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Effect of Pattern Overlap in Delay Feedback Control Method for Chaotic Neural Network

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Abstract— In this paper, we investigate an effect of pattern overlap in a delay feedback control method for a chaotic neural network (CNN) model with chaotic associative memories. In the memory search of recalling process, it is important to investigate the control method which could change the chaotic wandering state to a certain periodic state gradually and softly. As one of control methods, we have proposed a delay feedback control method for a chaotic neural network model. From computer experiments, we succeeded to stabilize chaotic orbits which wander among memory patterns by applying the delay feedback control. Furthermore, it is shown that a stabilized state depends on the overlap between itself and a state of the CNN model at the moment when the control signal was injected.

1. Introduction

Inspired by the investigations of the potentialities of chaotic dynamics in neural network model done by several pioneer workers [1]-[6], the possibility that chaos could play important roles in flexible information processing of biological systems has received much attention.

From their biological and computer experiments in the olfactory bulb, Skarda and Freemann presented an attractive idea in recalling process and learning process[4]: (i) During waiting for input signals, the dynamical response of the olfactory system falls into a highly developed chaotic attractor. (ii) When a certain memorized input is presented, suddenly the highly developed chaotic response shrinks into a weak chaotic attractor or a limit cycle. Thus, in recalling process, chaos could ensure rapid and unbiased access to previously trained patterns. From the theoretical viewpoints, Nara and Davis presented interesting results in complex memory search of neural network model with multi-cyclic memory patterns[6]. Kuroiwa, Nara et al. gave interesting results that chaotic dynamics enable rapid access to the target attractor of a memory fragment [7].

In realizing a complex memory search based on a chaotic wandering state, it is important to control chaotic dynamics gradually and softly. As one of control methods, we have proposed a delay feedback control method for a CNN model with chaotic associative memories[8, 9]. By applying the control signal, we have succeeded to stabilize the model into a certain periodic orbit corresponding to a memory pattern, which became unstable under bifurcation processes. However, we do not know where the system converges into *a priori* in a delay feedback control method. The stabilized state would depend on the overlap between itself and a state of the CNN model at the moment when the control signal was injected. Therefore, the purpose of this paper is to investigate an effect of pattern overlap in the delay feedback control method for a CNN model.

2. CNN Model

Let us present the CNN model proposed by Adachi and Aihara [5], briefly. The updating rule of ith neuron in the CNN model at time t is given as follows:

$$x_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1)) \tag{1}$$

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{N} w_{ij} x_j \tag{2}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i \tag{3}$$

where t represents a discrete time ($t = 0, 1, 2, \cdots$), the $x_i(t)$ is the output of the ith neuron at time t, the internal state variable of $\eta_i(t)$ is a feedback input from the other neurons in the CNN model which represents the effect of the associative memory, the internal state variable of $\zeta_i(t)$ is the refractoriness effect of the neuron at time t, a_i is a constant bias input of the ith neuron, and N is the number of neurons in the network. The parameter α is the refractory scaling of neuron. The parameters k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively.

The function $f(\cdot)$ is referred as the output function. In this paper, we employ sigmoid function with the steepness parameter ϵ as follows:

$$f(x) = \frac{1}{1 + \exp(-x/\epsilon)}. \tag{4}$$

The parameters w_{ij} are synaptic weights to the ith neuron from the jth neuron as given,

$$w_{ij} = \frac{1}{P} \sum_{p=1}^{P} (2\xi_i^p - 1)(2\xi_j^p - 1) \tag{5}$$

where ξ_i^p is the binary patterns stored as basal memory patterns in the CNN model and the ith component of the pth binary pattern takes 0 or 1. The parameter P is the total number of stored memory patterns. In this paper, we employ memory patterns as shown in figure 1. Each pattern consists of 10×10 pixels, indicating $N = 100$. The black boxes represent excited neurons which take 1, and the white boxes represent restraining neurons which take 0.

3. Delay Feedback Control Method in CNN Model

Let us explain a delay feedback control method in a CNN model, briefly[9]. The delay feedback control method is described as follows:

$$x_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1) + F_i(t+1)) \quad (6)$$

$$F_i(t+1) = k_d F_i(t) + \beta x_i(t-\tau) \quad (7)$$

where $F_i(t)$ is a control signal, β is the strength of the control signal, τ represents the delay time, and the parameter k_d is the decay parameters for the control signal. The variables of $\eta_i(t+1)$ and $\zeta_i(t+1)$ are the same ones as equations (2) and (3), respectively.

In a CNN model, the existence of the refractory scaling α and the steepness of the sigmoid function ϵ introduce to chaotic dynamics. In usual, the steepness of the sigmoid function ϵ is fixed. Thus, we could control chaotic dynamics by adjusting the effect of the refractory scaling α through the delay feedback control.

4. Computer Experiments

4.1. Parameter Dependence of Controlled Dynamics

In the delay feedback control method, the parameters are β, τ and k_d. Especially, the parameter dependence on β and k_d is important. If we can identify the system *a priori*, it is easy to determine the value of the parameters. In general, however, we could not know them *a priori*. Therefore, we investigate the parameter dependence of the controlled system dynamics on β and k_d.

