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A modified radiation model for human mobility: effects of distinct job-seeker expectation and job-offer benefit distributions

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Abstract—Predicting human mobility is fundamental to human societies, and various models have been introduced. One of the prevailing frameworks, the radiation model, interprets the job-hunting activities as the result that each individual accepts the geographically closest job, and predicts the mobility flow in a closed formula of population distribution. However, the same distributions of job-seeker expectations and job-offer benefits are assumed in the radiation model, which does not necessarily hold in general situations. In this research, we propose a modified radiation model based on the theoretical derivation in the case where the distributions of job-seeker expectations and job-offer benefits are different. Furthermore, we show the improved results of the predictions than the original radiation model for a flow data set between counties in the United States.

1. Introduction

Understanding human mobility, the spatio-temporal movement of individuals, is fundamental to human societies: it allows social planners to estimate transportation volume [1, 2], prevent the spread of epidemics [3, 4], etc. The prevailing frameworks in the field have been also applied in predicting trade flows between nations [5], and migration [6] in addition to human mobility flow.

Two theoretical frameworks, the gravity [7] and intervening opportunity [8] models occupy the main stream of research in characterizing mobility flow in the past 80 years. However, it is necessary to estimate the parameters in both models from flow data. In later research, a parameter-free model, the radiation model [9], has been proposed as an extension of the intervening opportunity model, which gives a better prediction for empirical observations compared with that from the gravity model.

Despite the merits of the radiation model including the absence of adjustable parameters and theoretical guidance from the stochastic process, some of the settings in the model could be improved, e.g., the assumption that distributions of job-seeker expectations and job-offer benefits are the same. In economics, Job-seeker and job-offer determine the labor market condition together. In real society, issues on skills mismatch between workers and jobs, that workers' skills exceed or lag behind the job requirements, have attracted much attention [10, 11]. In addition, the imperfect information on the labor market, especially uncertainty on the wages [12], raises individual's wrong expectations and leads to the mismatch of job-seeker expectation and job-offer benefit distributions. Therefore, in this study, we propose a model based on the radiation model with distinct job-seeker expectation and job-offer benefit distributions.

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2. Model

2.1. The original radiation model

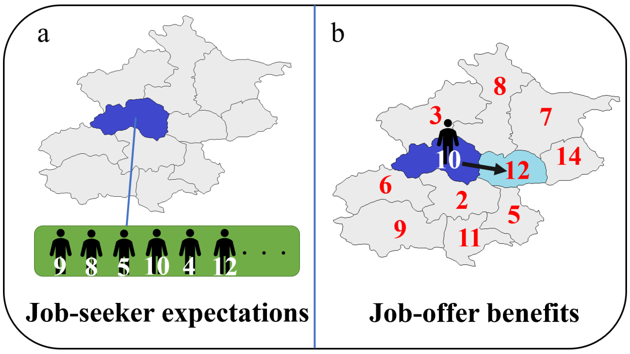

Figure 1: Spatial distributions of the job-seeker expectations and job-offer benefits. (a) Job-seekers hold their expected benefits for offers, represented by the number for each individual; (b) The red number represents the best offer in each region respectively. As hinted by the black arrow, this individual will accept the offer in light blue region with benefit 12.

In this subsection, we review the radiation model proposed by Simini et al. [9]. A job-seeker has his/her expected benefit of a job offer and searches for a job in all locations (Fig. 1a). The attractiveness of a certain location is represented by its offer benefit, which is the number on each location in Fig. 1b. A job-seeker always accepts the closest offer which provides a higher benefit than his/her expected one.

Given the job-seeker expectation and job-offer benefit distributions both as $\mathrm{P}(z)$, a random event that one job-seeker in location i accepts the offer in location j is the result of the following three independent events. In the following, m_i, m_j and s_{ij} represent the populations in location i and j, and the location within the circle centered at i and with radius d_{ij}, the geographical distance between i and j, respectively.

