2008 International Symposium on Nonlinear Theory and its Applications
NOLTA'08, Budapest, Hungary, September 7-10, 2008

Functional Sigma-Delta CNN

Hisashi Aomori and Mamoru Tanaka

Department of Information and Communication Sciences, Sophia University,
7–1 Kioi-cho, Chiyoda-ku, Tokyo 102–8554 Japan
Phone: +81–3–3238–3878, FAX: +81–3–3238–3321
Email: aomori@ieee.org

Abstract—A sigma-delta modulation is a well-known concept for analog-to-digital (A/D) converter. However, its underlying concept is limited to 1-D signals. The Sigma-Delta Cellular Neural Network (SD-CNN) is an efficient framework for a spatial domain sigma-delta modulation. Due to a CNN dynamics, each pixel of an image corresponds to a cell of a CNN, and each cell is connected spatially by the A-template. Therefore, the SD-CNN can be thought of as a very large-scale and super-parallel sigma-delta modulator. In this paper, we propose a novel functional sigma-delta modulation using SD-CNN. In order to provide new functions for SD-CNN, the essential conditions for constructing a spatial-domain sigma-delta modulation of CNN are reexamined. Multibit SD-CNN for high image reconstruction performance and SD-CNN with basic image processing ability for an integrated camera interface are proposed in this paper.

1. Introduction

Although the sigma-delta modulation [1] is a widely used and well-known technique for converting analog signals into pulse digital sequences, it is limited to 1-D signals. To deal with this difficulties, the sigma-delta cellular neural network (SD-CNN) [2] can be used for a sigma-delta modulation for 2-D signals. The SD-CNN is a new paradigm of CNN [3] for neural network based sigma-delta modulation, and this framework provides two important sigma-delta properties (noise shaping and image reconstruction properties) for 2-D signal processing tasks such as image processing, medical imaging, ultrasound imaging and so on. Its underlying concepts are based on the CNN.

The CNN has been applied to many applications such as image compression, filtering and recognition. Especially, image processing tasks are the best application for utilising nonlinear spatio temporal dynamics of CNN [4–6]. The nonlinear interpolative dynamics by the feedback A-template is one of the significant characteristics. By using the monotonically decreasing characteristic of its Lyapunov energy function, its applications can be implemented by solving the nonlinear optimization problem to minimize an objective function that defines the given application.

This paper presents a novel functional sigma-delta modulation using SD-CNN. In order to provide new functions for SD-CNN, the three SD-CNN formation conditions are reconsidered, and the essential conditions will be obtained. Then, multibit SD-CNN for high image reconstruction performance and SD-CNN with basic image processing ability for an integrated camera interface are proposed in this paper.

2. Functional Sigma-Delta Cellular Neural Network

2.1. The sigma-delta CNN

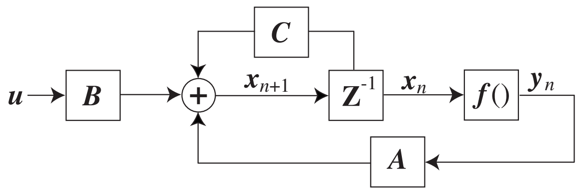

Figure 1: The sigma-delta cellular neural network.

In this section, the SD-CNN is discussed for finding the essential formation conditions. The functionality of the CNN is completely described by a number of small matrices called templates. Fig. 1 shows the block diagram of the SD-CNN. The state equation of SD-CNN is given by

$$\begin{aligned} x_{ij}(t+1) = & \sum_{C(k,l)\in N_r(i,j)} C(i,j;k,l)x_{kl}(t) \\ & + \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l)y_{kl}(t) \\ & + \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l)u_{kl}, \end{aligned} \tag{1}$$

$$y(t) = f(x(t)), \tag{2}$$

$$f(x) = \begin{cases} 1 & x \geq 1 \\ \hat{f}(x) & |x| \leq 1 \\ -1 & x \leq -1 \end{cases} \tag{3}$$

where $x_{ij}(t)$, $y_{ij}(t)$, $u_{ij}(t)$ and $N_r(i,j)$ are the state variable, output, input and $r-$neighborhood of a cell $C(i,j)$ as $N_r(i,j) = \{C(k,l)|\max\{|k-i|,|l-j|\} \leq r\}$, respectively. The template $C(i,j;k,l)$ indicates neighborhood connections coefficients between state variables, defined by

$$C(i,j;k,l) = \begin{cases} \eta & \text{if } k = i \text{ and } l = j, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where η is a positive constant value. Also, the energy function E [3] can be given by

