2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Basic Characteristics of Deterministic PSO with Rotational Dynamics

Ryosuke Sano†, Kenya Jin'no‡, Toshimichi Saito†

†EECE Dept., Hosei University
Tokyo, 184-8584 Japan
‡EE Dept., Nippon Institute of Technology
Saitama, 345-8501 Japan
Email: ryosuke.sano.9j@stu.hosei.ac.jp, jinno@nit.ac.jp, tsaito@hosei.ac.jp

Abstract—This paper discusses a basic characteristic of our proposed canonical deterministic particle swarm optimizer (abbr. CD-PSO). The phase-plane dynamics of the CD-PSO exhibits a rotation behavior. Since the rotation angle of the CD-PSO leads to the diversity of the searching, we can improve the performance to search optimum value of the evaluation function. We pay attention to this rotation behavior, an effective setting procedure of the rotation angle is proposed. We analyze the basic characteristic of the CD-PSO comparing the standard PSO. Also, we confirm the performance by using some numerical simulations.

1. Introduction

Particle swarm optimization (PSO) is a meta-heuristic algorithm for solving optimization problems, proposed by J. Kennedy and R. Eberhart [1][2]. PSO is applicable to variety of problems that neural networks, power electronics and such [3]-[4]. The PSO generates to search velocity towards known candidate solutions. The system contains random numbers as stochastic factor, therefore it is difficult to analysis of dynamics and parameter settings. In order to analyze the dynamics of PSO, M. Clerc, and J. Kennedy proposed Deterministic PSO (D-PSO) system, that removes the stochastic factor from the standard PSO [5]. D-PSO (and PSO) is difficult to set parameters, since stability is determined by the relationship of the parameters w, c_1 and c_2 [7]. We have proposed Canonical Deterministic PSO (CD-PSO) that transformed D-PSO into canonical form [6]-[8]. Its stability determined by only one parameter Δ.

On the other hand, PSO dynamics significantly change depending on method of setting random numbers. Similarly, CD-PSO dynamics depends on method of setting the parameter of rotation angle.

This paper discusses the differences of the dynamics between the standard PSO and the CD-PSO.

2. Standard PSO

The standard PSO in a D-dimensional search space is described as

$$\begin{array}{l} v_d^n \leftarrow v_d^n + c_1 r_{1d}^n (Pbest_d^n - x_d^n) + c_2 r_{2d}^n (Gbest_d - x_d^n) \\ x_d^n \leftarrow x_d^n + v_d^n \end{array} \tag{1}$$

$x_d^n \in \Re$ denotes the location and $v_d^n \in \Re$ denotes the velocity, where $n = 1 \sim N$ is index of the particle, and $d = 1 \sim D$ is index of the dimension. $Pbest_d^n \in \Re$ is called a personal best, it means the location of d-th dimension that gives the best value of the evaluation function of the n-th particle in the past history. $Gbest_d \in \Re$ is called a global best, it means the location of d-th dimension that gives the best value in the evaluation function of all particles. Standard PSO has three parameters of w, c_1 and c_2. w is inertia weight coefficient, c_1 and c_2 are acceleration coefficients. $r_{1d}^n \in [0, 1]$ and $r_{2d}^n \in [0, 1]$ are uniform random numbers and be independent of each other. They are following two definitions.

Scalar random number (PSOs)
The random number of each dimension is the same value in the n-th particle.

Vector random number (PSOv)
The random number of each dimension is different value in the n-th particle.

We adopted the parameters that are recommended in M. Clerc and J. Kennedy's paper [9][10].

