2008 International Symposium on Nonlinear Theory and its Applications
NOLTA'08, Budapest, Hungary, September 7-10, 2008

Maximum Flow Problem to be solved based on Unidirectional Cellular Neural Network

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abstract

In advanced of networked society by the internet, the way how to send data fast with a little loss becomes an important problem. It is a generalization maximum flow algorithm to give the best solution for the network-flow problem that which route is better to excange data. Therefore, the importance of the maximum flow algorithm is growing more and more. In this paper, we propose a novel neural network having a saturation characteristic for maximum flow problem. The proposed neural network can be realized by using nonlinear resistive network having a saturation characteristic of current. Especially, since neural network can independently process the operation in each neuron, it has a very high parallel processing performance. That is, the proposed method can be applied to the large-scale network-flow problem by a large-scale parallel processing.

1. Introduction

In the modern society, the spread of the internet is advanced more and more. Thus, the method how to send data fast with a little loss is very important, and to send data most efficiently, how much the communication charge is allocated in each route becomes a problem. This problem is well known as a network flow problem, and maximum flow algorithm gives the best solution for the problem which route is better to exchange data. Hence, the importance of the maximum flow algorithm is growing more and more.

The network flow problem [8][9] is a problem of throwing a lot of quantity of flow (communication charge) between the starting point and the terminal point on the network. The flow on the branch that satisfies the following conditions is called flow; Flow does not exceed the capacity of the branch (the capacity condition), and sum of flow of inflow is equal to sum of flow of outflow (the flow preservation condition). Some nodes exist between networks, and they are connected by the branch. The capacity of the branch (the capacity of communication) and the gain exist in the branch. The capacity of the branch shows the limit that can throw the flow, and the gain can enlarge or reduce the flowing the flow[7]. In this research area, by using abovementioned conditions, the following algorithms are developed for solving the maximum flow problem.

Ford-fulkerson algorithm [1][2] and preflow push algorithm [3][4] are well known as an algorithm to solve the maximum flow problem. The idea behind the Ford-Fulkerson algorithm is as follows; As long as there is a path from the source (start node) to the sink (end node), with available capacity on all edges in the path, we send the flow along one of these paths. Then we find another path, and so on. A path with available capacity is called an augmenting path. The idea behind the preflow push algorithm is as follows; The maximum flow is requested while maintaining preflow where the augmenting path dose not exist by using preflow that eases the flow preservation condition. Since ford-fulkerson algorithm is a repetition of an easy procedure, validity can be intuitively understood, but it takes a lot of computing time. The preflow push algorithm is high-speed and practicable algorithm now.

In this paper, we propose a novel neural network which has a saturation characteristic, and we show that the proposed neural network can suit to the maximum flow problem [5][6]. The weight between each neuron is based on Ohm's law, and nonlinear resistance with the saturation characteristic where the $I-V$ characteristic is described by the Sigmoid function is used in the proposed method. The starting point and the terminal are assumed to be a source of voltage and ground, respectively. When the voltage of a constant amount is applied to the starting point, the node voltage of each node is converged to equilibrium state. In such a condition, the current value between each neuron is equivalent to the flow, and the total of the current that flow out from the starting point are called maximum flow. Generally, the flow that flows between nodes is interactive. However, in this research, the direction of the flow is assumed to only one direction. The proposed method is described by the state equation concerning the node voltage of the neuron, and the behavior of the dynamical system is described as a differential equation. Since the network analysis by our method is equal to the problem of requesting the current distribution by the nonlinear resistive circuit analysis, it becomes possible to obtain the solution very high-speed. By the computer simulation, we show that the current distribution between nodes is equal to an analytical result by the conventional methods at equilibrium state.

2. Maximum Flow Algorithm

The theory of the network flow problem is one of the effective technique to solve the problem by linear programming. Especially, the theory is extremely effective to represent the model of the network, for example transportation problem, assignment problem and scheduling problem.

When a branch capacity c_k which has positive constant is allocated in each branch $b_k \in B(N)$ on network N as a graph, the N is called communication network or transportation network. And it is shown as $N = [V(N), B(N)]$ for nodes set $V[N]$. $C(N)$ is a set of branch capacity $\{c_k\}$.

A directed branch b_k connected from node v_i to v_j on N is denoted by $b_k = (v_i, v_j) = b_{ij}$.

