \\ R\sin\theta \end{bmatrix}, \text{ if } \sqrt{{y_j^t}^2 + {v_j^t}^2} < R. \quad (12)$$

Figure 2 illustrates an example of the reset behavior of the particle. In this case, the criterion value R is set as the half of the search range. Figure 2 displays the particle keeps the rotational radius. Therefore, we can say this method improves the global search ability. However, we confirmed that the efficiency of the local search is reduced because the criterion of the rotation radius is set [5]. To overcome this problem, we will propose a novel control method in the next section.

3. A multi-agent solution solving method

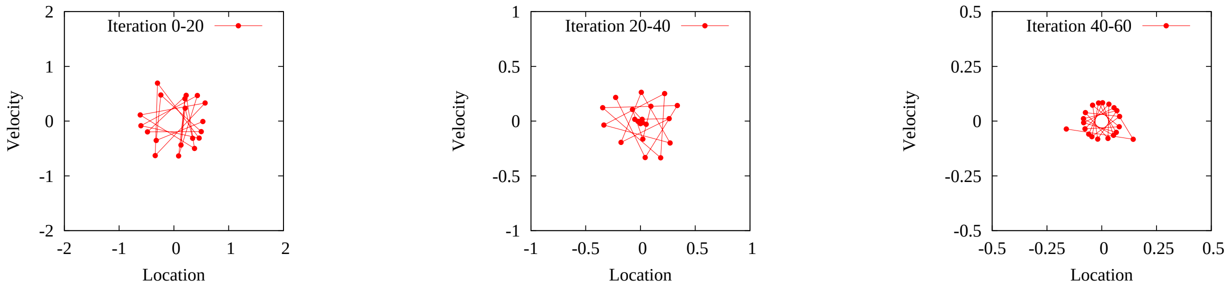
Since the criterion of the rotation radius is set, the CD-PSO can keep the global search, but the efficiency of the local search is reduced. Namely, the criterion value of the rotation radius prevents the local search. Therefore, we consider a method to vary the criterion adaptive.

The most important behavior of the particle of the CD-PSO is a damping vibration motion. Based on the damping vibration motion, we proposed a search system that consists of a plural agent. The concept of the proposed system is not based on the swarm intelligence, therefore, we call this system "multi-agent solution solving method" (abbr. MAS method) [8]. The dynamics of each agent of the MAS is desacribed by

$$x_{kd}^t = R\cos(\alpha t)\cos(\theta t) + p_{kd}^t, \quad (13)$$

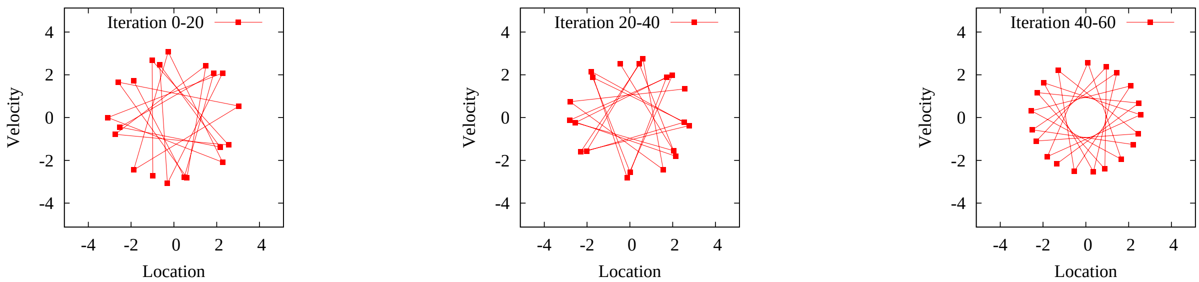
where x_{kd}^t denotes the d-th dimension location of the k-th agent on the t-th iteration, p_{kd}^t denotes the d-th dimension location of the k-th agent that is obtained the best evaluation value until the t-th iteration, and θ is a rotation angle for each iteration. Also, α is another rotation angle that controls the search range, and R denotes the maximum search range.

$R\cos(\alpha t)$ corresponds to the search range. If $\cos(\alpha t)$ is small, the MAS operates as a local search. Other hands, if $\cos(\alpha t)$ is large, the MAS operates as a global search.



Iteration:0 – 20th   Iteration:20th−40th   Iteration:40th−60th

Figure 1: The behavior of a particle of CD-PSO on the phase space when the rotation angle is the golden angle.



Iteration:0 – 20th   Iteration:20th−40th   Iteration:40th−60th

Figure 2: The behavior of a particle with a citerion rotation radius R

Therefore, α controls the variation speed of the search range.

4. Asynchronous MAS

The MAS is quite simple, and the solution search performance is similar to the CD-PSO. However, the performance of the MAS is worse than the conventional PSO. The reason is that the update of all dimension is a synchronous update manner. The synchronous update leads to the deterioration of the solution search performance. To overcome this problem, we propose a novel MAS whose update manner is asynchronous. The dynamics of the asynchronous MAS is descrobed by the following equation.

$$x^t_{kd} = R\cos(\beta_{kd}t)\cos(\theta t) + p^t_{kd}, \tag{14}$$

where β_{kd} is a rotation angle that is the different value on each dimension and each agent. β_{kd} is set as

$$\beta_{kd} = \alpha\,(kD + d - 1) \tag{15}$$

where D is a dimension of the objective function, and $\alpha = \pi/180$.

The different point between the MAS of Eq. (13) and the asynchronous MAS of (14) is the rotation angle β_{kd}. Namely, the rotation angle of the asynchronous MAS is the different value on each dimension and each agent. The asynchronous MAS strategy leads to the improvement of the solution search performance.

