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A study of robustness of PSO for non-separable evaluation functions

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Abstract—Recently, many researchers paid attention to the studies of meta-heuristics for the continuous value optimization problems. Especially, Artificial Bee Colony algorithm (abbr. ABC), Differential Evolution (abbr. DE), and Particle Swarm Optimization (abbr. PSO) are applied various optimization problems widely. In general, such meta-heuristics can obtain good solutions of multi-modal functions rapidly. However, solution search performance becomes worse in the case of non-separable functions. In this article, we focus on PSO to search an optimum solution of non-separable functions. We clarify the cause of the degradation of the search performance. Based on the above consideration, we propose a Norm linked PSO which solution search performance is robust for the dependencies among each design variable. We confirm the search performance of the norm linked PSO by numerical simulations.

1. Introduction

Optimization problems are important issue in various fields. In this paper, we consider a single object continuous value optimization problem. The purpose of this problem is finding an optimal design variable vector $\boldsymbol{x} = [x_1, \ldots, x_D]^\top \in \mathbb{R}^D$ that gives a minimum value of the objective function $f(\boldsymbol{x})$. Many evolutionary algorithms (abbr. EAs) have been proposed to solve the problems[1]. For example,, Artifitial Bee Colony algorithm (abbr. ABC)[2], Differential Evolution (abbr. DE)[3], and Particle Swarm Optimization (abbr. PSO)[4] are applied to various optimization problems widely.

In this article, we discuss the affect on the solution search performance when each design variable is non-separable. Based on the numerical simulation results of ABC, DE, and PSO, we investigate the effect of the non-separable design variables.

In Section 2, we introduce ABC, DE, and PSO. In Section 3, we clarify the relationship between the effect of the search performance and the strength of the dependencies among each design variable. In Section 4, we propose a norm linked PSO which solution search performance is robust for the dependencies among each design variable. The solution search performance is confirmed by numerical simulations. Finally, we will summarize the article in Section 5.

2. ABC, DE and PSO algorithm

2.1. Artifitial Bee Colony

ABC is an optimization method that is based on the foraging behavior by stoked the bee. The swarm of bees consists of employed bees, onlooker bees, and scout bees. The employed bees and the onlooker bees generate a trial solution $\boldsymbol{v}_{i,t}$ as the following equation.

$$v_{ij,t} = \begin{cases} x_{ih,t} + \phi(x_{ih,t} - x_{kh,t}) & \text{if} \quad j = h \\ x_{ij,t} & \text{if} \quad j \neq h \end{cases} \tag{1}$$

$x_{ij,t}$ denotes a location variable of the j-th dimension of the i-th bee at the t-th iteration. $\phi \sim U[-1, 1]$ is a uniform distributed random number. h is a random selected integer from $[1, D]$. k is an individual number different from i.

2.2. Differential Evolution

The search strategy of DE is described by the selecting base vector (*base*), the number of difference vector pairs (*num*), and the crossover method (*cross*).

The algorithm of DE/rand/1/bin is classified into two stages as follows.

Mutation: Three vectors $\boldsymbol{x}_{r1,t}$, $\boldsymbol{x}_{r2,t}$ and $\boldsymbol{x}_{r3,t}$, are selected randomly. The mutant vector $\boldsymbol{v}_{i,t}$ is created as the follows.

$$\boldsymbol{v}_{i,t} = \boldsymbol{x}_{r1,t} + F(\boldsymbol{x}_{r2,t} - \boldsymbol{x}_{r3,t}) \tag{2}$$

The mutation factor F is a constant.

Crossover: The trial vector $\boldsymbol{u}_{i,t}$ is created from the target vector $\boldsymbol{x}_{i,t}$ and the mutant vector $\boldsymbol{v}_{i,t}$ as the follows.

$$u_{ij,t} = \begin{cases} v_{ij,t} & \text{if} \quad r_{ij} \leq CR \quad \text{or} \quad j = k \\ x_{ij,t} & \text{if} \quad r_{ij} > CR \quad \text{and} \quad j \neq k \end{cases} \tag{3}$$

CR is a crossover rate, $r_{ij} \sim U[0, 1]$ is a uniform distributed random number, and k is a random integer from $[1, D]$.