A flow f from $s \in V(N)$ to $t(t \neq s) \in V(N)$ in the communication network N is defined by

$$\sum_{v_j \in \Gamma(v_i)} f(v_i, v_j) - \sum_{v_j \in \Gamma^{-1}(v_i)} f(v_j, v_i) = \begin{cases} F : v_i = s \\ -F : v_i = t \\ 0 : v_i \neq s, t, \end{cases} \quad (1)$$

$$0 \leqq f\left(v_i, v_j\right) \leqq c\left(v_i, v_j\right), \left(v_i, v_j\right) \in B(N), \quad (2)$$

where

$$\Gamma(v_i) = v_j | (v_i, v_j) \in B(N), \quad (3)$$

$$\Gamma^{-1}(v_i) = v_j | (v_j, v_i) \in B(N). \quad (4)$$

$F = F(f)$ in Eq. (1) is the value of flow f, and node s and t are the source and the sink respectively.

Let the left part of Eq. (1) be the flow that flows out from v_i, then the Eq. (1) represents the restriction, where the flow from source s is F and the flow from sink t is $-F$ and the flow from arbitrary $v_i \neq s, t$ is 0. Also, if we define that the flow $f(v_i, v_j)$ that flows in each $(v_i, v_j) \in B(N)$ is the branch flow, Eq. (2) represents the restriction where the branch flowing on each branch b_{ij} flows only in the direction from v_i to v_j and it does not exceed the branch capacity $c(v_i, v_j)$.

In the communication network N, a branch set (X, Y) is defined as

$$(X, Y) = \{(v_i, v_j) \in B(N) | v_i \in X, v_j \in Y\}, \quad (5)$$

where $X, Y \subset V(N)$. The branch class (X, Y) has source on X and sink on Y. For arbitrary flow f, the flow $f(X, Y)$ which flows (X, Y) is given by

$$f(X, Y) = \sum_{(v_i, v_j) \in (X, Y)} f(v_i, v_j). \quad (6)$$

In the flow f of the communication network N, a flow f_0 that gives the maximum value of $F(f)$ expressed by $F_0 = \max[F(f)]$ is called the maximum-flow.

3. Neural Network with Saturation Characteristic for Network-Flow Problem

3.1. Proposed Neural Network

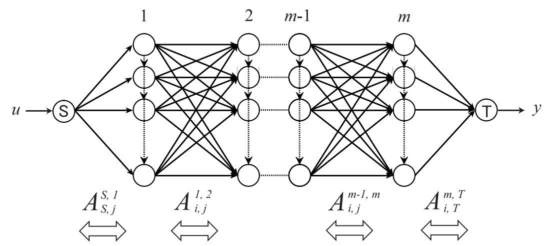

Figure 1: Network topology of proposed method

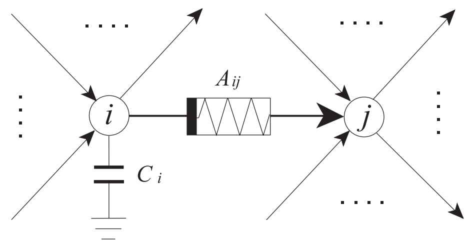

Figure 2: Association between neuron i and j

The network topology of proposed method is shown in Fig. 1. Input layer and output layer of the network are corresponding to start point S and terminal point T, and each point corresponds to a single neuron, and layer number of inner layer is m. Additionally, the propagation between the neighboring neurons is only one direction. In other words, the association from neuron v_i to neuron v_j is defined as b_{ij}, that is, if b_{ij} exists, b_{ji} dose not exist. When all the numbers of nodes except the start point S and the terminal point T are n, the maximum number of branches that meets the condition where connection between nodes is only one direction is $n(n-1)/2$, and layer number of inner layer is $m = 1$. The layer number of inner layer changes depending on how to connect the neuron. When we focus on the neuron v_i, Fig. 2 shows weight A_{ij} that exists between neuron v_j connected with neuron v_i. The proposed network has the saturation characteristic that the entire network converges to the equlibrium state if a certain amount of the current in starting point s goes out. The $I-V$ characteristic from neuron v_i to neuron v_j that shows the feature of this network is described by

$$I_{ij} = \frac{A_{ij}}{1 + exp(v_j - v_i)}, \quad (7)$$

where A_{ij} is maximum capacity $c(v_i, v_j)$ of the current in b_{ij} and a positive constant. If b_{ij} doesn't exist, then $A_{ij} = 0$. I_{ij} is a current that flows from neuron v_i to v_j, and u_i and u_j

