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# Memory State Evaluation of Spatio-Temporal Contextual Learning Memory Network Based on Output Spike Rate

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**Abstract**— The spatio-temporal contextual learning memory network model was proposed as a hippocampal memory model. This model has spatio-temporal learning rule synapses and Hebbian learning rule synapses, and can embed spatio-temporal information into the synaptic weight space in the network as a memory. However, no method has been proposed to evaluate embedded memory by using the output of the network. Therefore, we proposed a method to confirm the embedding of memory based on the output spike rate. In this study, we construct an extended spatio-temporal contextual learning memory network using two-variable spiking neurons. We evaluate memory state from output spike rates proposing as an efficient measure in a small-scale extended spatio-temporal contextual learning memory network.

## 1. Introduction

A hippocampal learning model was proposed [1], which is expected to be applied to edge devices. Originally, this model is for a continuous-time neural networks with spiking neurons, but due to calculation time issues, a discrete-time model has been used. To fully use the intrinsic performance in continuous-time regime, we have proposed an extended spatio-temporal contextual learning memory network (eSTCLMN) model [2] using spiking neurons with continuous-time. The model also takes mixed analog/digital circuit implementations into account to solve the calculation time problem. In the eSTCLMN model, a leaky integrate-and-fire (LIF) neuron model [3] was employed, which is simple and suitable for circuit implementation. However, it can only reproduce simple periodic spike trains.

The eSTCLMN model can embed the spatio-temporal information into the synaptic weight space in the network as a memory [4], but it is difficult to readout this memory on the output space [5]. Therefore, we proposed a method to use the output spike rate of the neurons in the output layer to evaluate the memory state embedded in the network [6].

In this study, we re-describe the eSTCLMN model employing a two-variable spiking neuron (TSN) [7], and verify its effectiveness through simple numerical experiments. TSN can reproduce a variety of spiking characteristics and has low power consumption. The TSN can generate chaotic spikes as an alternative to random input spikes, and convert spatio-temporal information into spike trains in the input layer of the network. The TSN can also be used like the LIF neuron in the output layer of the network.

Furthermore, we define an output spike rate (OSR), and confirm that the spatio-temporal information is not only memorized on the synaptic weight space in the network, but also can be read from the output spike rates on the output space using OSR.

## 2. Extended Spatio-Temporal Contextual Learning Memory Network Model

### 2.1. Network Model

We use a small eSTCLMN as shown in Fig. 1 to perform simple numerical experiments with TSNs. The eSTCLMN in the figure has two TSNs and one TSN in the input and output layers, respectively. In Fig. 1, blue double-lined rectangles indicate spatio-temporal learning rule (STLR) synapses, an orange rectangle indicates Hebbian learning rule (HBLR) synapse, and white circles indicate TSNs.

The input $u_1(t)$ is the sum of outputs from all its own synapses at time $t$, and defined by the following equation.

$$u_1(t) = \alpha^{\mathrm{S}} \sum_{j=1}^{2} w^{\mathrm{S}}_{1,j}(t) d^{\mathrm{S}}_{1,j}(t) + \alpha^{\mathrm{H}} w^{\mathrm{H}}_{1,1}(t) d^{\mathrm{H}}_{1,1}(t), \qquad (1)$$

where $\alpha^{\mathrm{S}}$ ($\geq 0$) and $\alpha^{\mathrm{H}}$ ($\geq 0$) are the gain coefficients for STLR and HBLR synapses, respectively. In some experiments [2, 4], it is preferable to set $\alpha^{\mathrm{S}}$ larger than $\alpha^{\mathrm{H}}$, therefore, we use $\alpha^{\mathrm{S}} = 0.95$ and $\alpha^{\mathrm{H}} = 0.05$ in this study.

