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A Globally Convergent Variable-Gain Homotopy Method for Solving Modified Nodal Equations

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Abstract— **Homotopy methods are known to be effective methods for finding DC operating points of nonlinear circuits with the theoretical guarantee of global convergence. There are several types of homotopy methods; as one of the most efficient methods, the variable-gain homotopy (VGH) method is well-known. However, the global convergence of the VGH method for modified nodal equations (that are used in SPICE) has not been theoretically guaranteed; actually, it sometimes fails to converge. In this paper, we propose a modified algorithm of the VGH method and prove its global convergence for modified nodal equations. An experimental result is given to show the validity of the theory.**

I. Introduction

Finding DC operating points of nonlinear circuits is an important and difficult task in circuit simulation. SPICE-like circuit simulators, which are widely used in LSI design, employ the Newton-Raphson (NR) method for solving modified nodal (MN) equations. However, the NR method or its variants often fails to converge to a solution unless the initial point is sufficiently close to the solution. Therefore, many circuit designers experience difficulties in finding DC operating points, especially for bipolar analog integrated circuits.

To overcome this convergence problem, globally convergent homotopy methods have been studied by many researchers from various viewpoints [1]–[18]. By these studies, the application of the homotopy methods in practical circuit simulation has been remarkably developed, and some of them succeeded in solving bipolar analog circuits with more than 20,000 elements with the theoretical guarantee of global convergence [7],[8],[10],[13].

There are several types of the homotopy methods; as one of the efficient methods for solving bipolar circuits, the variable-gain homotopy (VGH) method is well-known [2]–[6],[9],[11]. For this method, many studies have been performed from various viewpoints [2]–[6],[9],[11]. The VGH method has also been implemented in the circuit simulation package Sframe [3],[9]. In [5] and [11], it is written that the VGH method is one of the most efficient homotopy methods. However, the global convergence of the VGH method for MN equations has not been theoretically guaranteed; actually, it sometimes fails to converge.

In this paper, we propose a modified algorithm of the VGH method that is globally convergent for MN equations. Namely, we show that a simple modification of the homotopy function makes the VGH method globally convergent for MN equations. We prove a theorem that guarantees the global convergence of the proposed algorithm under mild conditions.

II. Preliminaries

In this section, we first review the relation between the MN equation and the modified cut-set (MC) equation [13], which is necessary in the discussion of this paper.

A. MN Equation

In DC analysis, various elements in IC's, such as bipolar junction transistors (BJT's), can be modeled with voltage-controlled current sources (VCCS's) in a wide sense [19],[20]. In this paper, for simplicity, we assume that the circuit consists only of VCCS's and independent sources. We also assume that there are no loops consisting only of independent voltage sources and no cut-sets consisting only of independent current sources [19].

Let the number of nodes in the circuit be $N+1$, the number of branches excluding the independent source branches be K, and the number of independent voltage sources be M. Such a circuit can be described by an MN equation of the form

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{H}_1 \boldsymbol{g}({\boldsymbol{H}_1}^T \boldsymbol{x}) + \boldsymbol{H}_2 \boldsymbol{x} + \boldsymbol{\sigma} = \boldsymbol{0}. \tag{1}$$

In (1) the variable vector $\boldsymbol{x} \in \boldsymbol{R}^n$ $(n = N+M)$ is represented as

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{i} \end{bmatrix} \tag{2}$$

where $\boldsymbol{v} \in \boldsymbol{R}^N$ denotes the node voltages to the datum node and $\boldsymbol{i} \in \boldsymbol{R}^M$ denotes the branch currents of the independent voltage sources. Also, $\boldsymbol{g} : \boldsymbol{R}^K \rightarrow \boldsymbol{R}^K$ is a VCCS-type continuous function representing the relation between the branch voltages $\boldsymbol{v}_b \in \boldsymbol{R}^K$ and the branch currents $\boldsymbol{i}_b \in \boldsymbol{R}^K$ of the branches excluding the independent source branches and is expressed as

$$\boldsymbol{i}_b = \boldsymbol{g}(\boldsymbol{v}_b). \tag{3}$$

In addition, $\boldsymbol{H}_1$ is an $n \times K$ constant matrix represented as