2.3. Particle Swarm Optimization

The PSO is an optimization method that is based on swarm intelligence. The PSO is described as

$$\begin{cases} v_{ij,t+1} = wv_{ij,t} + c_1 r_1 (pb_{ij,t} - x_{ij,t}) \\ \qquad\qquad + c_2 r_2 (gb_{j,t} - x_{ij,t}) \\ x_{ij,t+1} = x_{ij,t} + v_{ij,t+1} \end{cases} \tag{4}$$

w is an inertia weight coefficient, c_1 and c_2 are acceleration coefficients, and $r_1, r_2 \sim U(0,1)$ are uniform distributed random numbers.

Each particle has a personal best position $\boldsymbol{pb}_{i,t}$ that was found untill the current iteration t, and the global best position $\boldsymbol{gb}_{i,t}$ that is the best $\boldsymbol{pb}_{i,t}$ in the swarm.

3. Solution search performance of ABC, DE and PSO

3.1. Numerical simulation

Comparing with the solution search performance of ABC, DE and PSO, we carry out some numerical simulations by using well-known benchmark functions. The benchmark functions are shown in Table1.

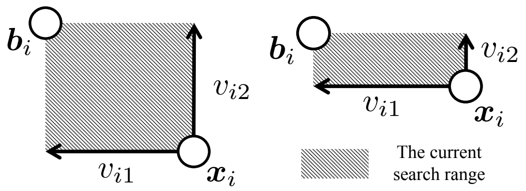
We evaluate the search performance by using the benchmark functions as shown in Table 1. Table 2 shows the solution search performance evaluation results of ABC, DE, and PSO. The dimension of the benchmark functions is 30, and the number of individuals is 30. In each simulation, the number of the maximum iteration is 3000, and the number of trials is 30. Also, the rotation matrix $\boldsymbol{M}$ and the shift vector $\boldsymbol{o}$ are the same in all trials. Table 3 shows the parameters used in the numerical simulations. Figure 2 shows the variation of the mean value of the obtained optimum value.

The numerical simulation results indicate that the solution search performance of ABC and PSO becomes worse in the case where the dependency among variables is strong. The solution search performance of DE becomes worse in the case of Shifted and Rotated Rastrigin function. However, the results of Rosenbrock function indicate that DE is resistant to the dependency among the all design variables. From Fig. 2, the solution search performance of ABC and DE depends on the initial state. The solution search performance of PSO is changed around 1500 iteration.

The solution search performance of ABC becomes worse because the only one dimension in the current solution vector is changed. This update strategy performs efficiently in the case where the local minima are arranged in a grid pattern as Rastrigin function. However, this update strategy does not perform efficiently in the case of rotated functions.

The solution search performance of DE is affected by the dependency among design variables is due to the crossover operation. If the crossover rate CR is a high value, the dependency is strong. On the other hand, if the crossover rate is low value, the dependency is weak. Thus, the lower crossover rate is recommended[5]. In our numerical simulations, we set high CR. Therefore, the results of DE indicate that the performance of Shifted and Rotated Rosenbrock function is better than one of Shifted Rosenbrock function. The adaptive DE which can eliminate the dependency among design variables is proposed[6][7].

Since the velocity of particle of PSO is determined in each dimension, the dependency among design variables affects the solution search performance. The velocity corresponds to the search range is determined by the difference between the current position and the best position as shown in Fig. 1. The search direction is along each axis. Therefore, the search performance of the rotated function is deteriorated[8]. Koguma and Aiyoshi have proposed a Linked Random Model to improve the performance of finding a diagonal direction by allowing link the random numbers r_1 and r_2 in each dimension. However, Koguma and Aiyoshi have reported that the solution search performance becomes worse because the diversity is reduced by the allowing link random numbers. The occurrence of such situation is shown in Fig. 1b that the inertia coefficients w prevent. However, the effect of the inertia coefficient is decreased if the velocity is reduced. In this case, the search range is restricted to the axes.