As shown in Fig. 1, $w^{\mathrm{S}}_{1,j}(t)$ is the weight values of the STLR synapses between the output layer neuron and the $j$-th input layer neuron, and $w^{\mathrm{H}}_{1,1}(t)$ is the weight value of the

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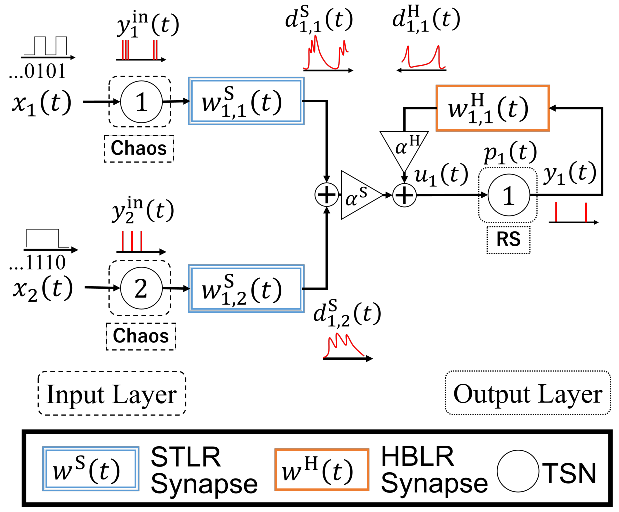

Figure 1: The small eSTCLMN [2] used in the simple numerical experiments (STLR synapses $M = 2$, and HBLR synapse $N = 1$).

HBLR synapse (self-feedback coupling). $d_{1,j}^{\mathrm{S}}(t)$ and $d_{1,1}^{\mathrm{H}}(t)$ are the outputs of the $j$-th STLR synapse and the HBLR synapse of the output neuron, respectively.

### 2.2. Two-variable Spiking Neuron Model

The eSTCLMN model conventionally used LIF neurons [2]. However, we instead introduce TSNs in the eSTCLMN.

Figure 2 shows the TSN circuit [7]. In the figure, the internal state voltage $p_i(t)$, and the recovery variable voltage $v_i(t)$ of the $i$-th TSN at time $t$, which are respectively given by

$$C_p \frac{\mathrm{d}p_i(t)}{\mathrm{d}t} = g_1(p_i(t)) - g_2(p_{\mathrm{leak}}) - g_3(v_i(t)) + u_i(t), \quad (2)$$

$$C_v \frac{\mathrm{d}v_i(t)}{\mathrm{d}t} = -g_4(v_i(t), a) + g_5(y_i(t), b), \quad (3)$$

where $C_p$ and $C_v$ are the values of capacitance that store the internal state charge and the recovery variable charge, respectively. $p_{\mathrm{leak}}$ is the voltage to adjust for charge leakage, and $u_i(t)$ is the input current. In Eqs. (2) and (3), $g_1(\cdot)$, $g_2(\cdot)$, $g_3(\cdot)$, $g_4(\cdot)$, and $g_5(\cdot)$ are conductance, which are functions that approximate circuit characteristics. In Eq. (3), $a$ is the voltage for adjusting the time constant of the recovery variable voltage $v_i(t)$, and $b$ is the voltage for controlling the current to the capacitance $C_v$ during neuron firing.

The output voltage $y_i(t)$ of $i$-th TSN is given by

$$y_i(t) = \frac{V_{\mathrm{DD}}}{1 + \exp\left(-(p(t) - p_{\mathrm{th}})/\varepsilon_{\mathrm{P}}\right)}, \quad (4)$$

where $V_{\mathrm{DD}}$ is the supply voltage, $p_{\mathrm{th}}$ is the threshold voltage, and $\varepsilon_{\mathrm{P}}$ is gain of the comparator at the output. When the neuron fires ($p_i(t) \geq p_{\mathrm{th}}$), the internal state voltage is reset as $p_i(t) = c$, where $c$ is the reset voltage.

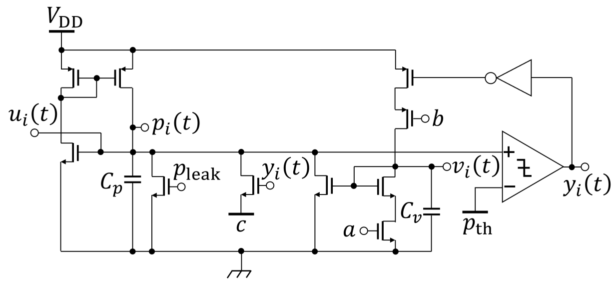

Figure 2: The schematic of the two-variable spiking neuron model [7].