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{D}_g \\ \boldsymbol{0} \end{bmatrix} \tag{4}$$

and $\boldsymbol{H}_2$ is an $n \times n$ constant matrix represented as

$$\boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}_E \\ {\boldsymbol{D}_E}^T & \boldsymbol{0} \end{bmatrix} \tag{5}$$

where $\boldsymbol{D}_g$ is an $N \times K$ reduced incidence matrix for the $\boldsymbol{g}$ branches and $\boldsymbol{D}_E$ is an $N \times M$ reduced incidence matrix for the independent voltage source branches. Moreover, $\boldsymbol{\sigma} \in \boldsymbol{R}^n$ is the source vector that is represented as

$$\boldsymbol{\sigma} = \left[\begin{array}{c} \boldsymbol{J} \\ -\boldsymbol{E} \end{array} \right] \tag{6}$$

where $\boldsymbol{J} \in \boldsymbol{R}^N$ is the current vector of the independent current sources and $\boldsymbol{E} \in \boldsymbol{R}^M$ is the voltage vector of the independent voltage sources. From (2) to (6), (1) can be written as follows:

$$\boldsymbol{f}_g(\boldsymbol{x}) = \boldsymbol{D}_g \boldsymbol{g}({\boldsymbol{D}_g}^T \boldsymbol{v}) + \boldsymbol{D}_E \boldsymbol{i} + \boldsymbol{J} = \boldsymbol{0} \tag{7a}$$

$$\boldsymbol{f}_E(\boldsymbol{v}) = {\boldsymbol{D}_E}^T \boldsymbol{v} - \boldsymbol{E} = \boldsymbol{0}. \tag{7b}$$

Most of the practical circuits, such as analog IC's, can be described by an MN equation of the form (7).

B. MC Equation

Next, we consider the MC equation. In the cut-set analysis [20] the voltages of the branches that compose a tree in the circuit are used as variables. The voltages of those branches are called the cut-set voltages and denoted as $\boldsymbol{u}$. Let the reduced incidence matrix for the tree branches be $\boldsymbol{T}$. Then, the following relation holds [20]:

$$\boldsymbol{u} = \boldsymbol{T}^T \boldsymbol{v}. \tag{8}$$

The MC equation is an extension of the cut-set equation to circuits containing independent voltage sources (as was done in the MN equation). By adding the branch currents of the independent voltage sources as variables such as

$$\boldsymbol{x} = \left[\begin{array}{c} \boldsymbol{u} \\ \boldsymbol{i} \end{array} \right] \tag{9}$$

we can formulate an equation of the form similar to (1). Such an equation is called the MC equation.

Since the reduced incidence matrix $\boldsymbol{T}$ of a tree is nonsingular, there exists $\boldsymbol{T}^{-1}$ [4]. Then, by applying the variable transformation $\boldsymbol{v} = (\boldsymbol{T}^{-1})^T \boldsymbol{u}$ to (7) and then multiplying (7a) by $\boldsymbol{T}^{-1}$ from the left, the MN equation (7) is transformed into the MC equation. Therefore, by considering $\boldsymbol{D}_g$ and $\boldsymbol{D}_E$ as the fundamental cut-set matrices and $\boldsymbol{J}$ as the cut-set current sources, (1) and (7) can be regarded as the MC equation.

Depending on the choice of the tree, the MC equation will result in different equations. A special case is the previously discussed MN equation. That is, by connecting imaginary branches with zero conductance between the datum node and all the other nodes, and by forming a tree with those imaginary branches, an MC equation that is essentially equivalent to the MN equation is obtained. Therefore, we can consider that the MC equation includes the MN equation [21].

C. Fundamental MC Equation

As another special case of the MC equation, we can consider a fundamental MC equation that is defined as follows. Consider a tree that consists of all independent voltage source branches and some of the $\boldsymbol{g}$ branches. Since it is assumed that there are no loops consisting only of independent voltage sources, there always exists a tree containing all independent voltage source branches. Then, formulate an MC equation for that tree. In this process, without loss of generality, we can rearrange the order of the branches so that the first N branches are the tree branches and, furthermore, the first M branches of the tree branches are the independent voltage source branches. The equation thus formulated is referred to as the fundamental MC equation.