$$\begin{cases} w = 0.729 \\ c_1 = c_2 = 1.49445 \end{cases} \tag{2}$$

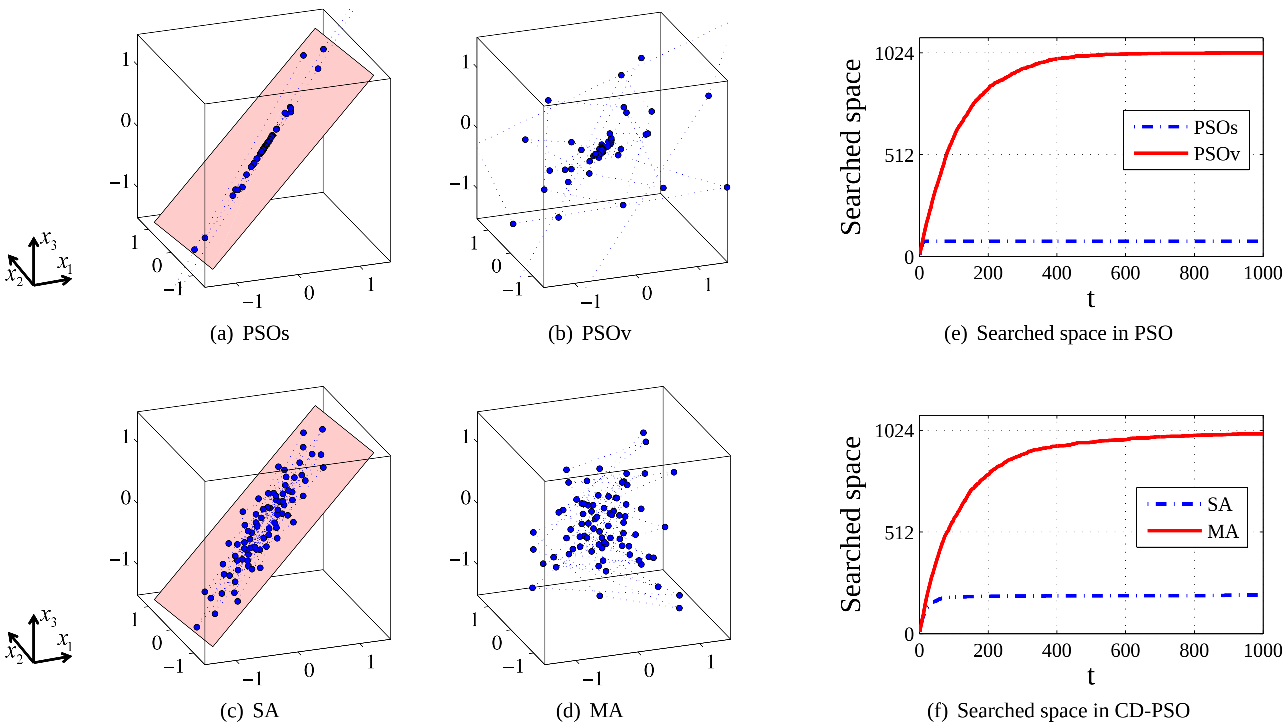
3. Canonical Deterministic PSO

The CD-PSO in a D-dimensional search space is described as

$$\begin{bmatrix} x_d^n \\ v_d^n \end{bmatrix} = \Delta \begin{bmatrix} \cos\theta_d^n & -\sin\theta_d^n \\ \sin\theta_d^n & \cos\theta_d^n \end{bmatrix} \begin{bmatrix} x_d^n - p_d^n \\ v_d^n \end{bmatrix} + \begin{bmatrix} p_d^n \\ 0 \end{bmatrix} \tag{3}$$

$$p_d^n \equiv \gamma\ Pbest_d^n + (1-\gamma)\ Gbest_d \tag{4}$$

where p_d^n denotes the target location that determined from the personal best and global best. CD-PSO has three parameters of γ, Δ and θ_d^n. The parameter γ controls the mixture rate of the personal best and global best, Δ is damping factor that controls convergence of particles. θ_d^n is rotation angle that controls the frequency and sampling interval of the search. We adopt two kinds of setting for rotation angle by using basis angle ϕ.



(a) PSOs (b) PSOv (e) Searched space in PSO

(c) SA (d) MA (f) Searched space in CD-PSO

Figure 1: Trajectory of three dimensional projection of ten dimensional variable space and searched space

Single rotation angle (SA)
The rotation angle of each dimension and each particle is the same value, where

$$\theta_d^n = \phi \tag{5}$$

Multiple rotation angle (MA)
The rotation angle of each dimension and each particle is different value, where

$$\theta_d^n = [\ \{\ (n-1)D + d\ \}\ \phi\ ] \bmod 360^\circ \tag{6}$$

In the case of D=3 and N=10, the parameters are as follows:

$$\begin{bmatrix} \theta_1^1 & \theta_2^1 & \theta_3^1 \\ \theta_1^2 & \theta_2^2 & \theta_3^2 \\ & \vdots & \\ \theta_1^{10} & \theta_2^{10} & \theta_3^{10} \end{bmatrix} = \begin{bmatrix} \phi & 2\phi & 3\phi \\ 4\phi & 5\phi & 6\phi \\ & \vdots & \\ 28\phi & 29\phi & 30\phi \end{bmatrix}$$