$$\begin{aligned} \mathrm{E} = & -\frac{1}{2}\sum_{(i,j)}\sum_{(k,l)} A(i,j;k,l) y_{ij}(t) y_{kl}(t) \\ & -\sum_{(i,j)}\sum_{(k,l)} B(i,j;k,l) y_{ij}(t) u_{kl} \\ & + (1-\eta)\sum_{(i,j)}\int_0^{y_{ij}(t)} f^{-1}(y)dy, \end{aligned} \tag{5}$$

where $f^{-1}(\cdot)$ is the pseudo-inverse function of $f(\cdot)$.

The SD-CNN framework requires some iterations for recovering the dynamic range of reconstruction image. If the nonlinear function is equal to a quantization function, then the state value of any dynamics iterations forcibly converges to stable regions of output function. Therefore, the essential convergence conditions of SD-CNN are given by

$$A(i,j;k,l) = A(k,l;i,j), \tag{6}$$

$$\hat{f}(x) = \begin{cases} \Delta\left(\left[\frac{x}{\Delta} + \frac{1}{2}\right]\right) & l \text{ is odd}, \\ \Delta\left(\left[\frac{x}{\Delta}\right] + \frac{1}{2}\right) & l \text{ is even}, \end{cases} \tag{7}$$

$$\Delta = \frac{2\xi}{l-1}, \tag{8}$$

where Δ is the width of the quantization step, l is a positive integer number ($l \geq 2$) that determines a number of the quantization level, and $[\cdot]$ denotes the Gaussian operator. Then, the approximated Lyapunov energy function can be obtained in matrix form as

$$\mathrm{E} = -\frac{1}{2}\boldsymbol{y}^T(\boldsymbol{A}-\boldsymbol{D})\boldsymbol{y} - \boldsymbol{y}^T\boldsymbol{B}\boldsymbol{u}, \tag{9}$$

$$\boldsymbol{D} = \mathrm{diag}\{\xi\eta, \cdots, \xi\eta\}, \tag{10}$$

where $\boldsymbol{a}^T$ means the transposed matrix of $\boldsymbol{a}$.

The objective function to be minimized in the CNN dynamics should represent a DAC condition for requirements of sigma-delta modulation. Hence, the A-template should work as a DAC, and the difference between the output digital image and the input analogue image should be small. Therefore the objective function is given by

$$\mathrm{obj}(\boldsymbol{y},\boldsymbol{u}) = \left\|\frac{1}{2}\boldsymbol{y}^T(\boldsymbol{G}\boldsymbol{y}-\boldsymbol{u})\right\|, \tag{11}$$

where $\boldsymbol{G}$ is the Gaussian filter.

Next, the evaluation function for image reconstruction will be introduced. Since the reconstruction image is obtained by the summation of multi-level pulse sequence images, the quality of the reconstruction image can be determined using the mean square error (MSE). The evaluation function is described as

$$\mathrm{eval}(\boldsymbol{y},\boldsymbol{u}) = \left\|\boldsymbol{u} - \sum_t \boldsymbol{G}\boldsymbol{y}(t)\right\|. \tag{12}$$

Since the evaluation function is for image reconstruction layer that has no CNN dynamics, the SD-CNN dynamics can be resolved from (9) and (11). By the comparison between above-mentioned equations, the A-template and B-template can be determined as

$$\begin{aligned} \boldsymbol{A} = & A(i,j;k,l), \qquad C(k,j) \in N_r(i,j) \\ = & -\frac{1}{2\pi\sigma^2}\exp\left(-\frac{(k-i)^2+(l-j)^2}{2\sigma^2}\right), \end{aligned} \tag{13}$$

where σ is the standard deviation of the Gaussian function, and the B-template is an arbitrary spatial filter.