In the fundamental MC equation $\boldsymbol{D}_E$ is represented by the following simple form:

$$\boldsymbol{D}_E = \left[\begin{array}{c} \boldsymbol{1}_M \\ \boldsymbol{0} \end{array} \right] \tag{10}$$

where $\boldsymbol{1}_M$ is an $M \times M$ identity matrix. Also, by separating the first M rows and the remaining $N-M$ rows and by separating the first $N-M$ columns and the remaining $K-(N-M)$ columns, $\boldsymbol{D}_g$ is represented as

$$\boldsymbol{D}_g = \left[\begin{array}{c} \boldsymbol{D}_{gE} \\ \boldsymbol{D}_{gg} \end{array} \right] = \left[\begin{array}{cc} \boldsymbol{0} & \boldsymbol{D}_{glE} \\ \boldsymbol{1}_{N-M} & \boldsymbol{D}_{glg} \end{array} \right]. \tag{11}$$

Therefore, the fundamental MC equation (7) is written in the following form:

$$\boldsymbol{f}_{gE}(\boldsymbol{x}) = \boldsymbol{D}_{gE} \boldsymbol{g}({\boldsymbol{D}_g}^T \boldsymbol{u}) + \boldsymbol{i} + \boldsymbol{J}_E = \boldsymbol{0} \tag{12a}$$

$$\boldsymbol{f}_{gg}(\boldsymbol{u}) = \boldsymbol{D}_{gg} \boldsymbol{g}({\boldsymbol{D}_g}^T \boldsymbol{u}) + \boldsymbol{J}_g = \boldsymbol{0} \tag{12b}$$

$$\boldsymbol{f}_E(\boldsymbol{u}_E) = \boldsymbol{u}_E - \boldsymbol{E} = \boldsymbol{0}. \tag{12c}$$

Here, the variable vector $\boldsymbol{u} \in \boldsymbol{R}^N$ is represented as

$$\boldsymbol{u} = \left[\begin{array}{c} \boldsymbol{u}_E \\ \boldsymbol{u}_g \end{array} \right] \tag{13}$$

where $\boldsymbol{u}_E \in \boldsymbol{R}^M$ and $\boldsymbol{u}_g \in \boldsymbol{R}^{N-M}$ denote the branch voltages of the independent voltage source branches and those of the $\boldsymbol{g}$ branches that compose the tree, respectively. Also, $\boldsymbol{J}_E \in \boldsymbol{R}^M$ and $\boldsymbol{J}_g \in \boldsymbol{R}^{N-M}$ are represented as

$$\boldsymbol{J} = \left[\begin{array}{c} \boldsymbol{J}_E \\ \boldsymbol{J}_g \end{array} \right]. \tag{14}$$

Since (12) has a simpler form than (7) it can be analyzed more easily than (7).

III. VGH Method

In this section, we review the VGH method for solving the MN equation (1) or (7).

For the simplicity of discussion, we assume that the relationship between the branch voltage vector $\boldsymbol{v}_q = (v_{\mathrm{be}}, v_{\mathrm{bc}})^T$ and the branch current vector $\boldsymbol{i}_q = (i_{\mathrm{e}}, i_{\mathrm{c}})^T$ of a bipolar junction transistor is described by the Ebers-Moll model:

$$\boldsymbol{i}_q(\boldsymbol{v}_q) = \left[\begin{array}{cc} 1 & -\alpha_r \\ -\alpha_f & 1 \end{array} \right] \left[\begin{array}{c} m_e(\exp(n_e v_{\mathrm{be}}) - 1) \\ m_c(\exp(n_c v_{\mathrm{bc}}) - 1) \end{array} \right]. \tag{15}$$

In the homotopy methods, we consider an auxiliary equation $\boldsymbol{f}^0(\boldsymbol{x}) = \boldsymbol{0}$ with a known solution $\boldsymbol{x}^0$ (or a solution easily obtained) and define a homotopy function:

