



# Synchronization in Chaotic Pulse-Coupled Neural Networks and its Application to Wireless Sensor Network

Hidehiro Nakano<sup>†</sup>, Akihide Utani<sup>†</sup>, Arata Miyauchi<sup>†</sup> and Hisao Yamamoto<sup>†</sup>

<sup>†</sup>Tokyo City University

1-28-1, Tamazutsumi, Setagaya-ku, Tokyo 158-8557 Japan

Email: {nakano, miyauchi}@ic.cs.tcu.ac.jp, {autani, yamahisa}@tcu.ac.jp

**Abstract**—This paper studies synchronous phenomena in a chaotic pulse-coupled neural network (CPCNN), and presents its application to a chaos-based data gathering scheme (CDGS) in wireless sensor networks (WSNs). In CPCNN, chaotic spiking oscillators (CSOs) are coupled to each other by their output spike-trains. Chaos synchronous phenomena and their breakdown phenomena in CPCNN are shown. In CDGS, each wireless sensor node has a timer characterized by a single CSO. The proposed CDGS can effectively gather sensor information in WSNs not only with a single sink node but also with multiple sink nodes. Through simulation experiments, we show effectiveness of the proposed CDGS and discuss its development potential.

## 1. Introduction

Pulse-coupled neural networks (PCNNs) have been studied extensively [1]-[8]. Basic PCNNs consist of simple spiking oscillators (SOs), and SOs are coupled to each other by their output spike-trains. PCNNs can exhibit various synchronous and asynchronous phenomena [1][2]. Based on these phenomena, PCNNs can be applied to associative memory [3], image processing [4][5], and so on. Since the dynamics of PCNN is very simple, synchronous phenomena in PCNN can be analyzed easily and can be observed from simple implementation circuits. However, conventional PCNNs consists of simple SOs can exhibit periodic synchronization only. In our previous works, chaotic PCNNs (CPCNNs) consisting of chaotic spiking oscillators (CSOs) have been proposed. CPCNNs have rich dynamics including partial and/or temporal synchronization of chaos [6]-[8].

As one of application examples of PCNN, a synchronization-based data gathering scheme (SDGS) in wireless sensor networks (WSNs) has been proposed [9]. In WSNs, hundreds or thousands of micro-sensor nodes with resource limitation are deployed without control in a region and used to monitor and gather sensor information of environments. In SDGS, a wireless sensor node or a sink node has a timer characterized by a single SO. The sink node transmits a stimulus spike signal periodically, and each wireless sensor node relays the signal. That is, the WSN itself forms a kind of PCNNs. Then, all the wireless sensor nodes are synchronized to the

sink node. Each wireless sensor node transmits or relays gathering sensor information to the sink node in the spike timings. Using the SDGS, the number of transmitting and receiving sensor information can be reduced effectively. Hence, energy consumption of each wireless sensor node can be saved. However, the conventional SDGS can realize periodic synchronization only. Therefore, the same sensor information tends to be relayed by many wireless sensor nodes. As discussed in Ref. [10], energy consumption of transceivers in transmitting sensor information is a dominant factor in WSNs. Such a redundant relay should be improved, and the total number of transmissions in WSNs should be reduced. In addition, the conventional SDGS considers WSNs only with a single sink node. In the viewpoints of practical applications, SDGS in WSNs not only with a single sink node but also with multiple sink nodes should be considered in more detail.

This paper studies synchronous phenomena in CPCNN and presents its application to data gathering scheme in WSN. This scheme is referred to as a chaos-based data gathering scheme (CDGS, [11]). In CPCNN, CSOs are coupled to each other by their output spike-trains. Chaos synchronous phenomena and their breakdown phenomena in CPCNN are shown. Next, CDGS using CPCNN is presented. In CDGS, each wireless sensor node has a timer characterized by a single CSO. The proposed CDGS can effectively gather sensor information in WSNs not only with a single sink node but also with multiple sink nodes. We evaluate the proposed CDGS using computer simulations. Through simulation experiments, we show effectiveness of the proposed CDGS and discuss its development potential.