In the computer experiment, we fix the delay time, $\tau = 1$, and change the value of the strength of the control signal, β, and the decay parameters for the control signal, k_d, variously. In the CNN model, we choose parameter value as follows, $k_r = 0.8$, $k_f = 0.8$, $a_i = 2$ and $\epsilon = 0.1$. We set the refractory scaling parameter to be $\alpha = 9.0$, where the CNN model reveals chaotic wandering states. All the simulations, we employ the same value of parameters. In order to avoid transient states, at 1000 steps after control signal has been injected, we investigate how many time period the controlled state takes.

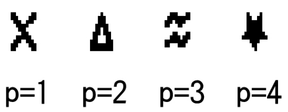

Figure 1: Memory pattern[5].

The result is given in figure 2. In the red color region of parameters of β and k_d, the controlled CNN model shows different patterns from memory patterns as shown in figure 1 and we could not find out periodic responses. On the other hand, in the green color region, the output of the controlled CNN model is one of memory patterns. The controllable parameter region is sufficiently large. In other words, we can control the CNN model without any knowledge about the value of system parameters, *a priori*. Thus, the delay feedback control method is practical in stabilizing the CNN into a certain memory pattern from chaotic wandering states.

4.2. Relationship between a Stabilized Pattern and a State of CNN model

We investigate a relationship between a stabilized pattern and a state of the CNN model at the moment when the control signal was injected. We calculate the Hamming distance between each memory pattern and the state of the CNN model, and evaluate how many times the controlled CNN model converges into the memory pattern whose attractor basins chaotic wandering state passes through at the moment. We perform the evaluation with the change of injecting time steps of the control signal from 10,000 steps to 20,000 steps.

Results are given in figure 3. As the Hamming distance becomes larger, the ratio becomes larger, indicating that the controlled CNN model converges into the memory pattern whose attractor basins chaotic wandering state passes through at the moment. On the other hand, as the Hamming distance becomes smaller, the controlled CNN model con-

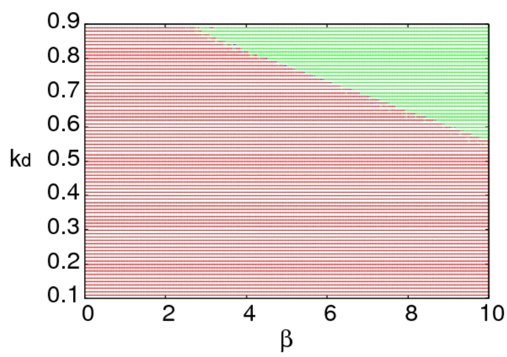

Figure 2: Parameter dependence of the controlled dynamics on β and k_d. Green color represents that the CNN model is stabilized into a certain memory pattern. Red color represents that the CNN is in chaotic wandering states.

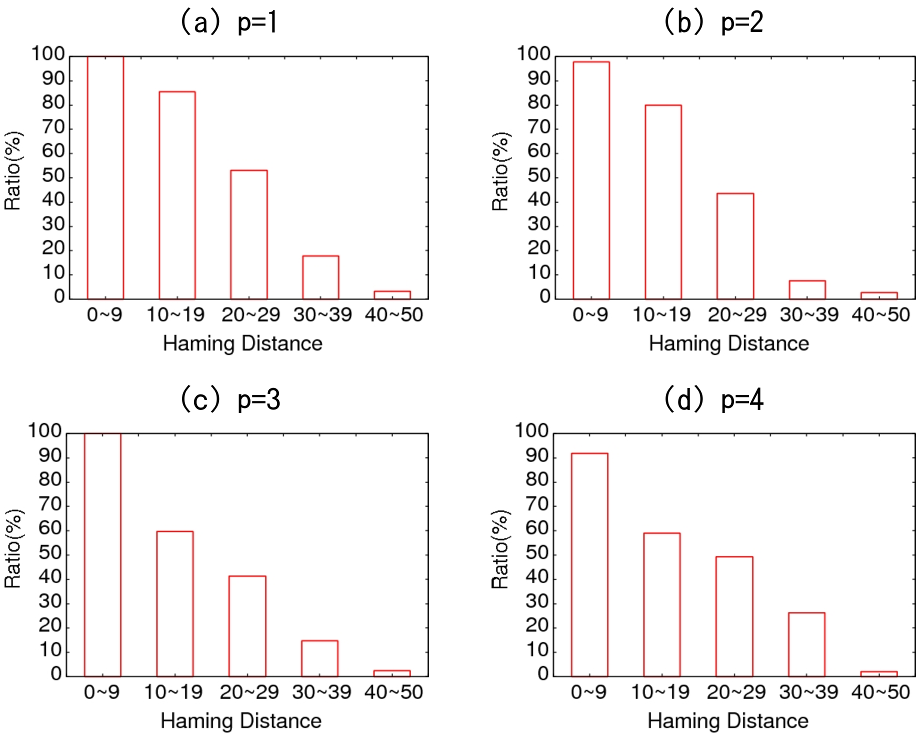

Figure 3: Relationship between a stabilized pattern and a state of the CNN model.

verges into a different memory pattern. At the next subsection, in the case that the converged state is different memory pattern, we investigate where the controlled CNN model converges.

