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Self-Organization of temporal network

Takaaki Aoki

Faculty of Education, Kagawa University
1-1 Saiwai-cho, Takamatsu, Kagawa 760–8521, Japan
Email: aoki@ed.kagawa-u.ac.jp

Abstract—We study a model of adaptive temporal networks that are regulated by human activity and vice versa. Thereby we seek to develop a unifying understanding of the mechanisms governing human social dynamics. We analyze the model using a master equation approach and show that the temporal and structural heterogeneities seen in real-world networks can emerge spontaneously from completely homogenous initial conditions. This theoretically tractable model will promote further studies to understand how our society is organized by the interplay between social relations and human activity.

1. Introduction

Human social behavior is a complex phenomenon that depends both on single individuals and on the interactions between them. In daily life, interactions between people create contact patterns that can be mathematically represented by networks, i.e. a set of nodes, corresponding to the people, connected by links, representing the contacts between the respective individuals [1]. In network terminology, the number of contacts of a given person is called the degree k of a node and thus the degree distribution p_k is the probability distribution that a randomly chosen node has degree k. Real-life networks have a high level of heterogeneity in the number of contacts per node. The empirical degree distributions are typically approximated by power-law [2, 3]. Contact patterns however are not static and more realistically nodes alternate between active and inactive states [4]. Nodes that have many contacts for example in one hour may be completely inactive in the following hours [5]. Recent studies suggest that this temporal activity, quantified by the inter-event intervals (IEIs), i.e. the time between two subsequent node activations, is highly heterogeneous and generates bursts of activity [6, 7, 4].

While several models have been proposed to explain the heterogeneity in the degree distribution [8, 9, 10, 11], a few studies have focused on modeling the burstiness of the temporal activity. In particular, Barabási proposed a priority-based model in which nodes prioritize tasks [6, 7]. Nodes first execute the high-priority tasks, i.e. these tasks are executed within a short time, while low-priority tasks have to wait longer times before leaving the queue. Every execution creates an activation of a node. Other models however are based on inhomogeneous Poisson processes on each node modulated by (daily and weekly) cycles of human activity [12, 13]. Combined, these processes generate the bursting or quiescent activities due to the changes of the activation rates of each node.

The shortcoming of the previous models is that they consider the network structure and node activity independently, and thus miss the fact that the microdynamics regulates the evolution of the network even if macroscopic network quantities, such as the degree distribution, are conserved [14, 5]. On the other hand, adaptive networks fill in this gap and combine the dynamics on and of the network in a single model [15, 16].

In this letter, we propose a model of temporal networks where the human activity and their connections are adaptively regulated by past interactions, as illustrated in Fig. 1(a). We analyze the master equation of the model using generating functions and show that, under suitable conditions, heterogeneous structural and temporal patterns spontaneously emerge from completely homogeneous initial conditions.

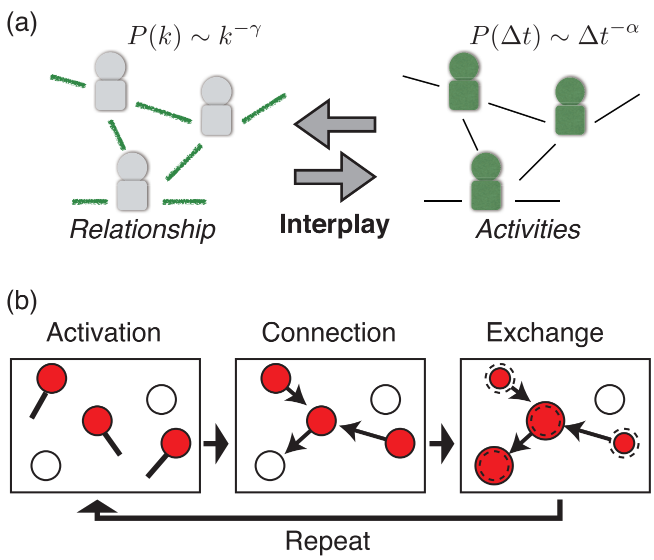

Figure 1: (a) Structural and temporal heterogeneities among people are interdependent. (b) Illustration of the interaction-regulated stochastic contact model. Within one time step, (i) nodes become activate, (ii) make random connections, (iii) exchange resources, and finally (iv) break down the links.