## 2. Chaotic Pulse-Coupled Neural Network

First, a chaotic pulse-coupled neural network (CPCNN) is introduced. Basic dynamics of the  $i$ -th CSO is described by the following equation.

$$\frac{d}{dt} \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} \Delta & \omega \\ -\omega & \Delta \end{bmatrix} \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix}, \quad (1)$$

$$\text{for } z_i(t) = 0, \text{ and } z_{N(i)}(t + \delta) = 1$$

$$z_i(t) = \begin{cases} 1, & \text{if } x_i(t) = 1 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

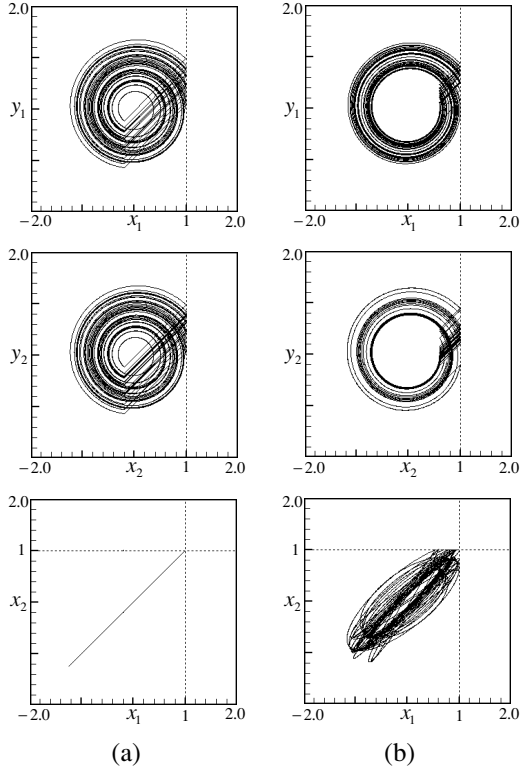


Figure 1: Typical phenomena from a master-slave CPCNN. Upper: Master attractors. Middle: Slave attractors. Bottom: Phase relationships.  $\Delta = 0.25$ ,  $\omega = 5$ ,  $p = 1$ ,  $a = 1$ ,  $\delta = 0$ . (a) Synchronization of chaos:  $q = -0.2$ . (b) Quasi-synchronization of chaos:  $q = 0.6$ .

$$\begin{bmatrix} x_i(t^+) \\ y_i(t^+) \end{bmatrix} = \begin{bmatrix} q \\ y_i(t) - p(x_i(t) - q) \end{bmatrix}, \quad (3)$$

if  $z_i(t) = 1$

$$\begin{bmatrix} x_i(t^+) \\ y_i(t^+) \end{bmatrix} = \begin{bmatrix} a \\ y_i(t) - p(x_i(t) - a) \end{bmatrix}, \quad (4)$$

if  $z_{N(i)}(t + \delta) = 1$

$$i = 1, 2, \dots, M$$

where  $\Delta$ ,  $\omega$ , and  $p$  denote a damping, a self-running angular frequency, and a slope, respectively.  $q$  and  $a$  denote base states for self-spike and compulsory-spike, respectively.  $N(i)$  is an index set of the neighbor and/or master CSOs to the  $i$ -th CSO.  $\delta$  is an offset time: The positive and negative values of  $\delta$  represents CPCNN with time-advance and time-delay couplings, respectively.

Fig. 1 shows typical phenomena from a master-slave CPCNN consisting 2 CSOs, where  $M = 2$ ,  $N(1) = \emptyset$  and  $N(2) = \{1\}$ . As shown in the figure, the 1-st (master) CSO exhibits chaotic attractors for both  $q = -0.2$  and  $q = 0.6$ . The 2-nd (slave) CSO is synchronized to the 1-st CSO for  $q = -0.2$ . That is, CPCNN exhibits master-slave synchronization of chaos. On the other hand, the 2-nd CSO is not synchronized but quasi-synchronized to the 1-st CSO for  $q = 0.6$ .

Since the dynamics of CPCNN is piecewise linear, these phenomena can be analyzed easily by using the exact piecewise solutions and return maps [6][7]. In CPCNN, CSOs are coupled to each other only by their output spike-trains. Therefore, in the master-slave CPCNN for  $\delta \neq 0$ , similar synchronous and quasi-synchronous phenomena with time-advance or time-delay can be observed. It should be noted that these phenomena can be observed easily because the spike time series  $z_i(t + \delta) = 1$  can be calculated by using the exact piecewise solutions.

