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Asymmetric collective decision making through quantum interference

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Abstract—Collective decision-making is important in recent information and communication systems. Therein, decision conflicts among multiple agents inhibit maximizing the potential utilities of the total system under study. It has been known that quantum processes can realize conflict-free joint decisions by utilizing entanglement of photons or quantum interference of orbital angular momentum of photons among two agents. However, the resultant joint decisions always result in a symmetric manner. Although this is a good aspect in view of ensuring equality, it is not sufficient to reduce disparities. Indeed, various forms of problematic inequalities are observed nowadays, such as educational disparity and gender gap, where preserving existing equality seems insufficient. In this study, we theoretically and numerically demonstrate conflict-free and asymmetric collective decision-making by utilizing entangled photons or quantum interference of photons carrying OAM. Whereas the asymmetry is indeed possible, photon loss is inevitable in the proposed models. The achievable range of asymmetry is analytically clarified.

1. Introduction

No matter how unstable the situation is, people need to make decisions by estimating and believing the choice will be beneficial. Multi-armed bandit problem models the decision-making process in uncertain environments where a player aims to maximize reward by predicting the best selection among a lot of slot machines, called arms, whose reward probabilities are uncertain. In a multi-armed bandit problem, exploration is needed to predict reward probabilities accurately, but too much exploration can diminish the total amount of obtained reward [1]. However, too short exploration can cause a miss of the best arm. Furthermore, when numerous players are involved in the game, decision conflicts can be another problem because multiple players choosing the same option can cause a bottleneck and impede the profits of the whole society [2],[3]. This problem is referred to as the competitive multi-armed bandit problem [4].

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Quantum properties of photons can help solve the problem of collective decision-making. Previous studies implemented quantum systems realizing conflict-free decision making among two players by utilizing entangled photons [5] and Hong-Ou-Mandel effects [4].

However, in these systems, affirmative actions are impossible because decisions made by them are always symmetric. Namely, the probability of player X choosing arm l and player Y choosing arm m is inevitably the same as the probability of player X choosing arm m and player Y choosing arm l. We call this property *symmetry*. Based on this symmetry, both players are always treated evenly.

The symmetric feature is appreciated if these players are equal at the beginning of the game because the equality is ensured at all times. On the other hand, if one player has much more advantages than the other prior to the game, this inequality cannot be solved by the previous studies' systems because of the symmetry (Figure 1). Namely, previous systems provides superiorities in terms of maintaining equalities, but they cannot reduce disparities.

To enable affirmative actions to resolve inequalities, decision-makings must be *asymmetric*. Asymmetry means the probability of player X choosing arm l and player Y choosing arm m can be different from the probability of player X choosing arm m and player Y choosing arm l. By employing asymmetry, it is possible to give advantages to underprivileged players. Indeed, the above-mentioned ini-

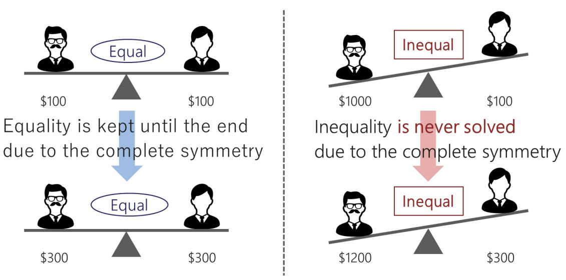

Figure 1: Why asymmetric decision-making is needed.

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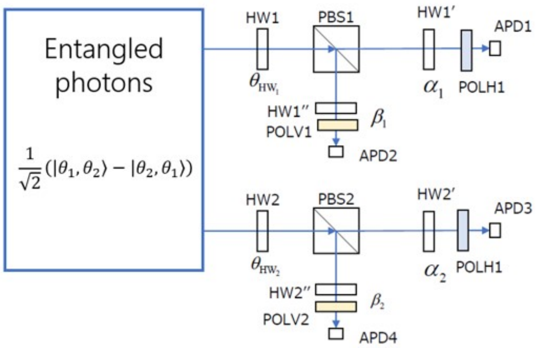


Figure 2: Architecture of the entangled-photon decision-maker. PBS = Polarizing Beam Splitter, HW = Half Wave plate, APD = Avalanche Photodiode, POLH = Polarizer (allowing horizontal polarization to pass), POLV = Polarizer (allowing vertical polarization to pass).