We confirm the effects of the asynchronous strategy by numerical simulations. In the numerical simulations, the number of agents is 30, and the dimension of the objective functions D is 30. The The maximam search range R is the half of the defined search range, and the rotation angle θ is set as the golden angle ϕ. The maximum iteration is 3000, and the number of trials is 30. We apply the well-known benchmark functions as shown in Table1.

To compare the performance, Table 2 shows the results of CD-PSO, MAS, and the asynchronous MAS. These results indicate that the solution search ability of the asynchronous MAS is excellent comparing with the CD-PSO and the MAS. The good performance towards CD-PSO Comparing the conventional CD-PSO and MAS but it can be confirmed that the performance is better than the CD-PSO by asynchronously. This will be seen that improved search performance by asynchronously from.

5. Conclusions

In this article, we proposed the novel asynchronous MAS that the rotation angle of the asynchronous MAS is the different value on each dimension and each agent. We confirmed the asynchronous MAS is effective to improve search ability by some numerical simulations. This setting strategy led to the excellent solution search performance. Therefore, the rigorous theoretical analysis of the asynchronous MAS is one of our future problems.

Table 1: Benchmark functions

Function	Definition	Domain	Optimum value
Shifted Sphere	$f(\boldsymbol{y}) = \sum_{i=1}^{D} y_i^2,\ \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	$[-100, +100]^D$	$f(\boldsymbol{o}) = 0$
Shifted Rastrigin	$f(\boldsymbol{y}) = 10D + \sum_{i=1}^{D} \left(y_i^2 - 10\cos(2\pi y_i)\right),\ \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	$[-100, +100]^D$	$f(\boldsymbol{o}) = 0$
Shifted Rosenbrock	$f(\boldsymbol{y}) = \sum_{i=1}^{D-1} \left(100(y_i^2 - y_{i+1})^2 + (1 - y_i)^2\right),\ \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	$[-100, +100]^D$	$f(\boldsymbol{1} + \boldsymbol{o}) = 0$
Shifted Griewank	$f(\boldsymbol{y}) = 1 + \frac{1}{4000}\sum_{i=1}^{D} y_i^2 - \prod_{i=1}^{D}\cos(\frac{y_i}{\sqrt{i}}),\ \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	$[-600, +600]^D$	$f(\boldsymbol{o}) = 0$
Rotated Rastrigin	$f(\boldsymbol{z}) = 10D + \sum_{i=1}^{D} \left(z_i^2 - 10\cos(2\pi z_i)\right),\ \boldsymbol{z} = \mathrm{M}\,\boldsymbol{x}$	$[-100, +100]^D$	$f(\boldsymbol{0}) = 0$
Rotated Rosenbrock	$f(\boldsymbol{z}) = \sum_{i=1}^{D-1} \left(100(z_i^2 - z_{i+1})^2 + (1 - z_i)^2\right),\ \boldsymbol{z} = \mathrm{M}\,\boldsymbol{x}$	$[-100, +100]^D$	$f(\boldsymbol{1}) = 0$

$\boldsymbol{o}$ denotes a random unform vector in the domain, M is rotation matrix of D × D

Table 2: Numerical simulation results

Function	Manner	Mean	Best	Worst	Std. Dev.
Shifted Sphere	CD-PSO	4.83e+04	2.30e+04	6.55e+04	8.58e+03
	MAS	7.50e+04	5.65e+04	9.93e+04	8.56e+03
	A-MAS	$\mathbf{1.39e+01}$	$\mathbf{4.66e+00}$	$\mathbf{2.87e+01}$	$\mathbf{6.73e+00}$
Shifted Rastrigin	CD-PSO	4.85e+04	2.64e+04	6.54e+04	8.59e+03
	MAS	7.52e+04	5.67e+04	9.95e+04	8.58e+03
	A-MAS	$\mathbf{3.24e+02}$	$\mathbf{2.50e+02}$	$\mathbf{4.12e+02}$	$\mathbf{4.22e+01}$
Shifted Rosenbrock	CD-PSO	1.53e+10	4.35e+09	3.38e+10	6.49e+09
	MAS	3.54e+10	1.00e+10	4.65e+10	7.70e+09
	A-MAS	$\mathbf{1.66e+04}$	$\mathbf{2.64e+03}$	$\mathbf{4.94e+04}$	$\mathbf{1.12e+04}$
Shifted Griewank	CD-PSO	1.31e+01	6.74e+00	1.74e+01	2.15e+00
	MAS	1.97e+01	1.51e+01	2.58e+01	2.14e+00
	A-MAS	$\mathbf{6.13e-01}$	$\mathbf{3.88e-01}$	$\mathbf{8.82e-01}$	$\mathbf{1.21e-01}$
Rotated Rastrigin	CD-PSO	1.65E+04	8.94E+03	2.31E+04	3.08E+03
	MAS	1.38E+04	6.60E+03	2.06E+04	4.22E+03
	A-MAS	$\mathbf{3.48E+02}$	$\mathbf{2.77E+02}$	$\mathbf{5.09E+02}$	$\mathbf{5.49E+01}$
Rotated Rosenbrock	CD-PSO	1.98E+09	6.43E+08	3.93E+09	8.34E+08
	MAS	1.10E+09	4.64E+08	2.84E+09	4.52E+08
	A-MAS	$\mathbf{7.65E+04}$	$\mathbf{1.12E+04}$	$\mathbf{4.07E+0}$	$\mathbf{8.44E+04}$

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