- The expected benefit of this job-seeker is z, which is the maximum value drawn from $\mathrm{P}(z)$ after m_i trials. The probability of this event is

$$\mathrm{P}_{m_i}(z) = \frac{d\mathrm{P}(z)^{m_i}}{dz}. \tag{1}$$

- The probability that at least one offer in location j drawn from $\mathrm{P}(z)$ after m_j trails has higher benefit than z is

$$\mathrm{P}_{m_j}(z) = 1 - \mathrm{P}(z)^{m_j}. \tag{2}$$

- The probability that all benefit values in rest locations within d_{ij} drawn from s_{ij} are less than z is

$$\mathrm{P}_{s_{ij}}(z) = \mathrm{P}(z)^{s_{ij}}. \tag{3}$$

The latter two events determine that the destination location j provides the best benefit value than any locations in between. Then the probability of one visit from location i to j is

$$\mathrm{P}_{\mathrm{Rad}}(1|ij) = \int_0^\infty \mathrm{P}_{m_i}(z)\mathrm{P}_{m_j}(z)\mathrm{P}_{s_{ij}}(z)dz. \tag{4}$$

Here we follow the natural assumption that the benefit z value is integrated from 0. For those job-seekers with negative expected benefits, they stay in their home area anyway and thus are not considered.

Substituting Eqs. (1) and (2) into Eq. (4), one can get

$$\mathrm{P}_{\mathrm{Rad}}(1|ij) \quad = \quad \frac{m_i m_j}{(m_i + s_{ij})(m_i + m_j + s_{ij})}. \tag{5}$$

See Ref. [9] for the detail of the derivation.

2.2. Our modified radiation model

As in Fig. 2, we extend the radiation model by introducing the distinct job-seeker expectation and job-offer benefit distributions. Without loss of generality, the job-offer benefit is assumed to follow the standard uniform distribution:

$$\mathrm{P_O}(z) = \begin{cases} 1 & 0 < z < 1, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

While the job-seeker expectation distribution takes the form:

$$\mathrm{P_S}(z) = \begin{cases} \frac{z-z_-}{z_+-z_-} & z_- < z < z_+, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

where z_+ and z_- jointly lead to the mismatch between individuals' expectations and benefits provided by the offers.

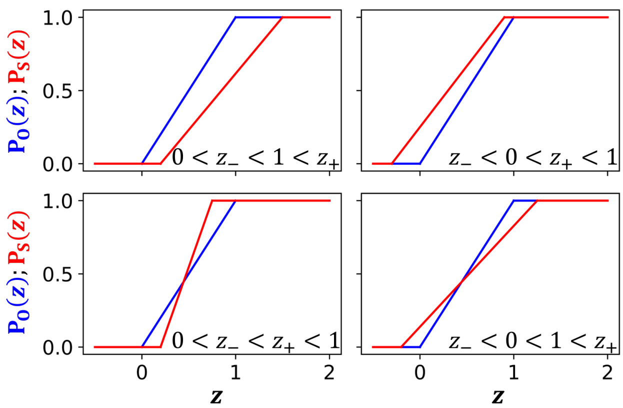

Figure 2: Four scenarios of job-offer benefit $\mathrm{P_O}(z)$ (blue) and job-seeker expectation $\mathrm{P_S}(z)$ (red) distributions.

In this model, the probability of one visit from location i to j is calculated as

$$\mathrm{P}(1|ij) = \int_0^\infty \mathrm{P_O}(z)^{s_{ij}}(1 - \mathrm{P_O}(z)^{m_j})d\mathrm{P_S}(z)^{m_i}. \tag{8}$$

To show the differences between job-seeker expectation and job-offer benefit distributions, we list four possible scenarios in Fig. 2, discussing the mismatch based on z_- and z_+. We first give the derivation with $0 < z_- < 1 < z+$ by substituting distributions in Eqs. (6) and (7) into (8):

$$\mathrm{P}_1(1|ij) = \frac{m_i}{(z_+ - z_-)^{m_i}} \int_{z_-}^1 (z^{s_{ij}} - z^{s_{ij}+m_j})(z - z_-)^{m_i-1}dz. \tag{9}$$