Table 1: Probabilities of decisions in the entangled-photon decision maker

Photon1	Photon2	Decisions	Probability
APD1	APD3	$(1,1)_{XY}$	$p_{11} = 0$
APD1	APD4	$(1,2)_{XY}$	$p_{12} = \sin\beta_2{}^2 \cos\alpha_1{}^2/2$
APD2	APD3	$(2,1)_{XY}$	$p_{21} = \sin\beta_1{}^2 \cos\alpha_2{}^2/2$
APD2	APD4	$(2,2)_{XY}$	$p_{22} = 0$

tial disparities are observed in a variety of societal issues ranging from earnings differentials, gender gaps, and educational inequalities. Besides such social problems, differentiation of services is a standard approach in industry. Furthermore, priority is inevitably important, especially concerning emergency and safety. Summing up, ensuring existing equality may not be sufficient.

This paper examines quantum models to address disparities by realizing *asymmetric* decision-making in the competitive multi-armed bandit problem. Two quantum models are investigated. Section 2 discusses the principle to realize the asymmetry by entangled photons, which is an extended version of the entangled-photon decision maker by Chauvet et al [5]. Section 3 examines asymmetry by using quantum interference effect of photons carrying orbital angular momentum (OAM), which is an enhanced principle of the method proposed by Amakasu et al [4].

2. Asymmetric collective decision making by entangled photons

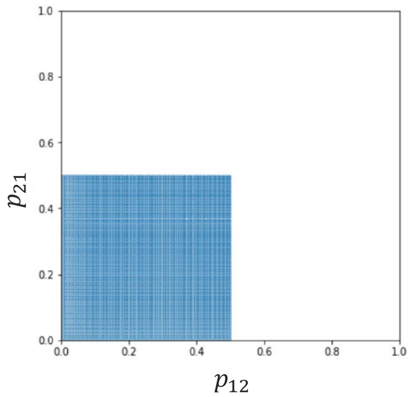
Here, we present the entangled-photon decision-maker that can achieve asymmetric decision-making. The experimental setup is shown in Fig. 2. This system is specifically for the 2-players-2-arms bandit problem. It makes the decision-making asymmetric by absorbing photons with specific probabilities at the polarizers before APDs. The existence of polarizers makes a difference between this system and one in the previous study [5]. Please note that we consider a specific input:

$$\frac{1}{\sqrt{2}}\left(|\theta_1,\theta_2\rangle - |\theta_2,\theta_1\rangle\right) \tag{1}$$

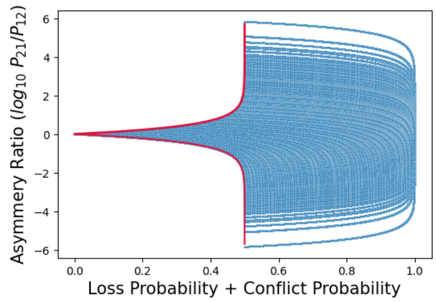
This input is a superposition of two states. One state is the entanglement of photon of phase θ_1 entering half-wave plate 1 and a photon of phase θ_2 into half-wave plate 2. Another is the entanglement of a photon of phase θ_2 into half-wave plate 1 and a photon of phase θ_1 into half-wave plate 2. The phases θ_1 and θ_2 fulfill the condition:

$$\theta_2 = \theta_1 + \frac{\pi}{2} \tag{2}$$

The probabilities of each pair of decisions are presented in Table 1. As Table 1 shows, a conflict never happens in this system, i.e. both players always choose different arms. On the other hand, as p_{12} and p_{21} cannot be more than 0.5, photons are lost unless $\alpha_1 = \alpha_2 = 0$, $\beta_1 = \beta_2 = \pi/2$.



(a) Possible pairs of p_{12} and p_{21} in the entangled-photon decision-maker



(b) The Relationship between Asymmetry Ratio and Loss Probability plus Conflict Probability

Figure 3: Result of the entangled-photon decision-maker

Figure 3a shows the possible pairs of p_{12} and p_{21} and it graphically represents that p_{12} and p_{21} are equal to or less than 0.5.

