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Periodic fluctuation observed in an ensemble firing of neurons in a cortical network model of correlated bidirectional connections

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Abstract—In cortical networks, it has been known that neurons generate self-sustained low-frequency firings in the absence of sensory stimuli. The so-called "spontaneous activity" typically gives rise to irregular and asynchronous firing among cortical neurons. Despite numerous theoretical attempts, the mechanism underlying the spontaneous activity remained unclear. A breakthrough has been recently made by Teramae *et al.* who proposed a neuronal network model, in which EPSPs (Excitatory postsynaptic potentials) obey a lognormal distribution. The model implied that the lognormal distribution of EPSPs in cortical networks was the key to sustain low-frequency firing of the neurons. Whereas their model mainly focused on the lognormal distribution, correlation of the EPSPs observed between bidirectionally coupled neurons has been disregarded. In our previous study, we added correlated EPSPs to the lognormal network and showed that the correlated EPSPs generated synchronized firing among the neurons. Extremely high firing frequencies were also observed in a group of neurons. The aim of this paper is to study further details of the neuronal dynamics in a network of correlated EPSPs. We show that periodic fluctuations exist in short-term population firing of the neurons. Such periodic fluctuations seem to arise from periodic spikings of neurons with high firing frequencies.

1. Introduction

The brain is composed of a huge number of neurons that communicate with each other by sending or receiving spikes. In the cortex, the so-called "spontaneous activity" of neurons that generate ongoing spikes even without external stimuli has been observed in both *in vitro* and *in vivo* experiments [1]. The dynamics of spontaneous activity are characterized typically by low-frequency [2], irregular [3], and asynchronous firings of the neurons [4]. The underlying mechanism of the spontaneous activity, however, remained unclear. To elucidate the origin of spontaneous activity, several models have been proposed, in which independent noise was generated internally within neurons or synapses [5]. Although such noise could sustain irregular firings and propagate various information in the neural network, the origin of the internal noise has not been well explained. To resolve this remaining issue, an alternative model composed of a large network of spiking neurons with conductance-based synapses has been studied [6]. That model, however, generated unexpectedly high firing frequencies and moreover weak input stimuli were needed to sustain the spontaneous activity. In contrast to the former studies, Teramae *et al.* [7] recently proposed a mathematical model focusing on a lognormal distribution of the excitatory postsynaptic potentials (EPSPs), which have been measured experimentally in local cortical circuit [8]. Without any internal noise, the model could successfully reproduce main features of the spontaneous firing activity including self-sustained low-frequency firings. Although their model provided a breakthrough in the field, their study was still preliminary in the sense that they model mainly focused on the lognormal distribution of EPSP and disregarded correlation of the EPSPs observed between bidirectionally coupled neurons [8]. In our previous study, we introduced correlated EPSPs to the lognormal network model and observed synchronized firing among the neurons as well as extremely high firing frequencies in a certain portion of neurons. The aim of this paper is to study further details of the neuronal dynamics in a network of correlated EPSPs. In particular, we show that periodic fluctuations exist in the population firings of the neurons, which may contribute to synchronized firings of the network dynamics.

2. Model and Analysis Method

2.1. Dynamics of a single neuron

The dynamics of individual neurons can be described by a leaky Integrate-and-Fire model:

$$\frac{dv}{dt} = -\frac{1}{\tau_m}(v - V_L) - g_E(v - V_E) - g_I(v - V_I), \quad (1)$$

where v represents the membrane potential. In our simulation, the membrane time constant τ_m was set to 20 ms for excitatory neurons and 10 ms for inhibitory neurons. The reversal potential of leak and excitatory and inhibitory postsynaptic currents were set to $V_L = -70$ mV, $V_E = 0$

mV, and $V_I = -80$ mV, respectively. The excitatory and inhibitory synaptic conductance values g_E and g_I change in time according to

$$\frac{dg_X}{dt} = -\frac{g_X}{\tau_s} + \sum_j G_{X,j} \sum_{s_j} \delta(t - s_j - d_j), \qquad X = E, I, \quad (2)$$

where the index X denotes either excitatory neuron ($X = E$) or inhibitory neuron ($X = I$). $\delta(t)$ stands for Dirac's delta function and $G_{X,j}$, d_j, and s_j are weight, delay, and spike timing of synaptic input from j-th neuron, respectively. The decay constant τ_s was set to 2 ms. The synaptic delays d_j were selected from an uniform random number in the range between $d_0 - 1$ and $d_0 + 1$ ms, where $d_0 = 2$ ms was for excitatory-to-excitatory connections and $d_0 = 1$ ms was for other connection types. The membrane potential threshold for a neuron to generate a spike was set to $V_{thr} = -50$ mV, where v was reset to $V_r = -70$ mV after the spiking. The refractory period was set to 1 ms.