The output of the SD-CNN dynamics becomes the input of image reconstruction layer. Since the output of SD-CNN dynamics can minimize (12), the optimal reconstruction image $\tilde{y}$ is given by

$$\tilde{y}_{ij} = \sum_{y_{kl}\in N_r(i,j)} \hat{B}(i,j;k,l)\hat{y}_{ij}, \tag{14}$$

where

$$\begin{aligned} \hat{\boldsymbol{B}} = & \hat{B}(i,j;k,l), \qquad C(k,j) \in N_r(i,j) \\ = & \frac{1}{2\pi\sigma^2}\exp\left(-\frac{(k-i)^2+(l-j)^2}{2\sigma^2}\right). \end{aligned} \tag{15}$$

2.2. Proposed functional SD-CNN

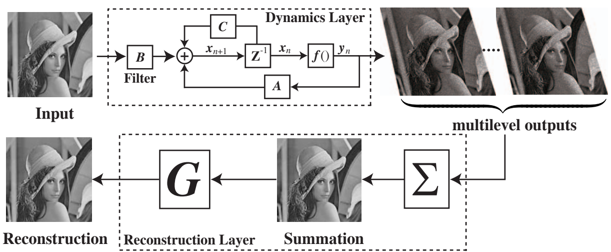

Figure 2: Proposed functional sigma-delta cellular neural network.

The proposed SD-CNN framework is composed by the two-layered system. Fig. 2 shows the proposed functional SD-CNN. In the first layer (CNN dynamics layer), the input image can be filtered by an arbitrary spatial filter, and is modulated by the SD-CNN dynamics. The pre-filter ability by the B-template enables not only sigma-delta modulation but also various image processing such as edge enhancement, noise reduction, smoothing and etc. by one SD-CNN device. This property is very useful for an integrated camera interface. The output nonlinear function (multi-bit quantizing function) is utilized for realizing an accurate A/D conversion via CNN dynamics. The quantization noises are shaped by the noise shaping property. The output of the first layer becomes the input of the second layer (image reconstruction layer). In this layer, the input digital pulse sequence image is added for the image reconstruction. After the transient of the first layer, the optimal reconstruction image is obtained. According as the sigma-delta characteristic, the reconstruction image gradually approaches the original image by the dynamics iterations.

Since each pixel of an image corresponds to the cell of CNN, the proposed system can be thought as very large scale and super parallel sigma-delta modulators. The spatio-temporal SD-CNN dynamics converts an input analogue image (continuous tone image can be thought as an analogue image) into multilevel pulse digital images, and each cell of CNN is equal to a sigma-delta modulator. Therefore the temporal processing is to generate the multilevel halftone image of input image. Besides, the A-template works as a DAC, and it provides spatial connections between each cell because of CNN characteristic. Therefore, the optimal predicted analogue value is obtained by the spatio processing. Then, the output binary sequences are added for original image reconstruction. This is an important property of sigma-delta modulation. The dynamic range of reconstruction image is recovered by adding multibit halftone image. Moreover, by using the low-pass filter, spatially distributed noises are reduced, and vanishing intensities of the summation image are interpolated.

3. Experimental Results

For evaluating the functionality and performance of the proposed functional SD-CNN, the system is implemented by the ANSI C++, and it is simulated in a computer. In this section, an arbitrary spatial filtering ability and effectiveness of multibit SD-CNN are examined.

3.1. Pre-filter ability for an integrated device

Although any spatial filter is selectable for advanced image processing tasks, the edge enhancement application by the 8-direction Laplacian edge detection filter, and the image sharping by the 8-direction Laplacian sharping filter are demonstrated in this section. Fig.3 shows an edge detected image and halftoned image by using a shaped input image. Since the halftoned image shown in Fig.3-(b) has very clear edge information due to the image sharping effect, the binarized image quality is high compared with that of non-shaped input image. Moreover, since this image processing task by pre-filter requires few iterations, this process is implementable at the end of sigma-delta dynamics iterations. Therefore, a sigma-delta modulation with some functionalities (future extraction, edge enhanced halftoning and so on) is possible.

3.2. Multibit sigma-delta modulation

We applied our system to standard gray-scale test images; “Aerial,” “Barbara,” “Boat,” “Couple,” “Crowd,” “Elaine,” “Goldhill,” “Lena” and “Milkdrop.”

(a) Edge Detection (b) Image Halftoning using shaped image

Figure 3: Examples of advanced image processing.

Owing to the dynamic range reconstruction, the convergence speed of the proposed system is limited. Therefore, it is necessary to determine the relevant condition for image reconstruction experimentally. Fig. 4 shows the relationship between dynamics iterations and peak signal to noise ratio (PSNR) of reconstruction images. This result suggests that the relevant condition of maximum iteration is given by $n_{max} \geq 32$.

For the simulation, the coding factor is decided experimentally; the $r-$neighborhood of cell $r = 2$, quantization step $l = 8$ (3-bit), and the standard deviation of Gaussian σ is decided like Table 1. The center value of the C-template η is experimentally obtained by $\eta = \frac{1}{3\pi\sigma^2}$.