### 2.3. STLR and HBLR Synapse Models

STLR synapses update their weight values, when signals are input to themselves, based on their coincidence factor with other synapses [1]. The STLR synapses shown in Fig. 1 are updated with the following equations.

$$\frac{\mathrm{d}w_{1,1}^{\mathrm{S}}(t)}{\mathrm{d}t} = \eta^{\mathrm{S}} h\left(p_1(t) - w_{1,2}^{\mathrm{S}}(t) d_{1,2}^{\mathrm{S}}(t)\right), \quad (5)$$

$$\frac{\mathrm{d}w_{1,2}^{\mathrm{S}}(t)}{\mathrm{d}t} = \eta^{\mathrm{S}} h\left(p_1(t) - w_{1,1}^{\mathrm{S}}(t) d_{1,1}^{\mathrm{S}}(t)\right), \quad (6)$$

where $\eta^{\mathrm{S}}$ ($\geq 0$) is the learning coefficient, and $h(\cdot)$ is a function that switches between long-term potentiation and long-term depression [2].

Similar to the general Hebbian learning rule [8], the HBLR synapses update the weight values according to the timing of the input and output as

$$\frac{\mathrm{d}w_{1,1}^{\mathrm{H}}(t)}{\mathrm{d}t} = \eta^{\mathrm{H}} d_{1,1}^{\mathrm{H}}(t) y_1(t), \quad (7)$$

where $\eta^{\mathrm{H}}$ ($\geq 0$) is the learning coefficient.

## 3. Simple Numerical Experiments

In this section, we perform simple numerical experiments on the eSTCLMN with TSNs to evaluate memory state based on the OSR.

### 3.1. Chaos and RS Mode of TSN

As shown in Fig. 1, in the input layer (the neurons enclosed by the dashed lines), TSNs are chaotic (Chaos mode) generating chaotic spike trains instead of random spike trains. In the Chaos mode, the TSN can convert the spatio-temporal information into chaotic spike trains. In this mode, the circuit parameters of the TSN are $a$ = 0.15 [V], $b$ = 0.11 [V], and $c$ = 0.115 [V], respectively.

On the other hand, in the output layer (the neuron enclosed by the dotted line), TSN works as a regular spike [9] neuron (RS mode) which generates periodic spike trains. In the RS mode, the TSN behaves like the LIF neuron. For this mode, the circuit parameters of the TSN are $a$ = 0.2 [V], $b$ = 0.1 [V], and $c$ = 0.05 [V], respectively.

### 3.2. Encoding of Input Spatio-Temporal Information

The small eSTCLMN in Fig. 1 learns a sequence consisting of 2-bit vectors as the spatio-temporal information. The input 2-bit vectors, $A_1$, $A_2$, $A_3$, and $A_4$ used in this study are $(x_1(t), x_2(t)) = (0,0),\ (0,1),\ (1,0)$, and $(1,1)$, respectively. Here, $x_1(t)$ and $x_2(t)$ are the inputs to the neurons in the input layer of the network, as shown in Fig. 1.

In addition, "0" and "1" are implemented in the circuit in Fig. 2 as $u_i(t) = 0$ [nA] and $u_i(t) = 80$ [nA], respectively.

The $l$-th learning sequence is denoted as $\mathscr{P}_l = (P(1),P(2),P(3),P(4))$. For example, $\mathscr{P}_1 = (A_1,A_2,A_3,A_4)$, $\mathscr{P}_2 = (A_1,A_2,A_4,A_3), \cdots, \mathscr{P}_{24} = (A_4,A_3,A_2,A_1)$. The time duration of one input vector $P(k)$ $(k \in \mathbb{N}, 1 \leq k \leq 4)$ is $T$ (6 [ms] in this study).

Figure 3 shows an example of converting spatio-temporal information into spike trains in the case of $\mathscr{P}_1$. In Fig. 3, $y_1^{\rm in}(t)$ and $y_2^{\rm in}(t)$ represent the outputs of the TSN in the input layer, respectively, as shown in Fig. 1. In this study, the total time length of the learning sequence of $\mathscr{P}_l$ is $4T = 24$ [ms].

### 3.3. Numerical Experimental Results

Figure 4 shows the histogram of the weight values of the STLR and HBLR synapses after learning by using the 24 different sequences $\mathscr{P}_l$. In the figure, the gray bars show the initial weight values of the STLR synapses ($w_{1,1}^{\rm S}(0) = 0.87$ and $w_{1,2}^{\rm S}(0) = 0.97$), and the blue and orange bars show the frequency of STLR and HBLR synapse weight values after learning ($t = 24$ [ms]) for 24 different sequences, respectively.