2. Model

The model consists in three steps: (i) activation of nodes, (ii) formation of pairs, and (iii) exchange of resources (Fig. 1(b)). We initially assign an internal state $x_i(t)$ for node i. The "resource" given by $x_i(t)$ triggers the activity of node i with a probability proportional to $x_i(t)$, and thus represents its willingness to contact other nodes in the network. At each time step t, a number of nodes N_A becomes active. Each active node at time t forms a directed link to another node in the network. For each link, the target node is uniformly selected from all nodes in the network with probability κ (contact to anyone in the network), and selected among the active nodes with probability $1-\kappa$ (contact to another active person, who has something to share). Therefore, links are generated only between the active nodes if $\kappa = 0$ [17], and with any network node if $\kappa = 1$ [18]. We thus extend previous models by considering intermediate situations by simply controlling the parameter κ.

The internal states are updated by transferring a fixed amount of resource D via each pair of directed links.

$$x_i(t+1) - x_i(t) = D\left[\sum_j a_{ij}(t) - \sum_j a_{ji}(t)\right], \qquad (1)$$

where $a_{ij}(t)$ is adjacency matrix at time t, i.e. $a_{ij}(t) = 1$ if there is a link between nodes i and j and 0 otherwise. This resource exchange means that the willingness to contact someone will decrease among the persons who made a contact to others, and it will increase among the persons who received the contacts. Finally, the links are broken and the entire routine is repeated in the next time step. We assume the initial state of all nodes to be completely identical and set $x_i(0) = 1$.

3. Results

To derive the master equation of the model, let us consider the node density $u_n(t)$ whose state $x_i(t) = nD$. We assume that the number of nodes N and the active nodes N_A, are large enough, and the total fraction of the active nodes N_A/N is fixed. The active fraction a_n of $u_n(t)$ is proportional to its resource amount $nDu_n(t)$. Therefore, $a_n u_n(t) = \frac{N_A}{N}\frac{nDu_n(t)}{\sum_n nDu_n(t)} = \frac{N_A}{N} nDu_n(t)$, where the total resource $\sum_n nDu_n(t)$ is preserved to 1. The active nodes are randomly connected to all nodes by $N_A\kappa$ links. In other words, all nodes have a chance of getting incoming links with $1/N(\equiv \rho_1)$ in $N_A\kappa(\equiv M1)$ trials. Moreover, the active nodes have a chance of getting incoming links with $1/N_A(\equiv \rho_2)$ in $N_A(1-\kappa)(\equiv M2)$ trials, while they have an outgoing link. The resource is transferred by a unit of D through the directed links. Using the Binomial probability $B(m,\rho,M)$ of m success with probability ρ in M trials, the master equation is:

$$\begin{aligned}\frac{du_n(t)}{dt} &= A(-1)a_{n+1}u_{n+1}(t) \\ &\quad - C_1 a_n u_n(t) - C_2 \bar{a}_n u_n(t) \\ &\quad + \sum_{m=1}^{N_A} a_{n-m}u_{n-m}(t)A(m) \\ &\quad + \sum_{m=1}^{N_A\kappa} \bar{a}_{n-m}u_{n-m}(t)B(m,\rho_1,M_1), \end{aligned} \qquad (2)$$

where $\bar{a}_n(=1-a_n)$ is the inactive fraction of $u_n(t)$, $A(m) = \sum_{m_1+m_2=m+1} B(m_1,\rho_1,M_1)B(m_2,\rho_2,M_2)$, and $C_1 = 1-A(0)$, $C_2 = \sum_{m=1}^{N_A\kappa} B(m_1,\rho_1,N_A\kappa)$.

Using a generating function $Q(t,x) = \sum_n u_n(t)x^n$, we finally we finally obtain the mean μ_x and the variance σ_x^2 of the stationary resource distribution $P(x)$ [1]:

$$\mu_x = 1, \qquad (3)$$

$$\sigma_x^2 = \frac{D}{2\kappa}\left[1 + 2\left(\frac{1}{D} - 1\right)\kappa + \left(1 - \frac{N_A}{N}\right)\kappa^2\right] + 1 - \frac{1}{D}. \qquad (4)$$

We find that the variance $\sigma_x^2 \to \infty$ as $\kappa \to 0$, while the mean μ_x is fixed.

