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Chaotic Associative Memory Dynamics of Chaotic Neural Network Model with Time Dependent System Parameter

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Abstract—In this paper, we investigate chaotic associative memory dynamics in a chaotic neural network model (referred as CNN hereafter) with a time-dependent system parameter. We have shown that an isolated chaotic neuron model with a time-dependent system parameter gives attractor coexistence behaviors depending on initial conditions. In this paper, we introduce a time dependent system parameter into Adachi & Aihara CNN with association recalling dynamics. Consequently, the system possesses two types of initial dependence, originating in (i) synaptic connections and (ii) a time-dependent system parameter. The purposes of this paper are (i) to show whether two types of initial dependence could coexist or not, and (ii) to investigate chaotic associative memory dynamics for various initial configurations. From computer experiments, two types of initial dependence can coexist and the system shows complex associative memory dynamics. In several parameter regions, the system reveals different chaotic associative memory dynamics depending on different initial configurations of memory patterns, that is, the difference originates in synaptic connections. On the other hands, for the other initial configurations which are slightly different from memory patterns, the system shows different periodic orbits. The difference between chaotic and periodic originates in a time-dependent system parameter.

1. Introduction

Based on the fact that chaotic phenomena have observed in biological systems, chaos would play important roles in information processing of biological systems [1]-[9]. Nara, Davis and their colleagues have presented fruitful results of chaotic wandering behaviors of in a neural network model [2, 3]. Related with complex memory search, they have investigated sensitive responses to memory pattern fragments in chaotic wandering behaviors. Kuroiwa and his colleagues have investigated possibilities in realizing a hierarchical memory search with chaotic wandering behaviors in CNNs [5]-[9]. They have investigated similarities and differences of sensitive responses to memory pattern fragments among three types of CNNs [5]-[8]. In addition, they have investigated how to construct hierarchical memory patterns for the hierarchical memory search [9].

Based on our investigations, however, sensitive and searchless access to target pattern could be realized. On the other hand, fuzzy processing like the person is impossible. A key point to realize the hierarchical memory search with fuzzy processing would employ a multistable chaotic system. One of examples of the multistable chaotic system is Logistic mapping with a time dependent system parameter, which reveals that different attractors can coexist. [10]. In this paper, therefore, we introduce a time dependent system parameter into Adachi & Aihara CNN [4], which could possess two types of initial dependence, originating in (i) synaptic connections and (ii) a time-dependent system parameter. Thus, the system could reveals different responses to almost same inputs, meaning of different initial dependence.

In Adachi & Aihara CNN with a time dependent system parameter, therefore, the purposes of this paper is (i) to show whether two types of initial dependence could coexist or not, and (ii) to investigate chaotic associative memory dynamics for various initial configurations.

2. Model Equations

2.1. Adachi & Aihara CNN

Let us present Adachi & Aihara CNN, briefly [4]. An internal state of each element in Adachi & Aihara CNN consists of two types of internal states, an associative term and a refractoriness term. Thus, the internal state of the ith element at time t, $u_i(t)$, is written by,

$$u_i(t) = \eta_i(t) + \zeta_i(t), \tag{1}$$

where $\eta_i(t)$ represents the associative term and $\zeta_i(t)$ denotes the refractoriness term.

The associative term of the ith element at time t is given by,

$$\eta_i(t+1) = k_\eta \eta_i(t) + \sum_{j=1}^{N} w_{ij} f(u_j(t); \beta_\eta) \tag{2}$$

where k_η and β_η represent a decay parameter and a control parameter of the steepness of the output function $f(\cdots)$ for the associative term, respectively, w_{ij} denotes a synaptic connection from jth element to ith one, and N describes the total number of elements. The $f(u_j(t); \beta_\eta)$ corresponds to an output for the associative term.

The refractoriness term of the ith element at time t is written by,

$$\zeta_i(t+1) = k_\zeta \zeta_i(t) - \alpha f(u_i(t);\beta_\zeta) + A_i, \qquad (3)$$

where k_ζ and β_ζ represent a decay parameter and a control parameter of the steepness of $f(\cdots)$ for the refractoriness term, respectively, α denotes a refractory scaling parameter, and A_i corresponds to a constant bias input.

The output function in this paper applies a sigmoidal one with steepness parameter β defined by,

$$f(x;\beta) = \frac{1}{1+\exp(-\beta x)}. \qquad (4)$$

In this paper, different values of the steepness parameter of β_η and β_ζ are employed to represent different output functions for the associative term and the refractoriness term, respectively. It should be noted that the output of the system corresponds to the output of the the associative term of $\{f(u_i(t);\beta_\eta)\}$.

2.2. Orthogonal learning method

In this paper, synaptic connections of $\{w_{ij}\}$ are calculated according to an orthogonal learning method,

$$w_{ij} = \sum_{a=1}^{P}\sum_{\mu=1}^{L} v_i^{(a)(\mu+1)} (v_j^{(a)(\mu)})^\dagger \qquad (5)$$

where $\boldsymbol{v}^{(a)(\mu)}$ denotes μth memory pattern vector among ath cycle, $(\boldsymbol{v}^{(a)(\mu)})^\dagger$ is a conjugate vector of $\boldsymbol{v}^{(a)(\mu)}$ and P represents the total number of cycle.