From this result, we confirm that the distribution of the weight values became wider as a result of learning. This implies that spatio-temporal information is separated, and stored in the weight space mostly due to the STLR.

### 3.4. Memory State Evaluation Based on Output Spike Rate

To investigate the memory state of the eSTCLMN in detail, we input a test sequence $\mathscr{P}_{\rm test}$ while the synaptic weight values are fixed after learning. $\mathscr{P}_{\rm test}$ is a random spike sequence whose duration is $4T$.

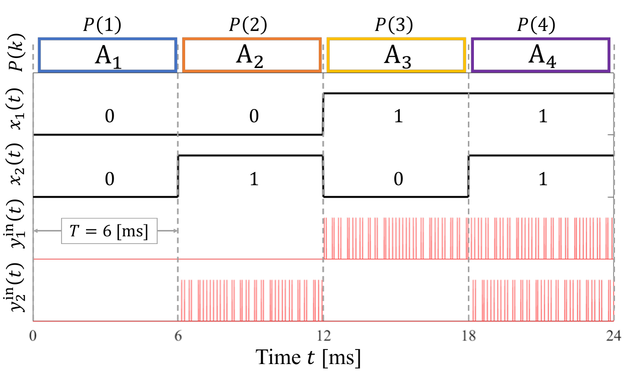


Figure 3: An example of the conversion of spatio-temporal information into spike trains in the input layer.

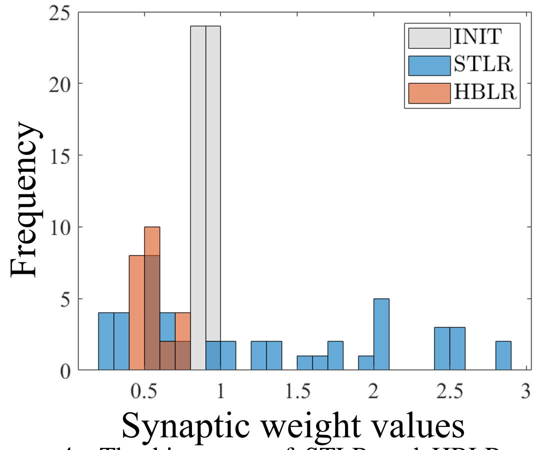


Figure 4: The histogram of STLR and HBLR synaptic weight values after learning.

The OSR of the neuron in the output layer is introduced as

$$\mathrm{OSR} = \frac{1}{\mathrm{IS}} \cdot \frac{1}{3T} \int_T^{4T} y_1(t_{\rm test})\, \mathrm{d}t_{\rm test}, \qquad (8)$$

where $t_{\rm test}$ is the time for the test sequence, and IS is the average frequency of the input spike train, which is 10 [kHz] in the simulations. To calculate OSR, $y_1(t_{\rm test})$ for $0 \leq t_{\rm test} < T$, i.e., 6 [ms] is excluded from the calculation to initialize the internal state $p_1(t_{\rm test})$. As shown in Eq. (8), the output spike rate is averaged over the time length of $3T$.

The histogram of OSRs, which were obtained after each $\mathscr{P}_l$ read by $\mathscr{P}_{\rm test}$, is shown in Fig. 5. The histogram are colored according to the first vector $P(1)$ in $\mathscr{P}_l$, such that $\mathscr{P}_1$ to $\mathscr{P}_6$ are blue ($P(1) = A_1$), $\mathscr{P}_7$ to $\mathscr{P}_{12}$ are orange ($P(1) = A_2$), $\mathscr{P}_{13}$ to $\mathscr{P}_{18}$ are yellow ($P(1) = A_3$), and $\mathscr{P}_{19}$ to $\mathscr{P}_{24}$ are purple ($P(1) = A_4$).

Figure 5 confirms that learning sequences with the common $P(1)$ show similar OSR values. In addition, it could be shown that the distribution characteristics of OSR are resembled to that of the synaptic weights values shown in Fig. 4.