### 3. Chaos-Based Data Gathering Scheme

Next, a chaos-based data gathering scheme (CDGS) in wireless sensor networks (WSNs) using CPCNN is explained. WSN consisting of wireless sensor nodes and sink nodes is considered. Each wireless sensor node or each sink node has a timer characterized by a single CSO, which controls timing to transmit and receive sensor information. A sink node  $S_c$  broadcasts “distance level  $l_c = 0$ ” as a beacon signal when  $z_c(t + \delta) = 1$ . If a wireless sensor node  $S_i$  can receive the signal directly, the distance level of  $S_i$  is adjusted as  $l_i = 1$ . At the same time, the state of  $S_i$  is changed based on Equation (4). Also,  $S_i$  forwards the beacon signal with own distance level  $l_i$  when  $z_i(t + \delta) = 1$ . If the other wireless sensor node  $S_j$  can receive the signal directly, the distance level of  $S_j$  is adjusted as  $l_j = l_i + 1 = 2$ . Repeating in this manner, distance levels of wireless sensor nodes correspond to a hop count to the nearest sink node.

As synchronization is achieved by the above explained manner, wireless sensor nodes having a large distance level can transmit sensor information earlier than those having a small distance level. As the offset time is set to sufficiently large value considered conflicts in MAC layer, the sensor information can be relayed sequentially from wireless sensor nodes far from the sink node.

### 4. Numerical Simulations

Fig. 2 shows a WSN model for the simulations. In the figure, 200 wireless sensor nodes are set at random on 8 concentric circles whose centers are  $(7, 0)$  or  $(-7, 0)$ , and 2 sink nodes are set on each center. In the simulations for a single sink node, let only the left center node be a sink node and let the right center node be a wireless sensor node. The radio range of each wireless sensor node and each sink node is set to 5. The radii of the concentric circles are set to 3, 6, 9 and 12, respectively.  $10n$  wireless sensor nodes are set on the  $n$ -th concentric circle from each center. Initial values of internal states in each wireless sensor node are set to random values. Parameters are fixed as  $\Delta = 0.25$ ,  $\omega = 5$ ,  $p = 1$ ,  $a = 1$ , and  $\delta = 0.2$ .

Fig. 3 shows spike time of each wireless sensor node. In the figure, horizontal axis denotes time, and vertical axis denotes the indexes of each wireless sensor node. Lower part of each figure denotes wireless sensor nodes and a

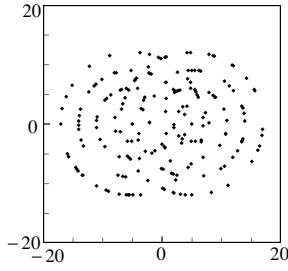


Figure 2: A model of a wireless sensor network.

sink node on the left concentric circles shown in Fig. 2, and upper part denotes them on the right concentric circles. In the case of a single sink node for  $q = -0.2$ , all the wireless sensor nodes are synchronized to each other with time difference depending on their own distance levels as shown in Fig. 3(a). We can also see that each series of interspike-intervals is chaotic. In the case of multiple sink nodes for  $q = -0.2$ , partial and temporal synchronization of chaos can be observed as shown in Fig. 3(b). It should be noted that it is also hard for the periodic SDGS to synchronize wireless sensor nodes in the case of multiple sink nodes. Because, frequency and/or phase of each sink node is not synchronized unless each sink node is coupled to each other. In the case of a single sink node for  $q = 0.6$ , quasi-synchronous spike-trains can be found as shown in Fig. 3(c). In the case of multiple sink nodes for  $q_i = 0.6$ , similar result can be confirmed as shown in Fig. 3(d).

In order to evaluate transmission efficiency in more detail, we evaluate the total number of wireless sensor nodes (TSN) which relay sensor information from a wireless sensor node to a sink node. Also, we evaluate average delivery ratio (ADR) for sensor information from a wireless sensor node to a sink node. We select 40 wireless sensor nodes  $S_k$  ( $k = 1, \dots, 40$ ) allocated on the most outside of the left concentric circles shown in Fig. 2.  $S_k$  transmits sensor information in every self-spike timing. We assume that only one wireless sensor node in  $S_k$  transmits sensor information 100 times, and we then evaluate TSN and ADR.

