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Basic Performances of Deterministic Particle Swarm Optimizer Networks

Tomoyuki Sasaki†, Hidehiro Nakano‡, Arata Miyauchi‡ and Akira Taguchi‡

†Tokyo City University, 1-28-1, Tamazutsumi, Setagaya-ku, Tokyo, 158-8557, Japan
Email: sasaki@ic.cs.tcu.ac.jp

Abstract—PSO network is the method of dividing population of PSO into multiple sub-swarm groups, and each sub-swarm group is connected based on network structure. In PSO network, the parameters which characterize network structure affect the search performances significantly. However, such an effect has not been investigated. In this paper, the deterministic PSO network with deterministic couplings is proposed. The proposed model does not have stochastic dynamics and has only deterministic dynamics. We investigate the characteristic of the D-PSON-D and perform the numerical experiments. We then clarify the suitable parameters for the network structure.

1. Introduction

Recently, various systems become large and complicated with developing technology. So, it is difficult to search optimum statements of the systems that satisfy constrained conditions within realistic time. Meta-heuristics which attract many researchers can search the acceptable solutions within realistic time. Particle Swarm Optimization (PSO)[1],[2] is one of the powerful meta-heuristics.

PSO has a lot of superior characteristics, such as low computational cost, little number of parameters, fast convergence characteristic and easy implementation, compared with other meta-heuristics. PSO is based on social behavior as a stylized representation of the movement of organisms such as a bird flock or fish school. These creatures determine the direction of movement by using their velocity information, position information, and direction of the swarm. In PSO, the particles corresponding to these creatures move to the direction to desired solutions in the search space.

Because particles share the best information in the swarm, if the best information is updated by a local minimum, all particles can move to and converge to the local minimum. Various method to improve the weak point have been proposed. In network structure PSO (NPSO)[3], each particle shares the information between neighbor particles. Then, it is able to avoid propagating the best information simultaneously and trapping into local minimum. The algorithms based on such a network topology between particles have been proposed actively[3]~[7]. Multi swarm methods have also been proposed[8]~[10].

In our previous work, we proposed PSO networks with deterministic couplings (PSON-D)[10]. In PSON-D, the population is divided into multiple sub-swarm groups. The multiple sub-swarm groups are connected to each other for purpose of communication. Each sub-swarm group connects to the other sub-swarm groups based on network structure. When the best information is updated in one of sub-swarm groups, the interprocess communication occurs between only connected sub-swarm groups. Even if one of the sub-swarm groups traps into a local minimum, the sub-swarm group can escape from the local minimum by the interprocess communication with neighbor sub-swarm groups. In PSON-D, network topology between sub-swarm groups and between particles affect the search performances. However, a investigation of these has not been performed sufficiently so far.

In this paper, we investigate the search performances of PSON-D. In order to elucidate the influence of the network topology, we propose the deterministic PSON-D (D-PSON-D). D-PSON-D is evaluated in the numerical simulations.

2. PSO networks with deterministic couplings (PSON-D)

In this section, the PSO networks with deterministic couplings (PSON-D) are explained. Figure 1 shows the example of PSON-D. In this figure, four sub-swarm groups are connected to two neighbor sub-swarm groups and six particles in each group are connected to two neighbor particles. In PSO, ith particle only shares the information between neighbor particles. The number of neighbor particles with which each particle shares the information is characterized as the Degree In the Group (DIG). In PSON-D, each sub-swarm group shares the information between neighbor sub-swarm groups. The number of neighbor sub-swarm groups with which each group can communicate to each other is characterized as the Degree Between the Groups (DBG). In PSON-D, the velocity of each particle is updated by using the personal best solution ($pbest$), the best solution ($lbest$) between the neighbor particles and the best solution ($glbest$) between the neighbor sub-swarm groups. The velocity and position are updated by following equations.

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1 r_1(\mathbf{pbest}_i^t - \mathbf{x}_i^t) + c_2 r_2(\mathbf{lbest}_i^t - \mathbf{x}_i^t) + c_3 r_3(\mathbf{glbest}_{g(i)}^t - \mathbf{x}_i^t) \quad (1)$$

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1} \quad (2)$$

where $\mathbf{x}_i^t$ and $\mathbf{x}_i^{t+1}$ denote the current and next position vectors of the ith particle, $\mathbf{v}_i^t$ and $\mathbf{v}_i^{t+1}$ denote the current and