$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \lambda \boldsymbol{f}(\boldsymbol{x}) + (1-\lambda) \boldsymbol{f}^0(\boldsymbol{x}) \tag{16}$$

where $\lambda \in [0, 1]$ is the homotopy parameter. Then, the solution curve (often called the path) of the homotopy equation:

$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{0} \tag{17}$$

is traced from the initial point $(\boldsymbol{x}^0, 0)$ at $\lambda = 0$. Such trace is often called the path following. If the solution curve reaches the $\lambda = 1$ hyperplane at $(\boldsymbol{x}^*, 1)$, then a solution $\boldsymbol{x}^*$ of (1) is obtained.

The VGH method [2]–[6],[9],[11] uses the following homotopy function termed the VGH :

$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{f}(\boldsymbol{x}, \lambda\boldsymbol{\alpha}) + (1-\lambda)\boldsymbol{G}(\boldsymbol{x} - \boldsymbol{a}) \tag{18}$$

where $\boldsymbol{\alpha}$ is a vector consisting of forward current gains α_f and reverse current gains α_r of transistors, $\boldsymbol{G}$ is an $n \times n$ diagonal matrix, $\boldsymbol{a}$ is a random vector that gives a bifurcation-free homotopy path (i.e., the homotopy path is bifurcation-free for almost all $\boldsymbol{a}$), and $\lambda\boldsymbol{\alpha}$ implies that the current gains of all transistors are multiplied by λ. The VGH method is a two-stage procedure. In phase 1, the initial point $\boldsymbol{x}^0$ that satisfies $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$ is computed by the modified NR method. In phase 2, the solution curve of $\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{0}$ is traced from $(\boldsymbol{x}^0, 0)$. In phase 1, the circuit described by $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$ contains diodes as only nonlinear elements, hence it has a unique solution.

The VGH method is efficient because it contains the excellent idea of variable-gain, which often makes the path following smooth. However, the initial point computed by the modified NR method is sometimes far from the solution [16]. In other words, the initial state at $\lambda = 0$ is sometimes far from the normal operation of transistor circuits. In [18], an efficient method is proposed, where we first determine a good initial point $\boldsymbol{x}^0$ and then determine $\boldsymbol{a}$ such that $\boldsymbol{x}^0$ becomes the solution of $\boldsymbol{h}(\boldsymbol{x}, 0) = \boldsymbol{0}$ using the SPICE-oriented approach. It is also shown in [18] that the VGH method can be easily implemented on SPICE without programming, although we do not know the homotopy method well.

IV. Proposed Method

It is easily seen that (18) is equivalent to

$$\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{f}(\boldsymbol{x}) + (1-\lambda)\tilde{\boldsymbol{f}}(\boldsymbol{x}) + (1-\lambda)\boldsymbol{G}(\boldsymbol{x} - \boldsymbol{a}) \tag{19}$$

where

$$\tilde{\boldsymbol{f}}(\boldsymbol{x}) \triangleq \begin{bmatrix} \boldsymbol{D}_g\tilde{\boldsymbol{g}}(\boldsymbol{D}_g^T\boldsymbol{v}) \\ \boldsymbol{0} \end{bmatrix}. \tag{20}$$

Here, the components $\tilde{g}_i$ $(i = 1, 2, \cdots, K)$ of $\tilde{\boldsymbol{g}} = (\tilde{g}_1, \tilde{g}_2, \cdots, \tilde{g}_K)^T$ are defined as follows:

1) If g_i and g_{i+1} are a pair of transistor branches, then the corresponding functions $\tilde{g}_i$ and $\tilde{g}_{i+1}$ are

$$\begin{bmatrix} \tilde{g}_i \\ \tilde{g}_{i+1} \end{bmatrix} = \begin{bmatrix} 0 & \alpha_r \\ \alpha_f & 0 \end{bmatrix} \begin{bmatrix} m_e(\exp(n_e v_{\mathrm{be}}) - 1) \\ m_c(\exp(n_c v_{\mathrm{bc}}) - 1) \end{bmatrix}. \tag{21}$$

2) If g_i is not a transistor branch, then $\tilde{g}_i = 0$.