2.2. Network structure of the cortical neuron model

The network model was composed of 2000 inhibitory neurons and 10000 excitatory neurons. The excitatory-to-excitatory connections were classified into bidirectional and unidirectional connections, whose coupling probabilities were set to $P_{uni} = 0.123$ and $P_{bi} = 0.0542$, respectively [8]. The EPSPs x obeyed a lognormal distribution:

$$p(x) = \frac{\exp[-(\log x - \mu)^2/2\sigma^2]}{\sqrt{2\pi}\sigma x}, \quad (3)$$

where the values $\mu - \sigma^2 = \log(0.2)$ and $\sigma^2 = 1.0$ were set based on the experimentally observed ones [8]. The corresponding weight value $G_{E,j}(j \in E)$ was determined in such a way that the membrane potential v reached to the EPSP value of x from the resting state v after a spike input was injected. We removed any unrealistic values of $G_{E,j}$ that gave rise to EPSP amplitude larger than 20 mV. In addition to the lognormally distributed EPSPs, correlation between bidirectionally coupled EPSPs has been observed in physiological experiment [8]. To introduce the correlation between EPSP strengths of bidirectionally connected excitatory neurons, we construct the EPSPs x_1 and x_2 as

$$\begin{aligned} x_1 &= \exp[\mu + \sigma(\sqrt{1-a}Y_1 + \sqrt{a}X)], \\ x_2 &= \exp[\mu + \sigma(\sqrt{1-a}Y_2 + \sqrt{a}X)], \end{aligned} \quad (4)$$

where X, Y_1, Y_2 are independent *Gaussian* random numbers. The parameter a controls the correlation R between x_1 and x_2. According to our numerical simulations, their relation is approximated by a polynomial equation of $R = 0.4252a^2 + 0.5579a$. The correlation value of $R = 0.36$ was reported in the physiological experiment [8].

Excitatory-to-inhibitory, inhibitory-to-excitatory, and inhibitory-to-inhibitory connections were set to have constant values of $G_{E,j}(j \in I) = 0.018$, $G_{I,j}(j \in E) = 0.002$, and $G_{I,j}(j \in I) = 0.0025$, respectively. The coupling probabilities between excitatory-and-inhibitory, inhibitory-and-excitatory, and inhibitory-and-inhibitory neurons were set to $P_{EI} = 0.1157$, $P_{IE} = 0.5785$, and $P_{II} = 0.5785$, respectively.

In the simulation of the neural network model based on Eqs. (1) and (2), we applied external *Poisson* spike trains to all neurons during the initial period of 100 ms [7]. The excitatory-to-excitatory synaptic transmissions failed at an EPSP amplitude-dependent rate of $p_E = b/(b + EPSP)$, where $b = 0.1$ mV.

2.3. Cross-correlogram

The level of synchronization between spike trains was evaluated by the cross-correlogram (CCG). The CCG is defined as a histogram of inter-spike intervals of 1000 randomly selected excitatory neurons among a total of 10000 neurons. The time lag was set to range between -20 ms and 20 ms with an increment of 1 ms. The CCG histogram was normalized by the maximum value to remove its frequency dependence. For spike trains with asynchronous firings, the normalized CCG appears with a flat structure, whereas it has a sharp peak at zero time lag for spike trains with synchronous firings.

2.4. Coefficient of Variation

The regularity of spike train was evaluated by the Coefficient of Variation (CV). The CV is defined as standard deviation of inter-spike intervals normalized by their mean value. For regular spike trains, the CV is close to zero, whereas it can be larger than one for irregular spike trains.

3. Result

3.1. Synchronization and periodic fluctuations in neural firings

This section provides simulation results of the network dynamics for $R = 0.5$, which is larger than the one observed in the physiological experiment [8]. Raster plot of Fig. 1A shows spike timings of the individual neurons. For the model with correlated connections, the observed firing dynamics showed two features. First, the spike trains appeared simultaneously among different neurons, indicating their synchronous firings (Fig. 1 A). This observation is consistent with the normalized CCG displaying a sharp peak at the zero time lag (Fig. 1B). Second, short-term average (time window of 1 ms) of population firing frequencies shows periodic fluctuations in both excitatory and inhibitory neurons (Fig. 2A). In two-dimensional representation of the firing frequencies of excitatory and inhibitory neurons, the orbit resembled a limit cycle oscillation (Fig. 2B). To confirm the limit cycle oscillations, fluctuations observed in the short-term average frequencies were analyzed by the fast Fourier transform for excitatory