We apply the binomial theorem

$$(z - z_-)^{m_i-1} = \sum_{k=0}^{m_i-1} \binom{m_i - 1}{k} z^k(-z_-)^{m_i-k-1}, \tag{10}$$

to split the benefit variable z in Eq. (9) into a polynomial to calculate the integration. The results is given as

$$\begin{aligned} \mathrm{P}_1(1|ij) =& \frac{m_i}{(z_+ - z_-)^{m_i}} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i - 1}{k} \times \\ & \Big[\frac{m_j z_-^{m_i-k-1}}{(s_{ij} + k + 1)(m_j + s_{ij} + k + 1)} - \frac{z_-^{s_{ij}+m_i}}{s_{ij} + k + 1} \\ & + \frac{z_-^{s_{ij}+m_j+m_i}}{s_{ij} + m_j + k + 1} \Big] \\ =:& \frac{m_i}{(z_+ - z_-)^{m_i}} S_1. \end{aligned} \tag{11}$$

Note that the radiation model [9] is obtained for $z_+ = 1$ and $z_- = 0$, i.e. $\mathrm{P_O}(z) = \mathrm{P_S}(z)$ holds, thus this framework is an extension of the radiation model.

Applying the same method, the probability of one visit from location i to j is solved also in each of other scenarios. For $z_- < 0 < z_+ < 1$, we have

$$\mathrm{P}_2(1|ij) = \frac{m_i}{(z_+ - z_-)^{m_i}} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \times \left(\frac{z_+^{s_{ij}+k+1}}{s_{ij}+k+1} - \frac{z_+^{s_{ij}+m_j+k+1}}{s_{ij}+m_j+k+1} \right) z_-^{m_i-k-1}. \tag{12}$$

We further obtain

$$\mathrm{P}_3(1|ij) = \frac{m_i}{(z_+ - z_-)^{m_i}} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \times \left[\left(\frac{z_+^{s_{ij}+k+1}}{s_{ij}+k+1} - \frac{z_+^{s_{ij}+m_j+k+1}}{s_{ij}+m_j+k+1} \right) z_-^{m_i-k-1} - \left(\frac{z_-^{s_{ij}+k+1}}{s_{ij}+k+1} - \frac{z_-^{s_{ij}+m_j+k+1}}{s_{ij}+m_j+k+1} \right) \right], \tag{13}$$

for $0 < z_- < z_+ < 1$, and

$$\mathrm{P}_4(1|ij) = \frac{m_i}{(z_+ - z_-)^{m_i}} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \times \frac{m_j z_-^{m_i-k-1}}{(s_{ij}+k+1)(m_j+s_{ij}+k+1)}, \tag{14}$$

for $z_- < 0 < 1 < z_+$.

Following the radiation model [9], the mobility flow from location i to j is given as

$$\langle T_{ij} \rangle = T_i \mathrm{P}_n(1|ij), \tag{15}$$

where $n \in \{1, 2, 3, 4\}$, and $T_i = \sum_{j \neq i} T_{ij}$ is the total outgoing flow from location i.

2.3. Approximation

Unfortunately, it is difficult to compute Eq. (15) numerically, because the populations m_i, m_j, and m_t, which can be as large as 9 million, appear in the power. This causes underflow which worsens the numerical precision. Therefore, we expand the $\mathrm{P}(1|ij)$ into Taylor series around $z_- = 0$ and $z_+ = 1$ to get the first-order approximation for Eq. (15). Namely, we have

$$\mathrm{P}_1(1|ij) \approx \mathrm{P_{Rad}}(1|ij) + c_{1+}(z_+ - 1) + c_{1-} z_-, \tag{16}$$

where $c_{1+} = \frac{\partial \mathrm{P}_1(1|ij)}{\partial z_+}\Big|_{z_-=0,z_+=1}$ and $c_{1-} = \frac{\partial \mathrm{P}_1(1|ij)}{\partial z_-}\Big|_{z_-=0,z_+=1}$ denote the first-order expansion coefficients. Since the results for the four scenarios in the first-order expansion are the same, we show the expansion on $\mathrm{P}(1|ij)$ below and omit the detailed derivation for other scenarios.