Based on our trial-and-error testing, the parameters are determined as

$$\begin{cases} \gamma = 0.0 \\ \Delta = 0.95 \\ \phi = 180(3-\sqrt{5}) = 137.51^\circ \end{cases} \tag{7}$$

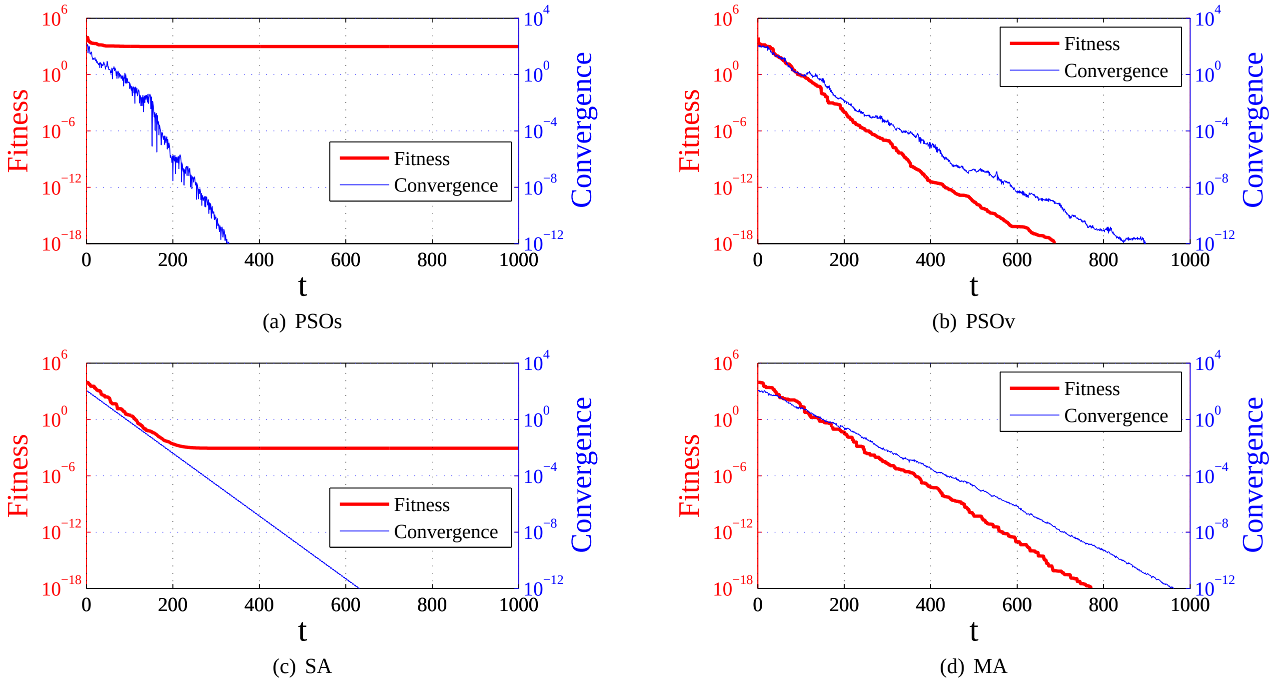
The value adopted in ϕ is the golden angle that often appears in nature and is known as a suitable angle to fill the circle. Perhaps most prominent example is a sequence of sunflower seeds.

4. Dynamics and bias of searching

In order to compare the difference of the dynamics of the algorithms, we perform some numerical simulations under the conditions that $Gbest_d$ and $Pbest_d^n$ are fixed to the origin. Figures 1(a), (b), (c) and (d) show a three dimensional projection of ten dimensional variable space in PSOs, PSOv, SA, and MA, respectively. The particles of PSOs and SA are constrained on certain hyper-plane as shown in Fig. 1(a) and Fig. 1(c). It is attributed to the fixed phase difference between each dimension at location and velocity. In the PSOv and MA, because the random number and rotation angle is different each dimension, the particle is searching without bias.