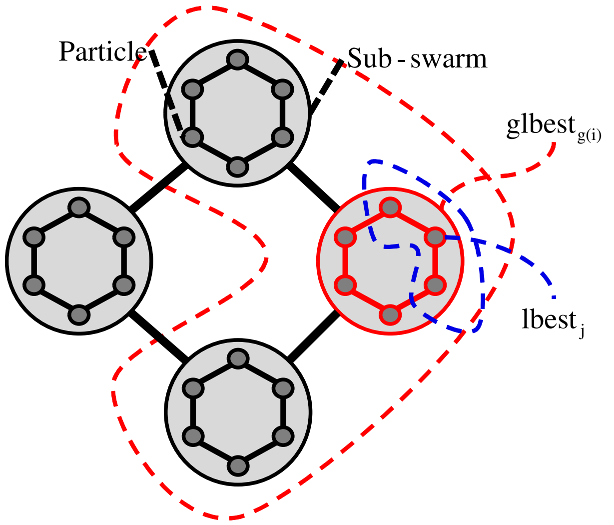

Figure 1: Example of PSON-D

next velocity vectors of the ith particle, and t denotes the search iteration. w is an inertia coefficient. c_1, c_2 and c_3 are acceleration coefficients, and r_1, r_2 and r_3 are uniform random numbers from 0 to 1. $g(i)$ denotes the index of the sub-swarm group to which the ith particle belongs. When the best "**lbest**" is updated in $g(i)$th sub-swarm group, $g(i)$th sub-swarm communicates to the neighbor sub-swarm groups and updates "$\mathbf{glbest}_{g(i)}$" by receiving "**lbests**" of connected sub-swarm groups. In PSON-D, even if one sub-swarm group traps into local minimum, the sub-swarm group can escape from the local minimum by the effect of "$\mathbf{glbest}_{g(i)}$".

The algorithm of PSON-D is described by the following procedures.

Step:1 Let $t = 0$. For all i, the vectors $\mathbf{v}_i^0$ and $\mathbf{x}_i^0$ are initialized at random. Then, the best solutions "$\mathbf{pbest}_i^0$" is decided.

Step:2 For all i, the ith particle updates "$\mathbf{lbest}_i^t$".

Step:3 If the best "$\mathbf{lbest}_i^t$" is updated, the $g(i)$th sub-swarm group communicates to the neighbor sub-swarm groups and updates "$\mathbf{glbest}_{g(i)}^t$"

Step:4 For all i, the vectors $\mathbf{v}_i^t$ and $\mathbf{x}_i^t$ are updated by Eq.(1) and (2).

Step:5 For all i, fitness of the ith particle are evaluated by the objective function and ith particle updates "$\mathbf{pbest}_i^t$".

Step:6 Let $t = t + 1$. Step 2 to 5 are repeated until $t = t_{max}$.

3. Deterministic PSON-D (D-PSON-D)

The search performances of PSO are affected by inertia coefficient, acceleration coefficients and *DIG*. In addition to these parameters, in PSON-D, *DBG* also affects the search performances. In order to analyze the search performances of PSON-D, it is necessary to investigate the effects of *DIG* and *DBG*. Since PSO uses random numbers, PSO can be regarded as a stochastic system. These random factors influence the diversity of search. *DIG* and *DBG* also influence the diversity of search. In order to investigate the effects of *DIG* and *DBG*, we propose Deterministic PSON-D (D-PSON-D) in which the random factors are removed from PSON-D. That is, we apply PSON-D to the ideas of deterministic PSOs[11],[12]. We investigate the search performances by using the D-PSON-D.

In PSON-D, the random factors are uniform random numbers from 0 to 1. The effect that the random factors give acceleration coefficients can be regarded as average 0.5. Then, in D-PSON-D, the random factors are fixed to 0.5. Hence, Eq. (1) can be transformed to Eq. (3)

$$\begin{aligned}\mathbf{v}_i^{t+1} &= w\mathbf{v}_i^t + 0.5c_1(\mathbf{pbest}_i^t - \mathbf{x}_i^t) + 0.5c_2(\mathbf{lbest}_i^t - \mathbf{x}_i^t)\\ &\quad +0.5c_3(\mathbf{glbest}_{g(i)}^t - \mathbf{x}_i^t)\end{aligned} \tag{3}$$

Let $\mathbf{g}_i^t$ be the balance point of **pbest**, **lbest** and **glbest**.

$$\mathbf{g}_i^t = \frac{c_1\mathbf{pbest}_i^t + c_2\mathbf{lbest}_i^t + c_3\mathbf{glbest}_{g(i)}^t}{c_1 + c_2 + c_3} \tag{4}$$

And, let ϕ be sum of all acceleration coefficients.

$$\phi = c_1 + c_2 + c_3 \tag{5}$$

Using these, Eq. (3) can be transformed to Eq. (6).

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + 0.5\phi(\mathbf{g}_i^t - \mathbf{x}_i^t) \tag{6}$$

We define $\mathbf{y}_i^t = \mathbf{x}_i^t - \mathbf{g}_i^t$. Then, Eqs. (2) and (6) become the following equations.