Hence, (18) becomes as follows:

$$\boldsymbol{f}(\boldsymbol{x}) + (1-\lambda)\tilde{\boldsymbol{f}}(\boldsymbol{x}) + (1-\lambda)\boldsymbol{G}(\boldsymbol{x} - \boldsymbol{a}) = \boldsymbol{0}. \tag{22}$$

In general, an $n \times n$ matrix

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_N & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_M \end{bmatrix} \tag{23}$$

is used as $\boldsymbol{G}$, where $\mathbf{G}_N$ and $\mathbf{G}_M$ are $N \times N$ and $M \times M$ diagonal matrices, respectively, with positive diagonal elements. However, this VGH method sometimes fails to converge for MN equations. In this paper, we use

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_N & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{G}_M \end{bmatrix} \tag{24}$$

as $\boldsymbol{G}$ and show that this simple modification makes the VGH method globally convergent for MN equations.

From (7), the homotopy function is written as

$$\begin{aligned} \boldsymbol{h}_g(\boldsymbol{x}, \lambda) &= \boldsymbol{D}_g\boldsymbol{g}(\boldsymbol{D}_g{}^T\boldsymbol{v}) + \boldsymbol{D}_E\boldsymbol{i} + \boldsymbol{J} \\ &\quad +(1-\lambda)\boldsymbol{D}_g\tilde{\boldsymbol{g}}(\boldsymbol{D}_g^T\boldsymbol{v}) \\ &\quad +(1-\lambda)\boldsymbol{G}_N(\boldsymbol{v} - \boldsymbol{a}_N) \qquad (25\text{a}) \\ \boldsymbol{h}_E(\boldsymbol{x}, \lambda) &= \boldsymbol{D}_E{}^T\boldsymbol{v} - \boldsymbol{E} - (1-\lambda)\boldsymbol{G}_M(\boldsymbol{i} - \boldsymbol{a}_M) \qquad (25\text{b}) \end{aligned}$$

where $\boldsymbol{h}_g : \boldsymbol{R}^{n+1} \to \boldsymbol{R}^N$, $\boldsymbol{h}_E : \boldsymbol{R}^{n+1} \to \boldsymbol{R}^M$, and

$$\boldsymbol{a} = \begin{bmatrix} \boldsymbol{a}_N \\ \boldsymbol{a}_M \end{bmatrix}. \tag{26}$$

We now define the following terminology [22].

Definition 1: A continuous function $\boldsymbol{g} : \boldsymbol{R}^K \to \boldsymbol{R}^K$ is said to be uniformly passive on $\boldsymbol{v}_b^0$ if there exists a $\gamma > 0$ such that $(\boldsymbol{v}_b - \boldsymbol{v}_b^0)^T(\boldsymbol{g}(\boldsymbol{v}_b) - \boldsymbol{g}(\boldsymbol{v}_b^0)) \geq \gamma\|\boldsymbol{v}_b - \boldsymbol{v}_b^0\|^2$ for all $\boldsymbol{v}_b \in \boldsymbol{R}^K$. □

A fairly general class of resistive elements including BJT's, diodes, tunnel diodes, and positive linear resistors are known to be uniformly passive on certain points [22]. Thus, the uniform passivity is a very mild condition. If all resistive elements contained in the circuit are uniformly passive, then the following theorem holds.

Theorem 1: Consider the VGH given by (25). Assume that $\boldsymbol{g}$ is Lipschitz continuous and there exists a $\boldsymbol{v}_b^0 \in \boldsymbol{R}^K$ such that $\boldsymbol{g}$ is uniformly passive on $\boldsymbol{v}_b^0$. Then, for any initial point $\boldsymbol{x}^0 \in \boldsymbol{R}^n$ the solution curve of $\boldsymbol{h}(\boldsymbol{x}, \lambda) = \boldsymbol{0}$ starting from $(\boldsymbol{x}^0, 0)$ reaches $\lambda = 1$. □

Since the proof of this theorem is very long, it is omitted here. By this theorem, the global convergence of the VGH method using the homotopy (25) is theoretically guaranteed for most of the practical circuits.