Figs. 4 and 5 show TSN and ADR, respectively. The horizontal axis denotes sorted indexes of the transmitting wireless sensor nodes. In the case of a single sink node for  $q = -0.2$ , all the wireless sensor nodes are synchronized to each other as shown in Fig. 3(a). Then, sensor information must be transmitted to the sink node without transmission miss as shown in Fig. 5(a). However, the sensor information is relayed by many wireless sensor nodes as shown in Fig. 4(a). In the case of multiple sink nodes for  $q = -0.2$ , each wireless sensor node is synchronized partially and temporally to each other as shown in Fig. 3(b). Then, TSN for each transmitting wireless sensor node decreases, compared with the case of a single sink node as shown in Fig. 4(a). However, it should be noted that ADR keeps high values for all the transmitting wireless sensor nodes as shown in Fig. 5(a). In the case of a single sink

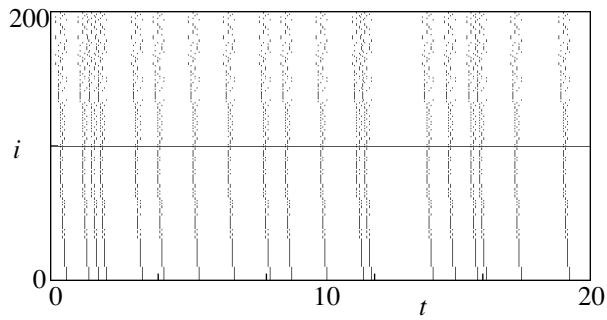
node for  $q = 0.6$ , each wireless sensor node is quasi-synchronized as shown in Fig. 3(c). Also, similar result can be found in the case of multiple sink nodes for  $q = 0.6$  as shown in Fig. 3(d). It should be noted that TSN shown in Fig. 4(b) can be reduced significantly, compared with TSN shown in Fig. 4(a). It can contribute to saving energy consumption of each wireless sensor node. Although transmission miss of sensor information occurs, ADR is kept as high values for all the transmitting wireless sensor nodes as shown in Figs. 5(b). Although a part of broken paths due to transmission miss exists, sensor information can be relayed to at least one sink node if at least one active path to the sink node exists. Therefore, quasi-synchronization in the proposed CDGS is significantly effective for prolonging WSN lifetime.

## 5. Conclusions

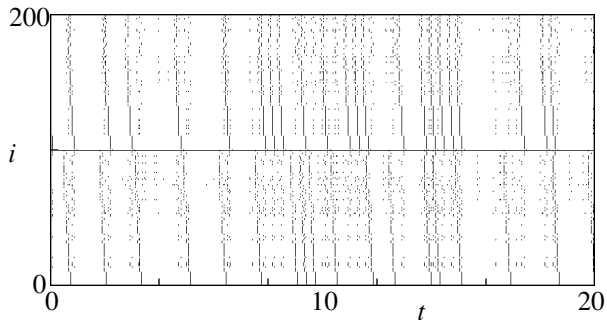
We have studied synchronous phenomena in a chaotic pulse-coupled neural network (CPCNN), and have presented its application to a chaos-based data gathering scheme (CDGS) in wireless sensor networks (WSNs). Through numerical simulations, we have shown that the proposed scheme can reduce the total number of the wireless sensor nodes which relay the same sensor information, keeping high delivery ratio. In addition, the proposed scheme can be applied easily to wireless sensor networks with multiple sink nodes and shows great performances in the viewpoints of prolonging the lifetime of wireless sensor networks. Future problems include evaluation of energy consumption and comparison with periodic SDGS in more detail.

## References

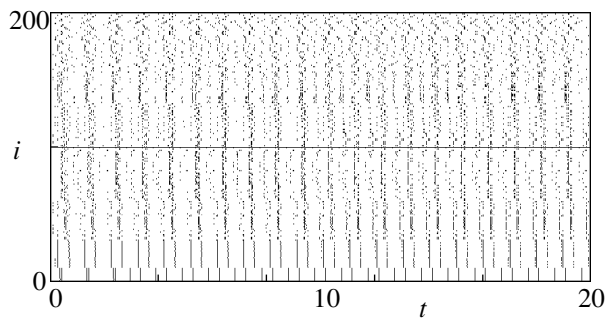
- [1] R. E. Mirollo & S. H. Strogatz, *SIAM J. Appl. Math.*, 50, 1645-1662 (1990)
- [2] E. Catsigeras & R. Budelli, *Physica D*, 56, 235-252 (1992)
- [3] E. M. Izhikevich, *IEEE Trans. Neural Networks*, 10, 3, 508-526 (1999)
- [4] J. J. Hopfield & A. V. M. Herz, *Proc. Natl. Acad. Sci.*, 92, 15, 6655-6662 (1995)
- [5] S. R. Campbell, D. Wang & C. Jayaprakash, *Neural Comput.*, 11, 1595-1619 (1999)
- [6] H. Nakano & T. Saito, *IEICE Trans. Funds.*, EA83-A, 5, 895-900 (2000)
- [7] H. Nakano & T. Saito, *IEEE Trans. Neural Networks*, 13, 1, 92-100 (2002)
- [8] H. Nakano & T. Saito, *IEEE Trans. Neural Networks*, 15, 5, 1018-1026 (2004)
- [9] N. Wakamiya & M. Murata, *IEICE Trans. Communications*, E88-B, 3, 873-881 (2005)
- [10] W. R. Heinzelman, A. Chandrakasan & H. Balakrishnan, *Proc. of the Hawaii International Conference on System Sciences*, 3005-3014 (2000)
- [11] H. Nakano, A. Utani, A. Miyauchi & H. Yamamoto, *IEICE Trans. Funds.*, E92-A, 2, 459-466 (2009)