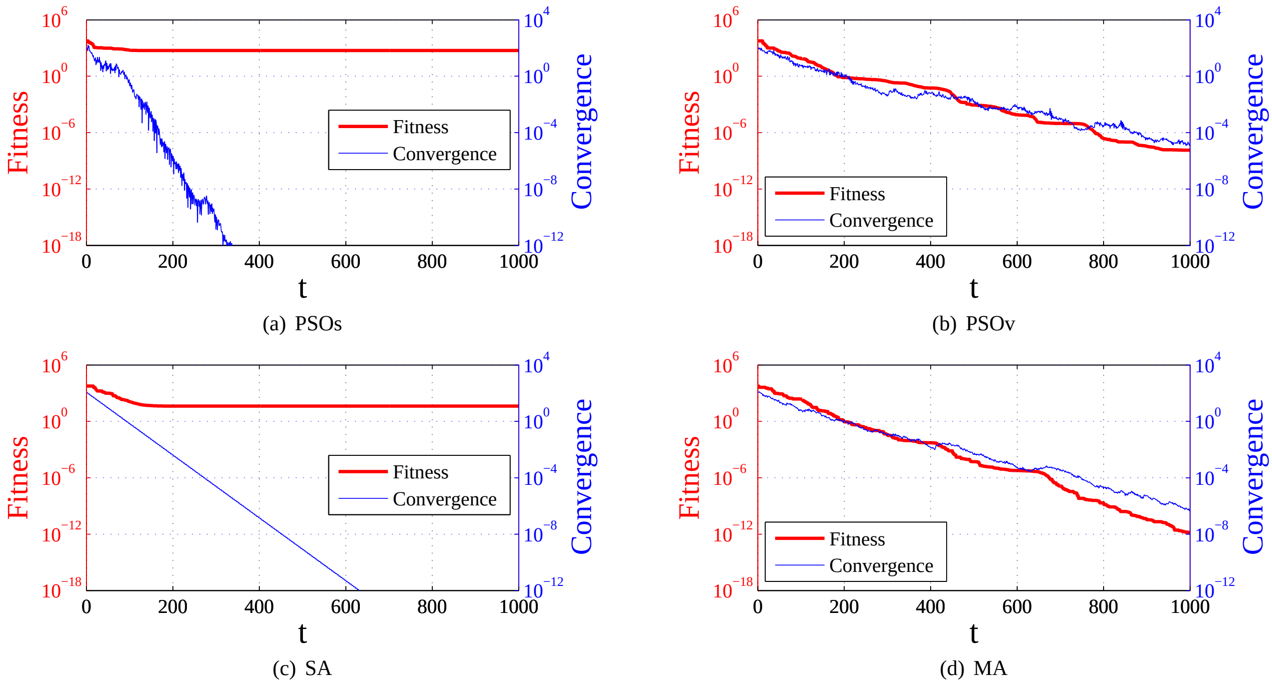
In order to quantify the bias of searching, we divided the search area into half-space relative to the origin in each dimension. Furthermore we measured whether particles searched it or not. Examples of three-dimensional space, it is divided into eight half-space. Namely, each half-space is defined as $\{(+,+,+),(+,+,-),(+,-,+),\ldots,(-,-,-)\}$.

Figures 1(e) and 1(f) illustrate the time fluctuation of the number of searched half-spaces. In this case, the evaluation function consists 10 dimensional variable, and the system has 10 particles. It is divided into 1024 half-spaces. In the PSOs and SA, increasing searched space is stop soon. In the PSOv and MA, the number of searched half-spaces is increased smoothly, and it converges to the maximum number. These results clearly indicate that there exists a bias in PSOs and SA searching. Furthermore demonstrate the similarity between each PSOs and SA, PAOv and MA.



(a) PSOs (b) PSOv (c) SA (d) MA

Figure 2: Search process of Sphere function (f_1)



(a) PSOs (b) PSOv (c) SA (d) MA

Figure 3: Search process of Rotated ellipsoidal function (f_2)

5. Numerical Experiments

We carry out the numerical simulation using some well-known benchmark test functions to confirm the effects of the bias of searching. We adopted the unimodal function to benchmark for two reasons. First, it can surely find a good solution if searching every neighborhood. Second, it unaffected by local solutions. The numerical simulations are carried out applying the two unimodal functions under the following conditions:

$$D = 10,\ N = 10,\ t_{max} = 1000,\ Trials = 1000$$

1) Sphere function (f_1)

$$f_1(\boldsymbol{x}) = \sum_{i=1}^{D} x_i^2 \tag{8}$$

$x_i \in [-64, 64]$, the global minimum is $\boldsymbol{x}^* = (0, \ldots, 0)$ with $f_1(\boldsymbol{x}^*) = 0$. It is unimodal and separable function.