Here we define the probability of both two players choosing the same arm as the *Conflict Probability* $p_{11}+p_{22}$, the *Asymmetry Ratio* of the decision-making as p_{21}/p_{12}, *Loss Probability* of the photons as $1-(p_{11}+p_{12}+p_{21}+p_{22})$. Figure 3b represents the relationship between the Conflict Probability plus Loss Probability and Asymmetry Ratio. The mathematical formula of the red-lined border is as follows:

$$y = \begin{cases} \dfrac{1}{1-2x} & \text{when } y \geq 1, \\ 1-2x & \text{when } y \leq 1. \end{cases} \tag{3}$$

Figure3b shows that we need to tolerate at most 50% loss of photons to get all possible Asymmetry Ratios. The loss is minimal for the symmetric case $p_{21} = p_{12} = 0.5$ and grows for more asymmetric cases. This means the entangled-photon decision-maker is suitable when it is desirable to keep an even treatment as much as possible so that the targeted inequality is relatively small.

3. Asymmetric decision-making using quantum interference of OAM

This section introduces how the decision-making system using orbital angular momentum (OAM) can realize asymmetric decision-making. The architecture of the system for the 2-player, n-arm bandit problem is represented in Fig. 4. OAM detected at X corresponds to the arm selected by player X, and OAM detected at Y corresponds to the arm chosen by player Y. The difference between this system and one in the previous study is one polarizing beam splitter [4].

The probability of player X choosing arm k_1 and player Y choosing arm k_2 is given by

$$\begin{aligned} &P(X:k_1, Y:k_2) \\ &= \frac{1}{4}a_{k_1}^2 b_{k_2}^2(\alpha-\beta)^4 + \frac{1}{4}a_{k_2}^2 b_{k_1}^2(\alpha+\beta)^4 \\ &\quad - \frac{1}{2}a_{k_1}a_{k_2}b_{k_1}b_{k_2}(\alpha-\beta)^2(\alpha+\beta)^2\cos(\theta_{k_1}-\theta_{k_2}). \end{aligned} \tag{4}$$

Therefore, the difference between the probability of player X choosing arm k_1 and player Y choosing arm k_2 and the probability of player X choosing arm k_2 and player Y choosing arm k_1 is given by

$$\begin{aligned} &P\,(X:k_1, Y:k_2) - P(X:k_2, Y:k_1) \\ &= 2\alpha\beta(a_{k_2}^2 b_{k_1}^2 - a_{k_1}^2 b_{k_2}^2). \end{aligned} \tag{5}$$

Hence, if the condition

$$\alpha\beta \neq 0,\ a_{k_2}b_{k_1} \neq \pm a_{k_1}b_{k_2} \tag{6}$$

holds, the difference given by Eq. (5) is not zero. Namely, $P(X:k_1, Y:k_2) \neq P(X:k_2, Y:k_1)$ is achieved, meaning that asymmetry in decision-makings is realized.

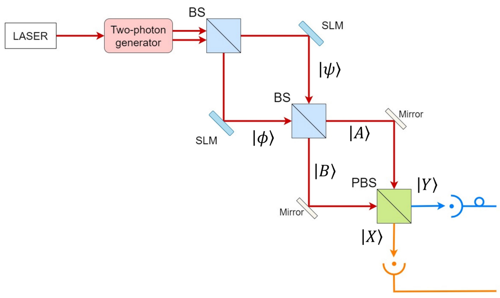


Figure 4: Decision-making system using OAM. PBS = Polarizing Beam Splitter, SLM = Spacial Light Modulator.

Table 2: Probabilities of decisions in decision-making system using OAM

Decisions	Probability
$(1,1)_{XY}$	$p_{11} = \alpha^2\beta^2 a_1^2 b_1^2$
$(1,2)_{XY}$	$p_{12} = \frac{1}{4}a_1^2 b_2^2(\alpha-\beta)^4 + \frac{1}{4}a_2^2 b_1^2(\alpha+\beta)^4$ $-\frac{1}{2}a_1a_2b_1b_2(\alpha+\beta)^2(\alpha-\beta)^2\cos(\theta_1-\theta_2)$
$(2,1)_{XY}$	$p_{21} = \frac{1}{4}a_2^2 b_1^2(\alpha-\beta)^4 + \frac{1}{4}a_1^2 b_2^2(\alpha+\beta)^4$ $-\frac{1}{2}a_1a_2b_1b_2(\alpha+\beta)^2(\alpha-\beta)^2\cos(\theta_2-\theta_1)$
$(2,2)_{XY}$	$p_{22} = \alpha^2\beta^2 a_2^2 b_2^2$