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t - 0.5\phi\mathbf{y}_i^t \tag{7}$$

$$\mathbf{y}_i^{t+1} = w\mathbf{v}_i^t + (1 - 0.5\phi)\mathbf{y}_i^t \tag{8}$$

If $\mathbf{v}_i^\infty = 0$, particles converge to a target point $\mathbf{g}_i^\infty$. Otherwise, particles diverge or oscillate.

Eqs. (7) and (8) can be transformed to matrix expression.

$$\begin{aligned}\begin{bmatrix}\mathbf{v}_i^{t+1}\\ \mathbf{y}_i^{t+1}\end{bmatrix} &= \begin{bmatrix} w & -0.5\phi\\ w & 1-0.5\phi\end{bmatrix}\begin{bmatrix}\mathbf{v}_i^t\\ \mathbf{y}_i^t\end{bmatrix}\\ &= A\begin{bmatrix}\mathbf{v}_i^t\\ \mathbf{y}_i^t\end{bmatrix}\end{aligned} \tag{9}$$

From Eq. (9), D-PSON-D can be regarded as a discrete system. In order for D-PSON-D to become asymptotic stability, eigenvalues which satisfy $|\lambda E - A| = 0$ need to be inside of the unit circle in the complex plane. Eigenvalues of matrix A are given by Eq. (10).

$$\lambda = \frac{(w + 1 - 0.5\phi) \pm \sqrt{(0.5\phi - 1 - w)^2 - 4w}}{2} \tag{10}$$

Therefore, the conditions of asymptotic stability of D-PSON-D are given by Eqs. (11) and (12).

$$0 < \phi < 4w + 4 \tag{11}$$

$$0 \leq w < 1 \tag{12}$$

Table 1: Benchmark Functions

Function Name		Range
Rastrigin's	$f_1(\mathbf{x}) = 10D + \sum_{i=1}^{D}(x_i^2 - 10cos(2\pi x_i))$	$[-5.12, 5.12]$
Rosenbrock's	$f_2(\mathbf{x}) = \sum_{i=1}^{D-1}\left(100(x_{i+1} - x_i^2) + (1 - x_i)^2\right)$	$[-2.048, 2.048]$

Table 2: The ratio of acceleration coefficients

No.	c_1	c_2	c_3
1	1	1/4	3/4
2	1	1/2	1/2
3	1	3/4	1/4
4	1/4	1	3/4
5	1/2	1	1/2
6	3/4	1	1/4
7	1/4	3/4	1
8	1/2	1/2	1
9	3/4	1/4	1

4. Numerical Experiment

In order to confirm the effect of each parameter, the numerical experiments are performed. The parameters of acceleration coefficients, *DIG* and *DBG* in D-PSON-D are varied. Then, the search performances of D-PSON-D are investigated.

4.1. Simulation Settings

Two benchmark functions with $D = 10$ shown in Table 1 are used in the experiments. In Table 1, f_1 is multimodal functions with numerous local minimum. f_2 is unimodal functions. The optimum solution of f_1 is $\mathbf{x} = (0, 0, \cdots, 0)$ and that of f_2 is $\mathbf{x} = (1, 1, \cdots, 1)$. In each function, the evaluation value of the optimum solution is "0".

The total number of particles is "40". Two variations of groups are performed in the experiments. First, the number of groups is "5" and the number of particles is "8". Second, the number of groups is "10" and the number of particles is "4".

4.2. Numerical experiment

The parameters of acceleration coefficients, *DIG* and *DBG* are varied. It is known that "$w = 0.729$" and "$c_1 = c_2 = 1.49445$" are good parameters to search solutions[2]. Then, in the experiments, w is fixed to "0.729" and the sum of acceleration coefficients are fixed to "1.49445 + 1.49445 = 2.9889". Also, based on the conditions (11) and (12), in the D-PSON-D, one acceleration coefficient is fixed to "1.49445" and the sum of the others are fixed to "1.49445". Table 2 shows the parameter sets for the acceleration coefficients used in the experiments, where each value is represented by the ratio to the parameter value "1.49445".

Table 3: The parameters of *DBG* and *DIG*

DBG		*DIG*	
5 groups	10 groups	8 particles	4 particles
1,2,4	1,2,4,9	1,2,4,7	1,2,3

Table 3 shows the parameters of *DBG* in each group and *DIG* in each particle. The experiments are performed for all combinations of these parameters shown in Tables 2 and 3.