2) Rotated ellipsoidal function (f_2)

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{D} \left(\sum_{k=1}^{i} x_k \right)^2 \tag{9}$$

$x_i \in [-64, 64]$, the global minimum is $\boldsymbol{x}^* = (0, \ldots, 0)$ with $f_2(\boldsymbol{x}^*) = 0$. It is unimodal and non-separable function.

To measure the degree of convergence of the particles, we used the following equation:

$$\begin{aligned} \mathrm{Convg} &= \frac{1}{N} \sum_{n=1}^{N} \sqrt{\sum_{d=1}^{D} \left[x_d^n - \bar{x}_d \right]^2} \\ \bar{x}_d &= \frac{1}{N} \sum_{n=1}^{N} x_d^n \end{aligned} \tag{10}$$

Convg represent the average of the Euclidean distance from the center of the particle swarm.

Figures 2 and 3 show an examples of search process on the f_1 and f_2. In the PSO and SA, we can see that the Convg decreases quickly, furthermore fitness stagnated soon. In contrast, in others, we can see that the Convg decreases slowly, furthermore fitness decrease to continues. This is consistent with the results expected from described above. Comparing the two functions, f_2 is more difficult to search the optimum value than f_1 since f_2 is non-separable. The convergence time of the PSOv and MA depend on the difficulty of the benchmark function, however other cases show the similar in each function.

The results are summarized in Table 1. The PSOv and MA can realize better fitness than the PSOs and SA. MA shows slightly better performance than PSOv, which suggests the usefulness of CD-PSO.

Table 1: Results

	f_1 Fitness	f_2 Fitness
PSOs	5.26 E+02	6.67 E+02
PSOv	2.14 E-21	3.83 E-04
SA	1.63 E+00	7.14 E+01
MA	3.92 E-22	1.83 E-07

6. Conclusions

This paper described the PSO dynamics by random number, and the CD-PSO dynamics by rotation angle. Our numerical simulation results indicated that the PSOv and MA are useful to search optimum solution than the PSOs and SA system. In summary, the searching diversity of the PSOv and MA depends on behavior of each dimension of the particle. Namely, the behavior is caused by the random factor and the rotation angle. In general, we can say that the random factor leads to the diversity. On the other hand, our CD-PSO is a deterministic system, namely, the system does not contain the random factor. Even in such deterministic system, the CD-PSO can create the diversity to control the rotation angle.

This paper represents bias of the search and benchmark on the unimodal functions. Further studies are needed in order to analysis of the global search on the multimodal function.

References

[1] J. Kennedy and R. Eberhart, ”Particle swarm optimization”, Proc. IEEE ICNN 1995, pp. 1942-1948, 1995.

[2] J. Kennedy, ”The particle swarm: Social adaptation of knowledge”, Proc. ICEC 1997, pp. 303-308, 1997.

[3] A. P. Engelbrecht, ”Fundamentals of Computational Swarm Intelligence”, Willey, 2005.

[4] H. Qin, J. W. Kimball and G. K. Venayagamoorthy, ”Particle swarm optimization of high-frequency transformer”, in Proc. 36th Annu. Conf. IEEE Ind. Electron. Soc., pp. 2908-2913, 2010.

[5] M. Clerc, and J. Kennedy, ”The Particle Swarm . Explosion, Stability, and Convergence in a Multidimensional Complex Space”, IEEE Trans. Evol. Comput. , vol. 6, no. 1, pp. 58-73, 2002.

[6] K. Jin'no, ”A novel deterministic particle swarm optimization system”, Journal of Signal Processing, 13, 6, pp. 507-513, 2009.

[7] K. Jin'no and T. Shindo ”Analysis of dynamical characteristic of canonical deterministic PSO”, in Proc. IEEE CEC, pp. 1105-1110, 2010

[8] T. Tsujimoto, T. Shindo, and K. Jin'no, ”The Neighborhood of Canonical Deterministic PSO”, 2011 IEEE CEC, 2011.

[9] M. Clerc and J. Kennedy, ”The particle swarm . explosion, stability, and convergence in a multidimensional complex space,” IEEE Trans.Evol. Comput. , vol. 6, no. 1, pp. 58-73, 2002.

[10] V. Kadirkamanathan, K. Selvarajah, and P. J. Fleming, ”Stabolity Analysis of the Particle Dynamics in Particle Swarm Optimizer,” IEEE Trans. Evol. Comput. , vol. 10, no. 3, pp. 245-255, 2006.