are the node voltages of neuron v_i and v_j respectively. Let $f(x)$ be

$$f(x) = \frac{1}{1 + exp(-x)}, \tag{8}$$

then, Eq.(8) can be rewritten by

$$I_{ij} = A_{ij} f(v_i - v_j). \tag{9}$$

The state equation with respect to the neuron v_i is given by

$$F_i(u_i) \equiv C_i \frac{du_i}{dt} = -\sum_{j=1}^{n} A_{ij} f(v_i - v_j), \tag{10}$$

where C_i is a capacitor that exists between neuron i and the ground. By solving the simultaneous differential equation concerning the v_i $(i = 1, 2, \cdots, n)$, the state of each neuron (node voltage) v_i is obtained. The potential difference between neurons which have the connections is assigned to Eq. (10), and by solving Eq. (10), the current value between each neuron is obtained.

3.2. Reproduction of Network by Nonlinear Resistive Network

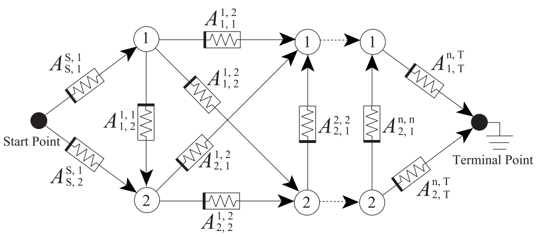

Figure 3: The resistive network using nonlinear resistances

In this paper, the proposed neural network is simulated by using the nonlinear resistive circuit shown in Fig. 3. The direction indicated by the arrow in figure is a direction where the current flows. Each branch (v_i, v_j) can be replaced with the element of nonlinear resistance. Each conductance between each node (v_i, v_j) is shown $c(v_i, v_j)$, and $c(v_i, v_j)$ corresponds to the amount of the maximum current that can flow in each branch. Each current value I is shown by $I - V$ characteristic given by Eq. (8). The voltage generator is set to starting point S, and terminal T is assumed to be a ground. Each node voltage $u_i(i = 1, 2, \cdots, n)$ is saturated at a certain time when the voltage from the voltage generator is stepped up, and the current value between nodes can be obtained in this saturated condition. Current value I_{ij} between each neuron in the state of equilibrium of this network can be considered to be flow f_{ij}, and the sum total of the current that flows out from starting point S is maximum flow F_0.

4. Simulation Results

In this research, we use a network as shown in Fig. 1, and the maximum flow of given network is obtained using our method. The number of neurons except the start point and the terminal are 80, and the layer number of inner layer is $m = 8$, and each inner layer has 10 neurons. The total of the branch is 720, and the weight of each branch is generated randomly. The proposed circuit is analyzed by a circuit simulator (SDP). Since the shigmoidal function can be approximated by the piecewise linear (PWL) function, we replace it with the PWL function.

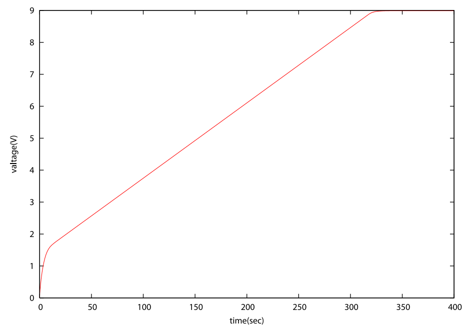

Figure 4: The time transient of node voltage of node 1

The simulation results are shown in Table 1 and Table 2. Table 1 shows analysis result of the flow that flows out from S, and Table 2 shows analysis result of the flow that flows out from Node1. From Table 1, maximum-flow F_0 is 480 ($F_0 = f(S, 1) + f(S, 2) + \cdots + f(S, 10)$). Additionally, at node 1, sum of the flow of inflow is 10 from Table 1, and sum of the flow of outflow is 10.001 from Table 2. That is, The flow preservation condition and the capacity condition are satisfied from Table 1 and Table 2.

The time transient of node voltage of node 1 is shown in Fig. 6. This time transient figure is a waveform using the normalized network where The capacitor of each neuron is $C_i = 1(F)$. Actually, when the integrated circuit is used, the convergence time becomes very short. That is, the analysis by the proposed method can obtain the solution at the very high speed.

5. Conclusion

In this paper, novel neural network with the saturation characteristic for the maximum flow problem was proposed. By using the proposed neural network, the equivalent result of the maximum flow algorithm is obtained. In addition, our method was an effective algorithm for obtaining the solution at the very high speed.