By tuning the parameter κ, this model is able to reproduce different structural and temporal patterns (Fig. 2). If $\kappa = 0$, the active individuals only contact those individuals that are also active, a dynamics that eventually lead to a closed group of a few nodes monopolizing the resource. Nodes outside this group however are left without any resources. In this situation, the diameter of the aggregated contact network (formed by all links collected during $T_s = 10^4$ time steps) shrinks and the temporal patterns of the nodes inside this group exhibit a Poisson-like dynamics, i.e. exponential inter-event times (Fig. 2(b)). In contrast, for larger κ, the active individuals may increasingly link to any uniformly chosen node. This situation leads to a homogeneous distribution of resources, which generates a Gaussian-like in-degree distribution (Fig. 2(e)) and an exponential (single-node) IEI distribution (Fig. 2(f)), the later a result of the quasi-homogenous Poisson process.

In the intermediate case, in which active individuals mainly contact other active individuals and occasionally link to those inactive, we observe the emergence of highly heterogeneous structural (Fig. 2(c)) and temporal patterns (Fig. 2(d)). The contact network has a power-law in-degree distribution with an exponential cutoff, similar to the resource distribution (Fig. 2(c)).

4. Conclusion

In this paper, we proposed a model of contact networks where the human dynamics are adaptively regulated by past

[1] See the reference [19] for details of the calculations and the related results.

interaction and exchange of resources between the individuals in the network. We analyzed the master equation of the model and found that structural and temporal heterogeneities, as observed in real-world temporal networks, can spontaneously emerge without any ad-hoc assumption on heterogeneity but by simply regulating the choice of contacts of the noes. In the model, these two heterogeneities are observed in an intermediate regime in which the active individuals typically communicate with other active people (rich-club) but occasionally connect to anyone in the network.

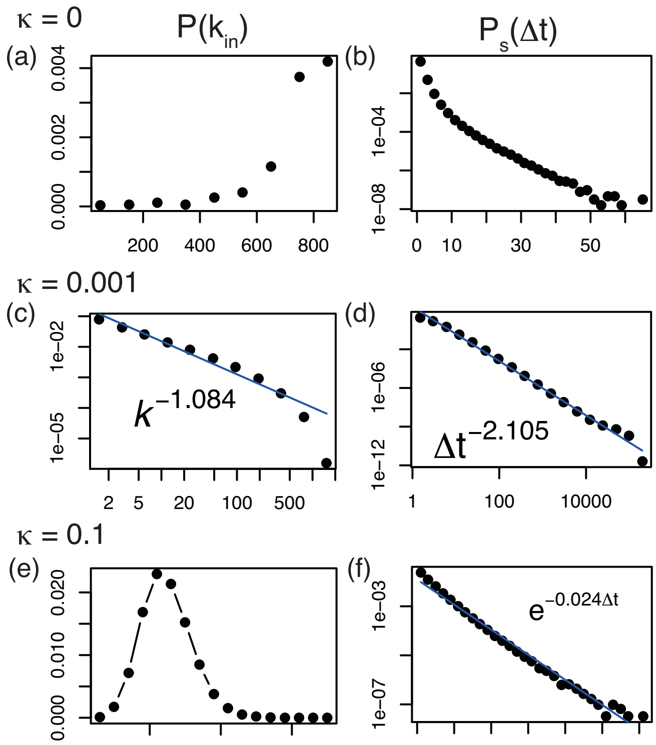

Figure 2: The degree distribution $P(k)$ and single-node IEIs distribution $P_s(\Delta t)$ for (a,b) $\kappa = 0$ ($P_s(\Delta t)$ is plotted in semi-log graph), (c,d) $\kappa = 0.001$ (both graphs are log-log), and (e,f) $\kappa = 0.1$ ($P_s(\Delta t)$ is semi-log), with $N_A = 1024$, $N = 2^{15}$, and $D = 0.01$.

Acknowledgments

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