2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Analysis of Simple Hysteresis Neural Networks for Basic Applications

Ryo Shibasawa† and Toshimichi Saito†

†Dept. of Electrical and Electronics Engineering, Hosei University
3–7–2 Kajino-cho, Koganei, Tokyo 184–8584, Japan

Abstract—This paper studies application of the hysteresis neural network to N-queen problems. In this application, the network exhibits oscillatory phenomena for some problem size. In order to suppress the oscillation, we introduce time-variant time-constant to the network. Performing basic numerical experiments, we have obtained an interesting result: applying the time-variant parameter to only one cell in the corner of the network, the oscillation can be suppressed and the desired solution can be obtained.

1. Introduction

The recurrent neural networks (RNN) have been applied to several optimization problems [1]-[3]. Classic RNNs have smooth activation functions and can realize effective parallel operation. In the applications, conditions for convergence to the target solutions and improvement of the processing performance have been considered.

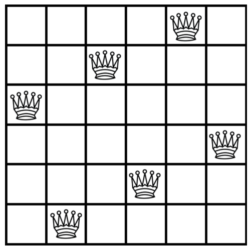
We have studied the hysteresis neural networks (HNN) with binary hysteresis activation function [4]-[8]. The HNN has important advantages: the system is piecewise linear and the dynamics can be analyzed speedily and precisely. Using the advantages, we have realized guaranteed storing desired memories to the HNN-based associative memories and have clarified periodic dynamics of a simple class of the HNNs.

This paper studies an improvement of the HNN dynamics in the application to N-queen problems presented in [8]. In this application, the HNN exhibits oscillatory phenomena for the case $N = 6$ etc. In order to suppress the oscillation, we introduce time-variant time-constant (TVTC) to the HNN. Performing basic numerical experiments, we have obtained an interesting result: applying the TVTC to only one cell in the corner of the HNN, the oscillation can be suppressed and the desired solution can be obtained. It goes without saying that we can suppress the oscillation by applying the TVTC to many cells.

It should be noted that the time-variant parameters are effective to improve performance of discrete-time associative memories [9], however, there exist not many works of time-variant parameters in continuous-time RNNs.

2. Hysteresis Neural Networks for N-Queen problem

The N-Queen problem is a typical optimization problems as shown in Figure1: placing problems of N chess queens on the $N \times N$ chessboard where the queens can not attack each other. In order to solve this problem, the following HNN has been applied [8]:



(a) One of solutions

0	0	0	0	1	0
0	0	1	0	0	0
1	0	0	0	0	0
0	0	0	0	0	1
0	0	0	1	0	0
0	1	0	0	0	0

(b) Neural expression

Figure 1: 6-Queen problem.

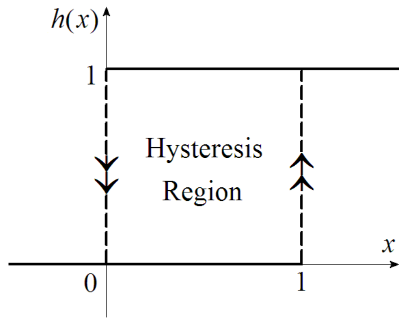


Figure 2: The binary hysteresis activation function

$$\lambda_{ij}\frac{dx_{ij}}{dt} = -x_{ij} + \left(1 - \sum_{m=1}^{N} y_{im}\right) + \left(1 - \sum_{m=1}^{N} y_{mj}\right) + a_{ij}\left(1 - \sum_{1 \le i-k, j-k \le N} y_{i-k,j-k} - \sum_{1 \le i-k, j+k \le N} y_{i-k,j+k}\right) + u_{ij} \quad (1)$$

$$y_{ij} = h(x_{ij}) = \begin{cases} 1 & \text{for} \quad 0 \le x_{ij} \\ 0 & \text{for} \quad x_{ij} \le 1 \end{cases}$$

where $i = 1 \sim N$ and $j = 1 \sim N$. The coordinates (i, j) corresponds to position on the chess board. The binary hysteresis activation $h(x)$ is switched from 0 to 1 (respectively, 1 to 0) if x reaches the right threshold 1 (respectively, the left threshold 0) as shown in Figure 2. For this HNN, the energy function is defined by

$$E(y) = \left| \left(1 - \sum_{m=1}^{N} y_{im}\right) + \left(1 - \sum_{m=1}^{N} y_{mj}\right) + a_{ij}\left(1 - \sum_{1 \le i-k, j-k \le N} y_{i-k,j-k} - \sum_{1 \le i-k, j+k \le N} y_{i-k,j+k}\right) \right| \quad (2)$$

The solutions correspond to the minimum value of $E(\boldsymbol{y})$. We have applied this HNN to N-queen problems for $N \in \{4, 5, \cdots, 15\}$ and λ_{ij}=1 for all i and j. The HNN exhibits oscillatory phenomena and the occurence rate is shown in Figure 3. In the figure the word "limit cycle generation rate (LCGR) " is used. The rate is calculated in 10^5 trials where the state is declared as a limit cycle if it does not converge to some equilibrium point within 10^4 switchings of $h(x_{ij})$. Figure 4 shows an example of limit cycle for $N = 6$. In these figures, we can see that the LCGR is extremely high for $N = 6$. Hereafter, we focus on the case $N = 6$ and consider to suppression of the limit cycles.