Acknowlegment

This research is supported by the fund of Open Research Center Project from MEXT of Japanese Government(2007-2011)

Table 1: Flow that flows out from S

Node Number of $m(=1)$	Nonlinear Resistance	Node Voltage of S (A)	Node Voltage of $m(=1)$ (B)	Potential Difference (A)-(B)	current value (Flow)
1	10 (Ω)	10 (V)	8.98796 (V)	1.01204 (V)	10(A)
2	20 (Ω)	10 (V)	9.0061 (V)	0.9939 (V)	19.878(A)
3	30 (Ω)	10 (V)	9.02518 (V)	0.97482 (V)	29.2446(A)
4	40 (Ω)	10 (V)	9.0452 (V)	0.9548 (V)	38.192(A)
5	50 (Ω)	10 (V)	9.06584 (V)	0.93416 (V)	46.708(A)
6	60 (Ω)	10 (V)	9.08682 (V)	0.91318 (V)	54.7908(A)
7	70 (Ω)	10 (V)	9.10782 (V)	0.89218 (V)	62.4526(A)
8	80 (Ω)	10 (V)	9.08861 (V)	0.91139 (V)	72.9112(A)
9	80 (Ω)	10 (V)	9.08861 (V)	0.91139 (V)	72.9112(A)
10	80 (Ω)	10 (V)	9.08861 (V)	0.91139 (V)	72.9112(A)
				Maximum Flow	479.9996(A)

Table 2: Flow that flows out from $Node1$

Node Number of $m(=2)$	Nonlinear Resistance	Node Voltage of Node1 (A)	Node Voltage of $m(=2)$ (B)	Potential Difference (A)-(B)	current value (Flow)
1	90 (Ω)	8.98796 (V)	8.9494 (V)	0.03856 (V)	3.4704(A)
2	10 (Ω)	8.98796 (V)	8.95688 (V)	0.9939 (V)	0.3108(A)
3	20 (Ω)	8.98796 (V)	8.9637 (V)	0.97482 (V)	0.4852(A)
4	30 (Ω)	8.98796 (V)	8.96904 (V)	0.9548 (V)	0.5676(A)
5	40 (Ω)	8.98796 (V)	8.97268 (V)	0.93416 (V)	0.6112(A)
6	50 (Ω)	8.98796 (V)	8.97461 (V)	0.91318 (V)	0.6675(A)
7	60 (Ω)	8.98796 (V)	8.97478 (V)	0.89218 (V)	0.7908(A)
8	70 (Ω)	8.98796 (V)	8.97321 (V)	0.91139 (V)	1.0325(A)
9	70 (Ω)	8.98796 (V)	8.97321 (V)	0.91139 (V)	1.0325(A)
10	70 (Ω)	8.98796 (V)	8.97321 (V)	0.91139 (V)	1.0325(A)
				Total Outflow of $Node1$	10.001(A)

References

[1] L. R. Ford, Jr. and D. R. Fulkerson, ”Maximal Flow Through a Network”, Canadian Journal of Mathematics, pp. 399-404, 1956.

[2] L. R. Ford, Jr. and D. R. Fulkerson, ”A Simple Algorithm for Finding Maximal Network Flows and an Application to the Hitchcock Problem”, Canadian Journal of Mathematics, pp. 210-218, 1957.

[3] B. V. Cherkassky and A. V. Goldberg, ”On implementing the push-relabeled method for the maximum flow problem”, Algorithmica, vol. 19, pp. 390-410, 1997.

[4] A. V. Goldberg and R. E. Tarjan, ”A new approach to the maximum flow problem”, Journal of the ACM, vol. 35, pp. 921-940, 1988.

[5] J. J. Hopfield and D. W. Tank,, ”Neural computation of decisions in optimization problems,”, Biolog. Cybern., vol. 52, no. 3, pp. 141-152, 1985.

[6] D. W. Tank and J. J. Hopfield, ”Simple neural optimization networks, an A/D converter, signal decision circuit, and a linear programming circuit”, IEEE Trans. Circuits Syst., vol. 33, pp. 533-541, May 1986.

[7] V. Ramachandran, ”The Complexity of Minimum Cut and Maximum Flow Problems in an Acyclic Network”, Networks, vol. 17, pp. 387-392, 1987.

[8] R. K. Ahuja, T. L. Maganti, and J. B. Orlin, ”Network Flows:Theory, Algorithms and Applications” Englewood Cliffs, NJ: Prentice-Hall, 1993.

[9] R. Ahlswede, N. Cai, S.-Y. Li, and R. W. Yeung, ”Network information flow”, IEEE Trans. Inf. Theory, vol. 46, no. 4, pp. 1204-1216, Jul. 2000.