4.3. Effect of Pattern Overlap

In order to investigate the problem, we focus on the overlap among memory patterns and dependence of converged patterns on the Hamming distance. In this paper, we evaluate overlap between two patterns of p and q as follows:

$$o^{pq} = \frac{1}{N}\sum_{i=1}^{N}(2\xi_i^p - 1)(2\xi_i^q - 1) \quad (8)$$

where, $\{\xi_i^p\}$ and $\{\xi_i^p\}$ represent pth and qth memory patterns, respectively. Results are given in table 1. For the 1st pattern, the 3rd pattern takes the largest overlap. For the 2nd, the 4th takes the largest overlap.

patterns	1-2	1-3	1-4	2-3	2-4	3-4
overlap	0.02	0.14	0.04	-0.04	0.10	0.10

Table 1: Overlap among memory patterns.

At last, we investigate the dependence of converged patterns on the Hamming distance for the 2nd pattern. Results are given in figure 4, where we evaluate how many times the controlled CNN model converges to each memory pattern under the situation that the chaotic wandering state passes through the attractor basins of the 2nd memory pattern at the moment when the control signal was injected, corresponding to the case of figure 3(b). In the case of the largest Hamming distance of 0 - 9 in figure 4(a), the controlled CNN model almost converges to the same attractor of the 2nd memory pattern, or rarely converges to the 4th memory pattern with the largest overlap with 2nd memory pattern. As the Hamming distance decreases, the ratio that the controlled CNN model converges to the 2nd memory pattern also decreases. On the other hand, the ratio to 4th pattern, to 3rd pattern, or 1st pattern gradually increases. The increasing tendency reflects the degree of the overlap. Thus, the converged pattern depends on the overlap among memory patterns and where attractor basins the chaotic wandering state passes through at the moment when the control signal was injected, that is, is the chaotic wandering state close to the attractor or distant?

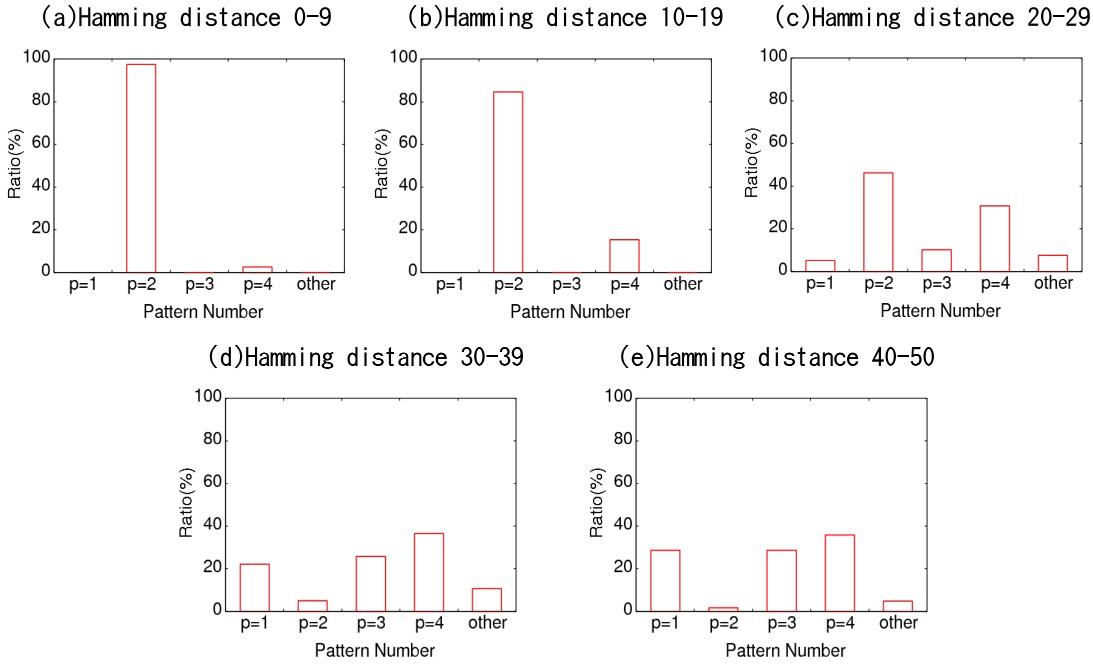

Figure 4: Dependence of Converged patterns on the Hamming distance for the memory pattern of $p = 2$.

5. Conclusions

In this paper, we investigate an effect of pattern overlap in a delay feedback control method for the CNN model. Results are as follows: As the chaotic wandering state is close to a certain attractor of the memory pattern, the controlled CNN model converges into the memory pattern. On the other hand, as the chaotic wandering state is close to the boundary of the memory pattern, the converged pattern depends on the overlap among memory patterns. Thus, if we construct hierarchical memory patterns by controlling overlaps, the delay feedback control method enable us hierarchical memory pattern search by means of chaotic wandering states.

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