(a) The bias is not occurred (b) The bias is occurred

Figure 1: The search range is changed by the relationship between the current position $\boldsymbol{x}_i$ and the best position $\boldsymbol{b}_i$. $\boldsymbol{v}_i$ denotes the velocity of the particle. Note that this figure does not consider the effect of the inertia coefficient and the acceleration coefficient.

Table 2: Simulation results of well-known benchmarck functions in Table 1

		ABC	DE	PSO	Proposed
f_1	Best	3.38E+00	8.32E+01	6.21E+00	2.34E+01
	Median	4.81E+00	2.10E+02	7.95E+01	2.91E+01
	Worst	1.08E+01	5.91E+07	3.61E+02	3.97E+03
f_2	Best	6.31E-03	1.89E+01	2.79E+01	3.88E+01
	Median	6.31E-03	2.82E+01	4.97E+01	6.57E+01
	Worst	1.40E-02	3.49E+02	8.56E+01	1.28E+02
f_3	Best	2.21E+01	1.15E+01	2.46E+01	2.44E+01
	Median	3.10E+01	2.55E+01	6.23E+01	2.87E+01
	Worst	3.36E+01	9.52E+05	4.51E+05	6.07E+04
f_4	Best	1.80E+02	2.59E+01	7.56E+01	3.38E+01
	Median	1.80E+02	3.89E+01	1.30E+02	6.45E+01
	Worst	3.35E+02	2.04E+02	2.69E+02	1.31E+02

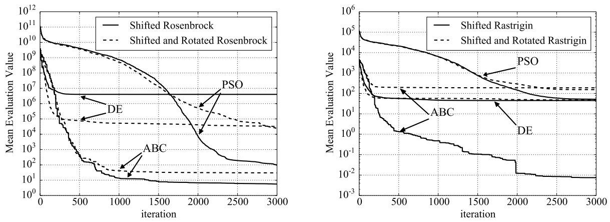
Table 3: The parameters setting of ABC, DE and PSO

ABC	The number of employed bees $Ne = 15$ The number of onlooker bees $No = 15$ The continuous search times upper limit 300
DE	$F = 0.5, CR = 0.9$
PSO	$w_t = w_2 + (w_1 - w_2)\frac{t_{\max}-t}{t_{\max}}$ $w_1 = 0.9, w_2 = 0.4,\ c_1 = c_2 = 2.0$

Table 1: Well-known benchmark functions

Names	Definition	Properties	Domain
Shifted Rosenbrock	$f_1(\boldsymbol{y}) = \sum_{i=1}^{D-1}[100(y_i^2 - y_{i+1})^2 + (y_i - 1)^2], \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	multimodal	$[-100, 100]^D$
Shifted Rastrigin	$f_2(\boldsymbol{y}) = \sum_{i=1}^{D}[y_i^2 - 10\cos(2\pi y_i) + 10], \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	multimodal	
Shifted and Rotated Rosenbrock	$f_3(\boldsymbol{x}) = f_1(z), z = \boldsymbol{M}\boldsymbol{y}, \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	multimodal	
Shifted and Rotated Rastrigin	$f_4(\boldsymbol{x}) = f_2(z), z = \boldsymbol{M}\boldsymbol{y}, \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$	multimodal	

$\boldsymbol{o}$ denotes a uniform random vector in the domain, $\boldsymbol{M}$ is a $D \times D$ rotation matrix.