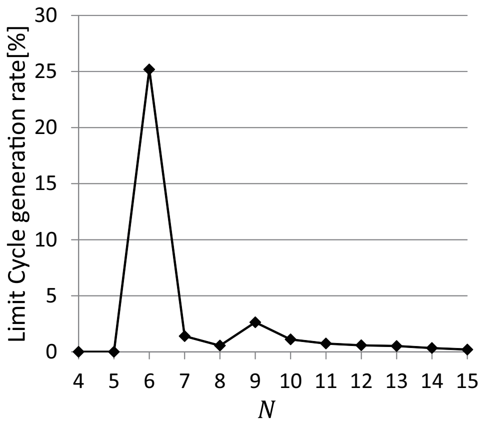


Figure 3: Limit cycle generation rate in 10^5 trials.

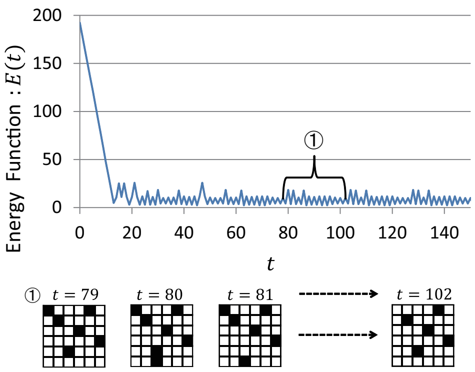


Figure 4: Example of limit cycles for 6×6 networks.

3. Numerical Experiments

Let τ denote the switching numbers of $h(x_{ij})$. The TVTC depends on τ. We apply the TVTC to all the cells:

$$\lambda_{ij}(\tau) = \begin{cases} \left|\cos\omega(\frac{36}{T}\tau + \alpha_{ij})\right| & \text{for } \tau = nT \\ 1 & \text{for } \tau \neq nT \end{cases} \tag{3}$$

where T is a positive integer that controls the switching interval. n is an nonnegative integer, $\alpha_{ij} = 6(i-1)+(j-1)$, i=1~6 and j=1~6. We refer to this case as HNN1. Figure 5 shows the LCGR for a_{ij}=0.3, u_{ij}=0.5 and ω=137°. Figure 6 shows convergence time to some solution. The average is calculated for 10^5 trials. We can see that the limit cycles are suppressed sufficiently and the variation of convergence time is small for T.

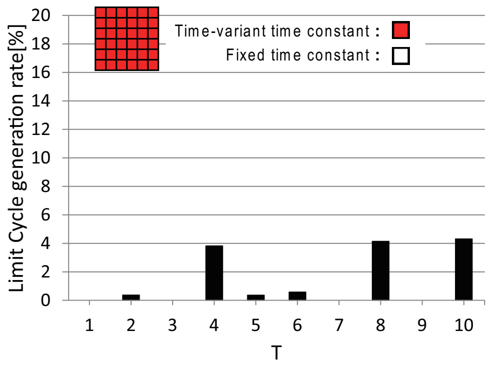


Figure 5: Limit cycle generation rate for HNN1

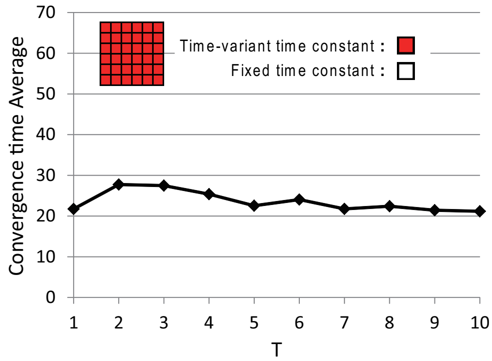


Figure 6: Convergence time for HNN1 (average in 10^5 trials).