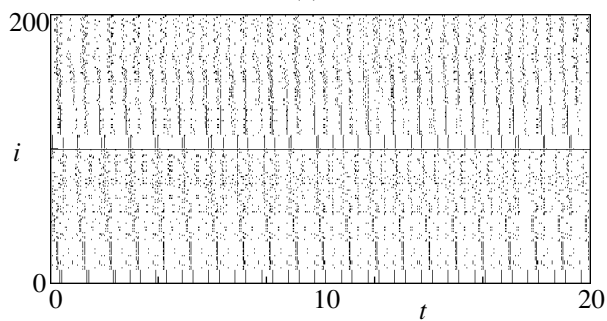
(a)



(b)

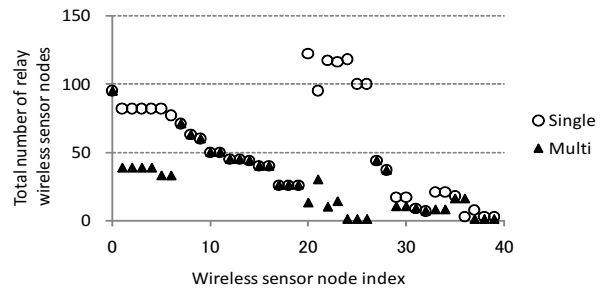


(c)

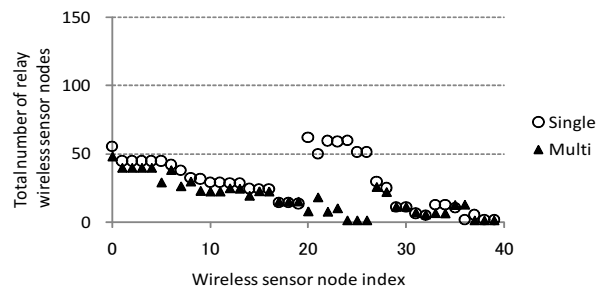


(d)

Figure 3: Spike time of each sensor node. (a) A single sink node ( $q_i = -0.2$ ). (b) Multiple sink nodes ( $q_i = -0.2$ ). (c) A single sink node ( $q_i = 0.6$ ). (d) Multiple sink nodes ( $q_i = 0.6$ ).

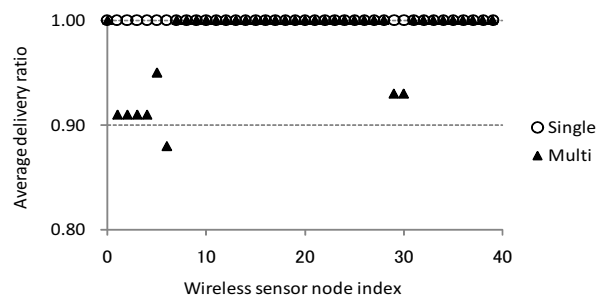


(a)

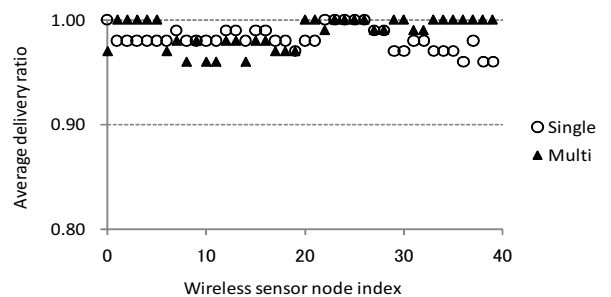


(b)

Figure 4: Total number of relay wireless sensor nodes (TSN). (a)  $q = -0.2$ . (b)  $q = 0.6$ .



(a)



(b)

Figure 5: Average delivery ratio (ADR). (a)  $q = -0.2$ . (b)  $q = 0.6$ .