The conjugate vector is defined as follows:

$$(v^{(a)(\mu)})^\dagger = \sum_{b=1}^{P}\sum_{\nu=1}^{L} (O^{-1})^{(a)(\mu)(b)(\nu)} v^{(b)(\nu)}, \qquad (6)$$

where O^{-1} is an inverse matrix of the overlap matrix calculated by,

$$O^{(a)(\mu)(b)(\nu)} = \sum_{k=1}^{N} v_k^{(a)(\mu)} v_k^{(b)(\nu)}. \qquad (7)$$

Note that applying the equation (5), P limit cycle memory patterns with a period of L are embedded in the system.

3. Adachi & Aihara CNN with time dependent system parameter

Let us present Adachi & Aihara CNN with time dependent system parameter. The internal state of ith element is given by,

$$u_i(t) = \varepsilon \eta_i(t) + \zeta_i(t), \qquad (8)$$

where ε controls a contribution weight of the associative term and the refractoriness term.

System parameters introducing chaotic associative memory dynamics are appropriate for the time dependent system parameter. Therefore, a candidate are α, β_ζ or A_i. In this paper, we employ β_ζ as a candidate of the the time dependent system parameter. Thus, the refractoriness term is rewritten by,

$$\zeta_i(t+1) = k_\zeta \zeta_i(t) - \alpha f(u_i(t);\beta_\zeta(t)) + A_i, \qquad (9)$$

where $\beta_\zeta(t)$ is given by,

$$\beta_\zeta(t) = \begin{cases} \beta_1 & (t = \text{even}) \\ \beta_2 & (t = \text{odd}). \end{cases} \qquad (10)$$

By switching β_1 and β_2 at time t, the refractoriness term involves initial dependence originating in the time dependent system parameter. On the other hand, the associative term includes initial dependence originating in synaptic connections. Therefore, we expect that two types of initial dependence could coexist by controlling the contribution weight of ε

4. Computer Experiments

4.1. Purposes and Method

In this paper, we investigate whether two types of initial dependence could coexist or not, and chaotic associative memory dynamics for various initial configurations. Therefore, we investigate output sequences for various β_1 and β_2 by direct observation. In addition, we employ visiting measure to characterize chaotic associative memory dynamics.

The visiting measure is evaluated by counting which basins of memory patterns an orbit of the chaotic associative memory dynamics in Adachi & Aihara CNN passes at each time step. In order to determine which basins the orbit passes, we set outputs of Adachi & Aihara CNN at each time step among its updating as an initial configuration of a recurrent neural network model (referred as RNN), and we check which memory patterns RNN converges into within 100 steps. We evaluate the visiting measure with use of T different points of the orbit of the chaotic associative memory dynamics from T_0+1 to T_0+T steps.

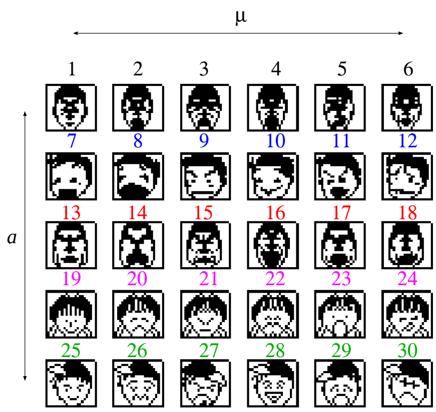

Figure 1: Multi-cycle memory patterns [2].