For more detail analyses in the case of $P(1) = A_1$, $P(2)$ and $P(3)$ are also colored with different colors as in Fig. 6(a) and Figs. 6(b)–(d), respectively. In Fig. 6(a), $\mathscr{P}_1$ and $\mathscr{P}_2$ are orange ($P(2) = A_2$), $\mathscr{P}_3$ and $\mathscr{P}_4$ are yellow ($P(2) = A_3$), and $\mathscr{P}_5$ and $\mathscr{P}_6$ are purple ($P(2) = A_4$). In Figs. 6(b)–(d), $\mathscr{P}_3$ and $\mathscr{P}_5$ are orange ($P(3) = A_2$), $\mathscr{P}_1$ and $\mathscr{P}_6$ are yellow ($P(3) = A_3$), and $\mathscr{P}_2$ and $\mathscr{P}_4$ are purple ($P(3) = A_4$).

These results show that OSRs can give different values depending on the temporal context in the learned sequence $\mathscr{P}_l$ when the $\mathscr{P}_{\rm test}$ is applied. For example, if the OSR $\approx 11$ [%], we could consider that learned sequence is $\mathscr{P}_2 = (A_1, A_2, A_4, A_3)$. In conclusion, by $\mathscr{P}_{\rm test}$, it would be possible to retrieve the spatio-temporal memory information using OSRs.

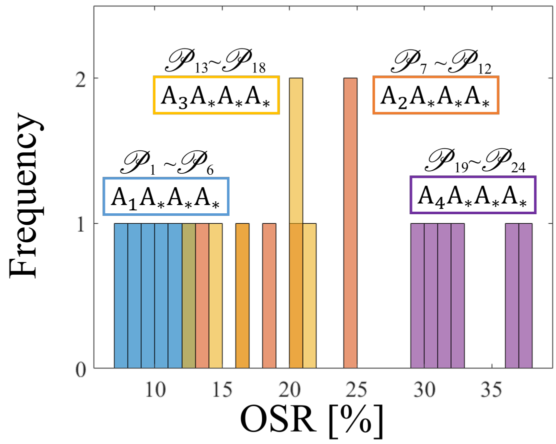


Figure 5: The histogram of OSRs when the test sequence $\mathscr{P}_{\text{test}}$ is applied. Each first input vector $P(1)$ in $\mathscr{P}_l$ is colored as; $A_1$: blue, $A_2$: orange, $A_3$: yellow, $A_4$: purple. $A_*$ represents an arbitrary vector.

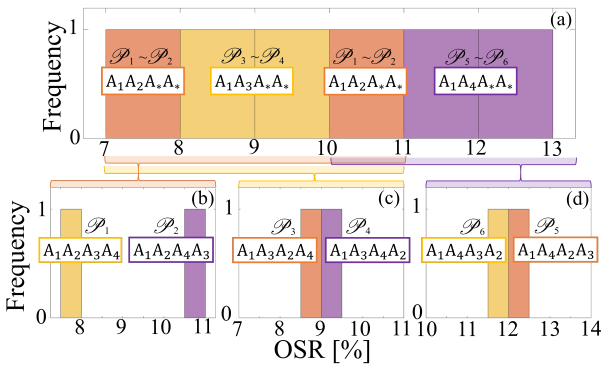


Figure 6: The histogram depicted for learning sequences $\mathscr{P}_1$ to $\mathscr{P}_6$ of which $P(1) = A_1$ in Fig. 5. (a) The bars are colored according to $P(2)$ in $\mathscr{P}_l$. Results (b) with learned sequence of $(A_1, A_2, A_*, A_*)$, (c) with learned sequence of $(A_1, A_3, A_*, A_*)$, and (d) with learned sequence of $(A_1, A_4, A_*, A_*)$.

## 4. Conclusion

We proposed the method to evaluate the memory state of the eSTCLMN, in which spatio-temporal information is embedded as a memory, by employing OSR as a novel measure. As a result, similar distributional characteristics of OSRs to that of the synaptic weight values after learning were confirmed.

In the previous studies, it was suggested that the spatio-temporal information is embedded in the synaptic weight space. However, how to retrieve the memory is not clear yet. This study gave one method to read out some memory information using OSRs.

In the future, we will implement hardware of the eST-CLMN with TSNs.


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