Table 4 shows the experimental results of *Rastrigin's*. Table 5 shows the experimental results of *Rosenbrock's*. In Tables 4 and 5, "*Average*" represents the best results in the combinations of *DBG* and *DIG*. "*Small*" means ring topology. In the case of "*Small*", a information can be propagated slowly. "*Big*" means star topology or near star topology in which a particle or sub-swarm group connects more than 40% of the whole particles or sub-swarm groups. In the case of "*Big*", a information can be propagated quickly. "*Middle*" means the intermediate topology between "*Small*" and "*Big*".

Tables 4 and 5 show *DIG* from "*Small*" to "*Middle*" makes good search performances. By smaller *DIG*, the ith particle propagates own information to few neighbors. Then, the best information is propagated to all particles slowly. Hence, each particle can search solution space more diverse and find good solution, although convergence speed is slow. While, larger *DBG* makes good search performances. When the jth sub-swarm group traps into a local minimum, the best solution information which is better than "$\mathbf{glbest}_j$" is needed. Communicating with many sub-swarm groups can increase the probability of updating "**glbest**". Even if a sub-swarm group traps into a local minimum, the sub-swarm group can escape from the local minimum by communicating with many sub-swarm groups. Moreover, these results show that trends of *DBG* and *DIG* do not almost depend on acceleration coefficients. It can be said that "*Small*" and "*Middle*" *DIG* and "*Big*" *DBG* can lead good search performances to PSON.

5. Conclusion

This paper has investigated the effect of parameters such as acceleration coefficients, *DIG* and *DBG* in PSON-D. In order to confirm the effect of these parameters, we have proposed Deterministic PSON-D (D-PSON-D). The various parameters are varied. Then, search performances of D-PSON-D are evaluated. The simulation results show that larger *DBG* and relatively smaller *DIG* can lead good search performances. Moreover, the trends of *DBG* and *DIG* do not almost depend on acceleration coefficients.

Future problems are followings: (1) D-PSON-D is applied to various benchmark functions, and (2) D-PSON-D is compared with PSON-D which includes random factors.

Table 4: The results for *Rastrigin's*

5 groups and 8 particles

DBG	*DIG*	C_1	C_2	C_3	*Average*
Big	*Small*	1/4	1	3/4	**1.83E + 001**
Big	*Small*	1/4	3/4	1	1.97E+001
Big	*Small*	1/2	1	1/2	2.18E+001
Big	*Small*	1	1/4	3/4	2.72E+001
Big	*Middle*	1/2	1/2	1	2.28E+001
Big	*Middle*	3/4	1/4	1	2.65E+001
Big	*Middle*	1	1/2	1/2	2.89E+001
Big	*Middle*	1	3/4	1/4	3.12E+001
Big	*Middle*	3/4	1	1/4	3.84E+001

10 groups and 4 particles

DBG	*DIG*	C_1	C_2	C_3	*Average*
Big	*Small*	1/4	3/4	1	**1.99E + 001**
Big	*Small*	1/4	1	3/4	2.19E+001
Big	*Small*	1/2	1/2	1	2.31E+001
Big	*Small*	1/2	1	1/2	2.87E+001
Big	*Small*	1	1/4	3/4	2.94E+001
Big	*Small*	1	1/2	1/2	3.20E+001
Big	*Small*	3/4	1	1/4	3.53E+001
Big	*Small*	1	3/4	1/4	3.87E+001
Big	*Middle*	3/4	1/4	1	2.55E+001

Table 5: The results for *Rosenbrock's*

5 groups and 8 particles

DBG	*DIG*	C_1	C_2	C_3	*Average*
Big	*Small*	3/4	1	1/4	**5.05E + 000**
Big	*Small*	1/2	1	1/2	5.06E+000
Big	*Small*	1/4	1	3/4	6.51E+000
Big	*Small*	1	1/2	1/2	8.60E+000
Big	*Middle*	1	3/4	1/4	5.43E+000
Big	*Middle*	1/4	3/4	1	9.59E+000
Big	*Middle*	1	1/4	3/4	1.69E+001
Big	*Big*	1/2	1/2	1	1.58E+001
Big	*Big*	3/4	1/4	1	2.03E+001

10 groups and 4 particles

DBG	*DIG*	C_1	C_2	C_3	*Average*
Big	*Small*	1/2	1	1/2	**4.41E + 000**
Big	*Small*	1	3/4	1/4	4.42E+000
Big	*Small*	3/4	1	1/4	4.84E+000
Big	*Small*	1/4	1	3/4	5.95E+000
Big	*Small*	1	1/2	1/2	8.26E+000
Big	*Small*	1/4	3/4	1	8.79E+000
Big	*Small*	1/2	1/2	1	1.23E+001
Big	*Middle*	1	1/4	3/4	1.47E+001
Big	*Middle*	3/4	1/4	1	1.89E+001

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