At the same time, however, conflicts can occur with a specific probability. The probability of the conflict happening with arm k is as follows:

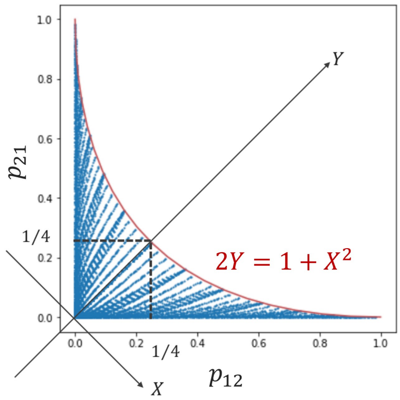
$$P(X:\ k,\ Y:\ k) = \alpha^2\beta^2 a_k^2 b_k^2. \tag{7}$$

Here we examine the details concerning 2-player, 2-arm situations. The probabilities of each pair of decisions are presented in Table 2. Figure 5a shows the possible pairs of p_{12} and p_{21}. There are impossible pairs such as $(p_{12}, p_{21}) = (0.5, 0.5)$. The border between the possible pairs and impossible pairs, the possible zone boundary, is shown by the red line in Fig. 5a. The formula of this border is as follows:

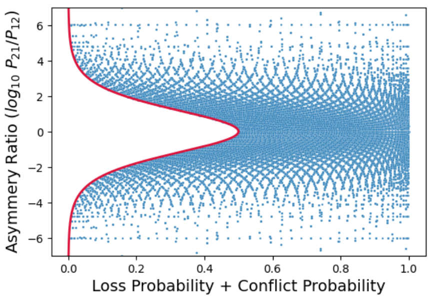
$$2(p_{12}+p_{21}) = 1 + (p_{12}-p_{21})^2 \tag{8}$$

Figure 5b shows the relationship between the Conflict Probability plus Loss Probability and Asymmetry Ratio. The red-colored border curve in Fig. 5b, minimum-loss-plus-conflict boundary, is represented as follows:

$$y = \begin{cases} \dfrac{(1+\sqrt{1-2x})^2}{(1-\sqrt{1-2x})^2} & \text{when } y \geq 1 \\ \dfrac{(1-\sqrt{1-2x})^2}{(1+\sqrt{1-2x})^2} & \text{when } y \leq 1 \end{cases} \tag{9}$$



(a) Possible pairs of p_{12} and p_{21} in the decision-making system using OAM. $X = p_{12} - p_{21}$, $Y = p_{12} + p_{21}$



(b) The Relationship between Asymmetry Ratio and Loss Probability plus Conflict Probability

Figure 5: Result of the decision-making system using OAM

Like the entangled-photon decision-maker, 50% loss or conflict is necessary to obtain any Asymmetry Ratio. When a lower rate of loss or conflict is attractive, we get extreme Asymmetry Ratios, such as more than 100 or smaller than 0.01. Therefore, the decision-making system using OAM is more appropriate when inequality is so severe that more powerful affirmative actions are needed.

4. Conclusion

We examined asymmetric collective decision making by quantum attributes of photons to enable affirmative actions to reduce disparities. Asymmetry in decision-making have been successfully demonstrated by either of the two model systems utilizing entangled photons and quantum interference of OAM, whereas the previous studies are limited to symmetric decision-making. With the asymmetric joint decisions, inequalities among agent, which may be inherent prior to the games, can be attenuated.

However, in both systems, there were impossible pairs of p_{12} and p_{21}. Also, loss of photons or conflict could occur with a certain probability. We show that by admitting the loss of photons or conflict occurring with 50% or higher, any level of asymmetry can be accomplished. When the intended inequality is relatively small, entangled-photon-based approach is preferable since a minor loss of photons and a lower conflict rate is possible. In cases when a larger degree of inequality is demanded, OAM-based system provides a superior performance with low conflict rate and photon loss.

This paper paves the way toward extending photonic and quantum collective decision making to include a broader sense of equality and social welfare.

Acknowledgments

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