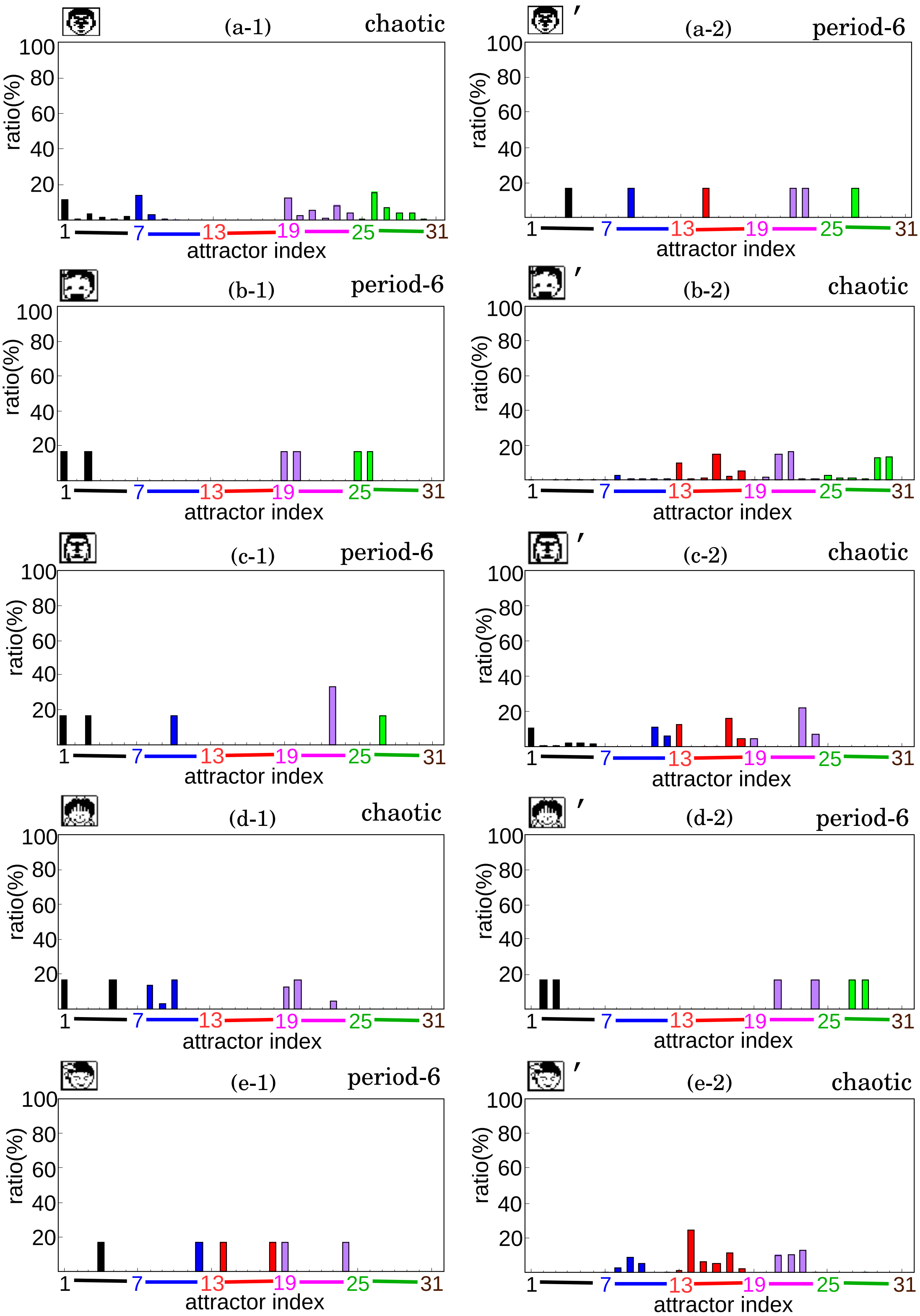

Figure 2: Visiting measure.

Through all the simulations, we set parameters as follows, $\alpha = 3.3$, $A_i = 0.3$, $k_\eta = 0.3$, $k_\zeta = 0.8$, $\varepsilon = 0.5$, and $\beta_\eta = 100.0$. In the calculation of the visiting measure, we choose $T_0 = 20,000$ and $T = 10,000$.

4.2. Results

We observed output sequences and visiting measures with the change of β_1 and β_2 variously. We have succeeded to confirm coexistence of two different initial dependence in output sequences of Adachi & Aihara CNN for various pairs of β_1 and β_2, where chaotic associative memory dynamics and periodic one, periodic one and the other periodic one, or chaotic one and the other chaotic one coexist.

In this paper, we present typical example with $\beta_1 = 10.0$ and $\beta_2 = 38.7$. In Fig. 4.1, the visiting measure is given. A relation of the figure indexes from (a) to (e) represents initial dependence originating in synaptic connections, where different face patterns belonging in different cycles are employed as initial configurations. On the other hand, a relation of the figure indexes of (1) and (2) represents initial dependence originating in time dependent system parameter, where quite similar initial configurations belonging in the same face pattern are applied, that is, the difference is decimal value of the pattern. A distribution of the visiting measure is completely dissimilar among initial conditions, suggesting all the orbits are entirely different. Thus, two types of initial dependence can coexist. In Fig. 4.2, output sequences are given. Even though quite similar initial configurations belonging in the same face pattern are applied, output sequences are different.

5. Conclusions

In this paper, we investigate chaotic associative memory dynamics in Adachi & Aihara CNN with the time-dependent system parameter. The system can possess two types of initial dependence, originating in (i) synaptic connections and (ii) a time-dependent system parameter. Starting from different initial face patterns, the system reveals different dynamics, suggesting the initial dependence originating in synaptic connections. On the other hand, starting from quite similar initial configurations belonging in the same face pattern, the system also shows different dynamics, suggesting the initial dependence originating in the time dependent system parameter. The difference of dynamics where two types of initial dependence coexist or not is a future problem.

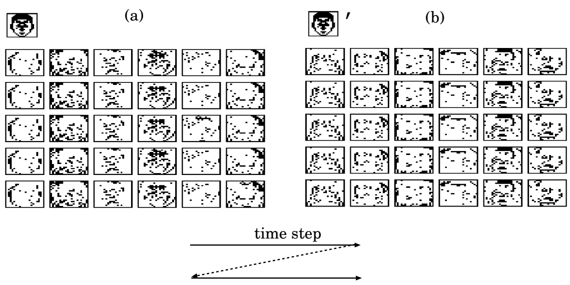

Figure 3: Typical examples of output sequences.

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