(a) Rosenbrock (b) Rastrigin

Figure 2: The variation of the mean best evaluation value of the benchmark functions by using ABC, DE and PSO. ($D = 30, N = 30$, Trial number: 30)

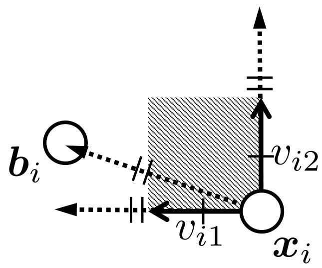
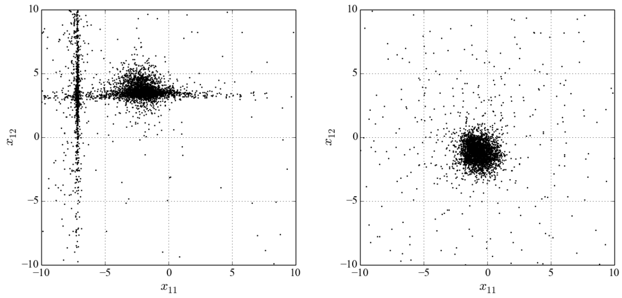


Figure 3: The search range of Norm linked PSO



(a) Conventional PSO (b) Norm linked PSO

Figure 4: The search trajectories of the particles for Rotated Rastrigin function. ($D = 30, N = 30, t_{\max} = 10000$)

4. Norm linked PSO

4.1. Proposal of Norm linked PSO

The factor of PSO solution search performance is affected dependencies among variables is that the velocity of particles is determined in each dimension. To overcome this problem, we propose a novel PSO unaffected dependency among all design variables. In this article, this system calls Norm linked PSO. The dynamics of Norm linked PSO is described by the following equations.

$$\begin{cases} v_{ij,t+1} = wv_{ij,t} + s_1 r_1 \cdot \mathrm{sign}(pb_{ij,t} - x_{ij,t}) \\ \qquad\qquad + s_2 r_2 \cdot \mathrm{sign}(gb_{j,t} - x_{ij,t}) \\ x_{ij,t+1} = x_{ij,t} + v_{ij,t+1} \end{cases} \tag{5}$$

$$s_1 = \alpha_1 \|\boldsymbol{pb}_{i,t} - \boldsymbol{x}_{i,t}\|_2 \tag{6}$$

$$s_2 = \alpha_2 \|\boldsymbol{gb}_t - \boldsymbol{x}_{i,t}\|_2 \tag{7}$$

where w is an inertia weight coefficient, c_1 and c_2 are acceleration coefficients, and $r_1, r_2 \sim U(0, 1)$ are uniform distributed random numbers. α_1 and α_2 are constants, and s_1 and s_2 correspond to the euclidean norm.

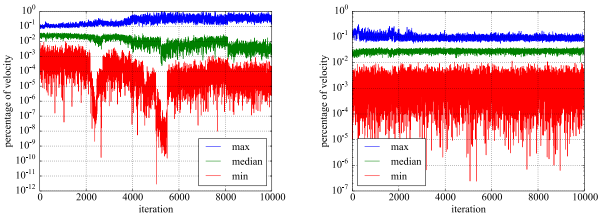
Norm linked PSO determines the moving direction by the sign function, and the velocity by the euclidean norm between the current position and the best position. This strategy prevents the occurrence of bias in the velocity of the particles in each dimension. In Norm linked PSO, s_1 and s_2 are the same value in each dimension. Since all dimension of the velocity makes the same, the moving direction becomes an oblique direction, and the diversity of the search is reduced. However, the random numbers r_1 and r_2 maintain the diversity of the search. Figure 3 shows conceptual diagram of Norm linked PSO.

Figure 4 shows the search trajectories of Norm linked PSO and conventional PSO. The search direction of the conventional is constrained to the coordinate axes. In contrast, the search direction of Norm linked PSO is not constrained to the axis.