V. Example

We implemented the proposed algorithm on SPICE3 using the SPICE-oriented approach proposed in [18]. We have applied the proposed algorithm to several practical circuits and have obtained good results. In this section, we show an example to confirm the validity of the theory.

Consider the regulator circuit shown in Fig. 1 that is used in bipolar LSI's. Note that this circuit cannot be solved by the DC analysis of SPICE3. We applied the standard VGH method using (23) as $\boldsymbol{G}$ and the proposed VGH method using (24) as $\boldsymbol{G}$ where $\boldsymbol{G}_N$ and $\boldsymbol{G}_M$ are $N \times N$ and $M \times M$ identity matrices, respectively. Fig. 2 shows the solution curves of the standard method and the proposed method starting from the initial points $\boldsymbol{x}^0$ such that $\boldsymbol{v}_q^0 = (0.7, 0)^T$ for npn transistors and $\boldsymbol{v}_q^0 = (-0.7, 0)^T$ for pnp. It is seen that the standard method diverged but the proposed method converged to the solution.

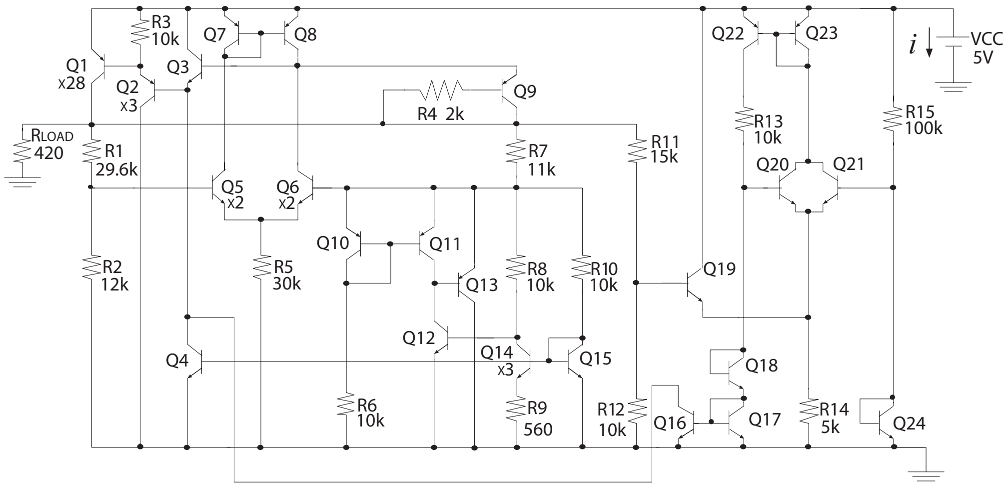

Fig. 1. Regulator circuit.

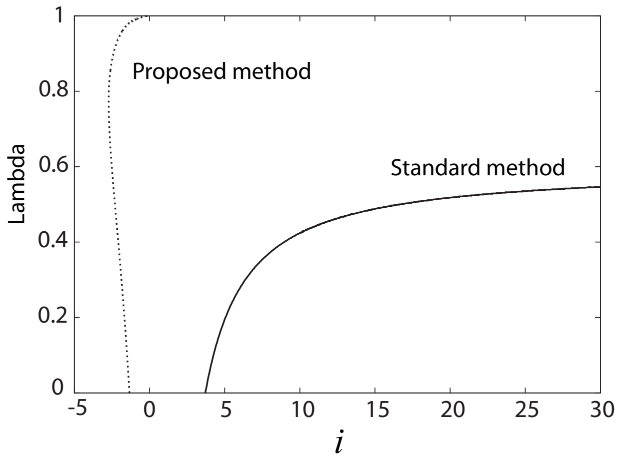

Fig. 2. Solution curves.

VI. Conclusion

In this paper, we have shown that the VGH method becomes globally convergent for MN equations by using (24) as $\boldsymbol{G}$. The experimental result shows the validity of the theory.

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