Table 1 shows the PSNR of 1-bit quantizer (128 iterations) and that of 3-bit quantizer (32 iterations). The PSNR is described as the differences between the input image and reconstruction image of each image. Fig. 5 shows the input, the binarized and reconstruction of Lena. As shown in Table 1 and Fig. 5, our proposed method has a excellent reconstruction performance.

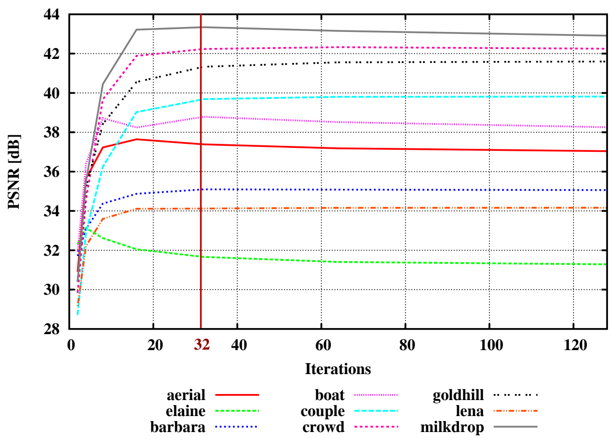

Figure 4: PSNR vs. dynamics iterations.

(a) Original Lena (b) Reconstruction Image via SD-CNN with 3-bit quantizer

Figure 5: Original image and reconstructed image using 3-bit quantizer.

Table 1: Reconstruction Performance: PSNR of Each Image.

Image	bit(s)	σ	PSNR
Aerial	3-bit	0.63	**37.39**
	1-bit	0.705	33.70
Barbara	3-bit	0.53	**35.10**
	1-bit	0.710	31.78
Boat	3-bit	0.60	**38.79**
	1-bit	0.755	34.45
Couple	3-bit	0.68	**39.69**
	1-bit	0.735	36.20
Crowd	3-bit	0.71	**42.24**
	1-bit	0.985	34.29
Elaine	3-bit	0.45	31.66
	1-bit	0.750	34.69
Goldhill	3-bit	0.65	**41.33**
	1-bit	0.765	36.64
Lena	3-bit	0.66	**43.34**
	1-bit	0.795	38.18
Milkdrop	3-bit	0.63	**39.90**
	1-bit	0.74	33.32

4. Conclusion

The functional sigma-delta cellular neural network has been proposed in this paper. By introducing a pre-filter process and a multibit quantizing function as the nonlinear output function of CNN, the SD-CNN can perform not only sigma-delta modulation with high reconstruction performance but also some image processing tasks. This property is very useful for an integrated camera interface. Especially, since the halftoned image by using shaped input image has very clear edge information, the binarized image quality is high compared with that of non-shaped input image.

The experimental results show that the proposed method using multibit quantizing function has excellent reconstruction performance.

Acknowledgment

This research was partially supported by the Grant-in-Aid for Young Scientists (B) of Ministry of Education, Culture, Sports, Science and Technology, Japan (MEXT), and the fund of the Open Research Center Project from MEXT (2007–2011).

References

[1] J. C. Candy and G. C. Temes, Eds., *Oversampling Delta-Sigma Data Converters: Theory, Design, and Simulation*. IEEE Press, 1992.

[2] H. Aomori, T. Otake, N. Takahashi, and M. Tanaka, “Sigma-detal cellular neural network for 2d modulation,” *Neural Networks*, vol. 21, no. 2–3, pp. 349–357, Mar. 2008.

[3] L. O. Chua and L. Yang, “Cellular neural networks: Theory,” *IEEE Trans. Circuits and Syst.*, vol. 35, no. 10, pp. 1257–1272, Oct. 1988.

[4] K. R. Crounse, T. Roska, and L. O. Chua, “Image halftoning with cellular neural networks,” *IEEE Trans. Circuits Syst.*, vol. 40, no. 4, pp. 267–283, Apr. 1993.

[5] N. Takahashi, T. Otake, and M. Tanaka, “Nonlinear interpolative effect of feedback template for image processing by discrete-time cellular neural network,” *Journal of Circuit Syst. Comp.*, vol. 12, no. 4, pp. 505–518, Aug. 2003.

[6] K. Yokosawa, T. Nakaguchi, Y. Tanji, and M. Tanaka, “Cellular neural networks with output function having mulple constant rgions,” *IEEE Trans. Circuits Syst.*, vol. 50, no. 7, pp. 847–857, July 2003.