From Eq. (11), the partial derivative of $\mathrm{P}_1(1|ij)$ with respect to z_+ is obtained as

$$c_{1+} = \frac{\partial \mathrm{P}_1(1|ij)}{\partial z_+}\Big|_{z_-=0,z_+=1} = \frac{-m_i^2}{(z_+ - z_-)^{m_i+1}} S_1 \Big|_{z_-=0,z_+=1}. \tag{17}$$

Since only the $k = m_i - 1$ term survives, Eq. (17) is simplified as

$$c_{1+} = \frac{-m_i^2 m_j}{(s_{ij}+m_i)(s_{ij}+m_i+m_j)}. \tag{18}$$

Similarly, we get

$$\begin{aligned} c_{1-} &= m_i \left[\frac{m_i S_1}{(z_+ - z_-)^{m_i+1}} + \frac{1}{(z_+ - z_-)^{m_i}} \frac{\partial S_1}{\partial z_-} \right] \Big|_{z_-=0,z_+=1} \\ &= m_i \left[\frac{m_i m_j}{(s_{ij}+m_i)(s_{ij}+m_i+m_j)} + \frac{-(m_i-1)m_j}{(s_{ij}+m_i-1)(s_{ij}+m_i+m_j-1)} \right]. \end{aligned} \tag{19}$$

3. Results

3.1. Datasets

We applied the above equations to predict the county-to-county flow of the United states in 2000. The county-level commuting trips are recorded based on the questions which county individuals work in, and provided by the Census Bureau [13]. County-wise population and county-pair distance are available at [14] and [15], respectively.

3.2. Comparison with the radiation model

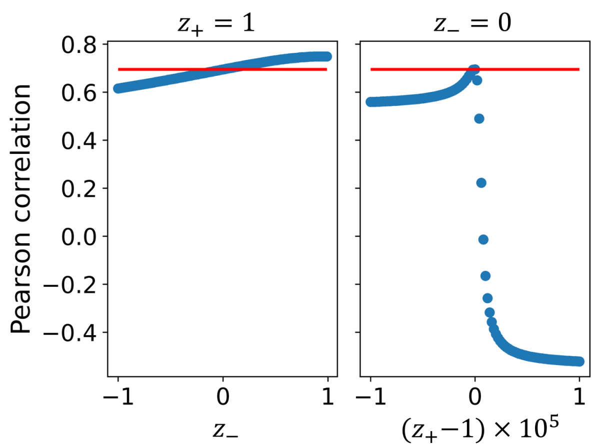

Figure 3: The dependence of the Pearson correlation coefficient on z_- (left) and z_+ (right). Here the Pearson correlation coefficient is the one between the flow counts from data and the estimations from the modified model. The Pearson correlation of the original radiation model is hinted by the red horizontal line.

We make the prediction based on the first-order approximation in Eq. (16) and show the dependence of the Pearson correlation coefficient on z_+ and z_- respectively in Fig. 3.

The dependence of coefficient on z_- shows an increasing trend and slightly decreases in the value close to $z_- = 1$. This is well explained by the over-estimating of job-seekers on themselves. Individuals always have higher expected benefits than the actual ones provided by the offers. The mismatch of expectation and offer benefit leads to a better performance of a larger difference between job-seeker expectation and job-offer benefit distributions.

At the same time, we observed the dependence of the Pearson correlation coefficient on z_+ while fixing $z_- = 0$, which is however very sensitive to the change of z_+.

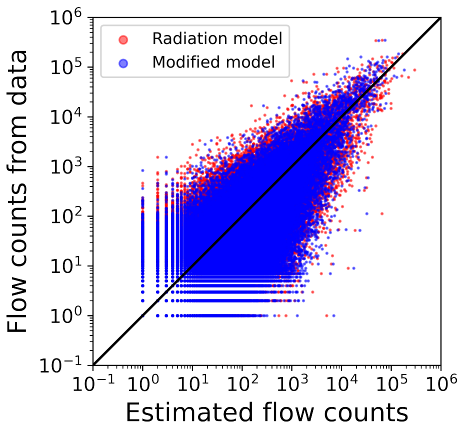

Figure 4: Predictions on the flow counts by the radiation model and the modified model are plotted in x-axis by red and blue dots respectively, while the y-axis is the real flow counts from the data. Here $z_- = 0.8$ and $z_+ = 1.0$ are assumed in the modified model.

As an example in Fig. 4, the Pearson correlation coefficient is 0.75 from the modified model while that from the radiation model is 0.69. A better prediction is observed that results from the modified model appear to be narrower than that from the radiation model.

Acknowledgments

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