The percentage of each dimension of velocity is defined by the following equation .

$$\mu_{ij,t} = |x_{ij,t}| \Big/ \sum_{k=1}^{D} |x_{ik,t}| \tag{8}$$

Figure 5 shows the time variation of $\mu_{ij,t}$ in case of Rotated Rastrigin function. Variations in the percentage increase in the conventional PSO, the maximum value as overlapping iterations becomes large. On the other hand, the percentage of each dimension in the proposed method to remain roughly constant.



(a) Conventional (b) Proposed

Figure 5: The variation of percentage of velocity $\mu_{ij,t}$ ($D = 30, N = 30$)

Table 4: The parameters setting of Norm linked PSO

$w_t = w_2 + (w_1 - w_2)\frac{t_{\max}-t}{t_{\max}}$
$w_1 = 0.9, w_2 = 0.4, \ \alpha_1 = \alpha_2 = 0.194$

4.2. The solution search performance of Norm linked PSO

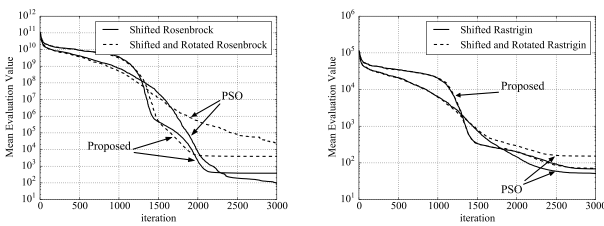
We carry out numerical simulations using the benchmark functions in Table 1 to investigate the solution search performance of Norm linked PSO. The parameters are shown in Table 4, and the numerical simulation results is shown are Table 2. Figure 6 shows the variation of the mean value of the obtained optimum value.

These results indicate that the solution search performance of Norm linked PSO is not affected by the dependency among design variables comparing with the conventional PSO.

5. Conclusions

The solution search performances of ABC, DE and PSO were investigated by numerical simulations. We confirmed the performances that is affected by the strength of the dependency among the design variables. We clarified the cause is that only one dimension was changed in the update strategy of ABC. Also, we clarified that the optimal parameters of DE depend on the strength of the dependency. This fact is the cause of the search performance of DE. In the PSO, we clarified that the imbalance of the search range is generated by the relationship between the current position and the best position. And, we confirmed that the imbalance affects the search performance of PSO.

To eliminate the deviation of the search range of the conventional PSO, we proposed Norm linked PSO to determine the velocity of the particles by Euclidean norm. And we investigated the solution search performance of Norm linked PSO by numerical simulations. As the results, we clarified that the solution search performance of Norm linked PSO is not affected by the strength of the dependency among design variables comparing with the conventional PSO.



(a) Rosenbrock (b) Rastrigin

Figure 6: The variation of the mean of the obtained optimal evaluation value ($D = 30, N = 30$, Trial number: 30)

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References

[1] M. Kubo and J. P. Pedroso, "Mathematics of Metaheuristic", Kyoritsu Shuppan (2009).

[2] D. Karaboga, Technical Report TR06, aaa (2005).

[3] R. Storn and K. Price, Technical report, International Computer Science Institute, **11**, TR-95-012, pp. 1–15 (1995).

[4] J. Kennedy and R. Eberhart, Neural Networks, 1995. Proceedings., IEEE International Conference on, **4**, pp. 1942–1948 vol.4 (1995).

[5] J. Montgomery and S. Chen, IEEE Congress on Evolutionary Computation, IEEE, pp. 1–8 (2010).

[6] J. Brest, S. Greiner, B. Boskovic, M. Mernik and V. Zumer, IEEE Transactions on Evolutionary Computation, **10**, 6, pp. 646–657 (2006).

[7] J. Zhang and A. C. Sanderson, IEEE Transactions on Evolutionary Computation, **13**, 5, pp. 945–958 (2009).

[8] Y. Koguma and E. Aiyoshi, IEEJ Transactions on Electronics, Information and Systems, **130**, 1, pp. 29–38 (2010).