Next, we apply the TVTC only to one cell on the corner. For simplicity, we consider the TVTC to the cell of (1, 1):

$$\begin{aligned} \lambda_{11}(\tau) &= \begin{cases} \left|\cos\omega\frac{\tau}{T}\right| & \text{for } \tau = nT \\ 1 & \text{for } \tau \neq nT \end{cases} \\ \lambda_{ij}(\tau) &= 1 \quad \text{for } (i, j) \neq (1, 1) \end{aligned} \tag{4}$$

where n is a nonnegative integer. We refer to this case as HNN2. Figure 7 shows the LCGR for a_{ij}=0.3, u_{ij}=0.5 and ω=137°. We can see that the limit cycles are suppressed sufficiently for $T = 5$ and $T = 9$. Figure 8 shows convergence time to some solution: it varies depending on T. We have confirmed similar results in the case where either of the other corner cells has the TVTC.

Outline of these results are summarized in Table1 that shows relation between the LCGR and convergence time.

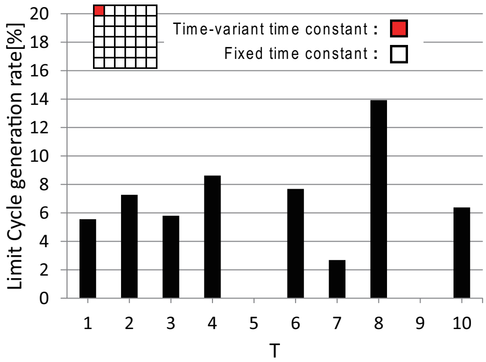


Figure 7: Limit cycle generation rate for HNN2.

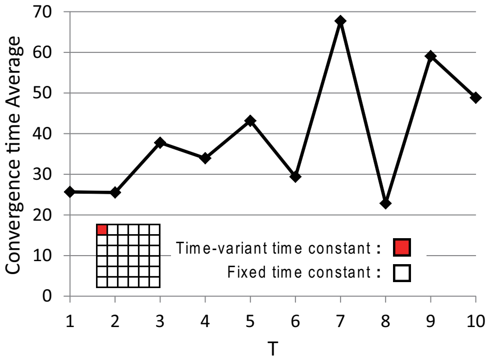
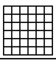
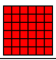
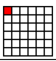


Figure 8: Convergence time for HNN2 (average in 10^5 trials).

Cell that changes time constant	Limit cycle generation rate [%]		Convergence time	
	Best	Worst	Best	Worst
	25.2		0.16	58.9
	0 (T=1,3,7,9)	4.3 (T=10)	0.05 (T=1)	334.4 (T=2)
	0 (T=5,9)	13.9 (T=8)	0.16 (T=2)	1242 (T=10)

Time-variant time constant : Fixed time constant :

Table 1: The relation between the limit cycle generation rate and convergence time.

4. Conclusion

We have considered performance improvement of HNN in application to N-Queen problems. Applying the TVTC to the case of 6-Queens, un-desired limit cycles can be suppressed and desired solution is obtained. Especially, we have confirmed that such a suppression is possible even if the TVTC is applied to one corner cell only. Future problems include analysis of dynamics of HNN with TVTC and application to wider problems.

References

[1] D. W. Tank and J. J. Hopfield, ”Simple ”Neural” optimization networks: An A/D converter, signal decision circuit, amd linear programming circuit,” IEEE Trans. Circuits & Syst. , vol. 33, pp. 533-541, 1986.

[2] M. P. Keneddy and L. O. Chua, ”Neural networks for nonlinear programming,” IEEE Trans. Circuit & Syst. , vol. 35,pp. 554-562, 1988.

[3] A. Rodrígues-Vázques, R. Domíngues-Castro, A. Rueda, J. L. Huertas, and E. Sánches-Sinencio, ”Nonlinear switched-capacitor ”Neural” networks for optimization problems,” IEEE Trans. Circuit & Syst. , vol. 37, pp. 384-397,1990.

[4] K. Jin’no and T. Saito, Analysis and synthesis of a continuous-time hysteresis neural network, Proc. of IEEE/ISCAS, pp. 471-474, 1992.

[5] K. Jin’no and T. Saito, Analysis of a simple hysteresis network and its application for an efficient associative memory, Proc. of IEEE/ISCAS, pp. 2172-2175, 1993.

[6] K. Jin’no and T. Saito, Analysis of periodic attractors from a simple hysteresis network, IEICE Trans. Fundamentals, E79-A, 6, pp. 873-882 (1996)

[7] K. Jin’no, T. Nakamura and T. Saito, Analysis of bifurcation phenomena in a 3 cells hysteresis neural network, IEEE Trans. Circuit Syst. I, 46, 7, pp. 851-857, 1999.

[8] T. Nakaguchi, K. Jin’no and M. Tanaka, Hysteresis Neural Networks for *N*-Queens Problems, IEICE Trans. Fundamentals, E82-A, pp. 1851-1859, 1999.

[9] K. Araki and T. Saito, An associative memory including time-variant self-feedback, Neural Networks, 7, 8, pp. 1267-1271, 1994.