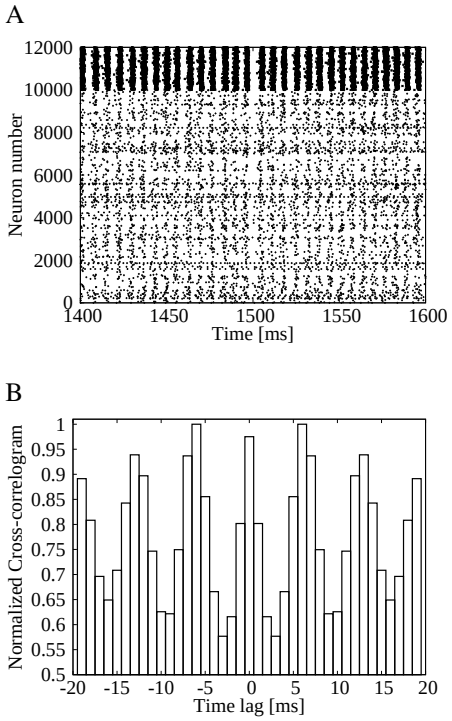

Figure 1: Simulation results with correlated EPSPs (R = 0.50). (A) Raster plot. Spike timings are indicated for excitatory neurons (number from 0 to 10000) and inhibitory neurons (number from 10000 to 12000). (B) Histogram of the normalized cross-correlogram (CCG). Time lag is in the range from −20 ms to 20 ms with a bin size of 1 ms.

(Fig. 2C) and inhibitory neurons (Fig. 2D). The result indicates that their fluctuations indeed shows a strong periodicity at around 150 Hz.

3.2. Effect of regular firings induced by high firing frequencies

Fig. 3A shows distributions of the firing frequencies of excitatory neurons. The firing frequency was computed by counting the number of spikes generated from each neuron for duration of 10 s. About 6 % of neurons shows firing frequencies higher than 10 Hz (Fig. 3A). According to our visual inspection of Fig. 1A, neurons with high firing frequencies tend to generate regular spikes. To evaluate the level of regularity in the spikes, the CV was computed for two neurons, which showed a frequency of about 60 Hz. Fig. 3B shows time series (duration of 10 s) of the CV and firing frequency computed in a short time window of 500 ms. When neurons show high frequency firing, small values of CV were observed (Fig. 3B: solid line from t = 2500 ms to t = 4000 and dotted line around t = 3000 ms). Conversely, large CV values were observed when the neurons showed low frequencies (Fig. 3B: solid line from t = 4000 ms to t = 6500 and dotted line from t = 6500 ms to t = 9000). Indeed, as shown in a two-dimensional plot (firing frequencies *v.s.* CVs) of Fig. 3C, the CVs tend to be inversely proportional to the firing frequencies. This implies that neurons fire with a strong periodicity as their frequencies get high.

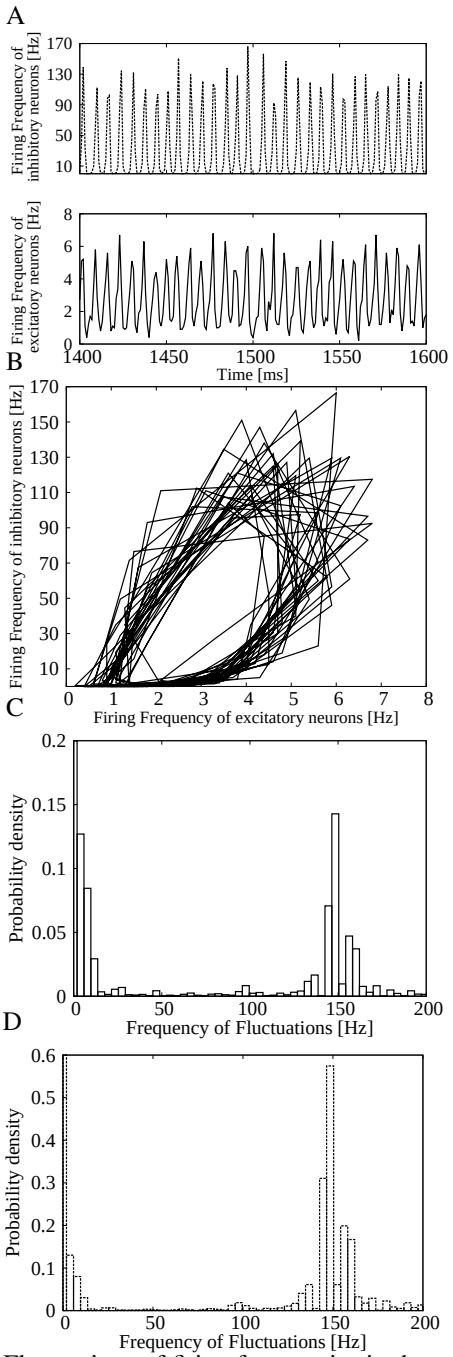

Figure 2: Fluctuations of firing frequencies in the network of correlated couplings (R = 0.50). (A) Time series of averaged population firing frequencies for excitatory (bottom) and inhibitory (top) neurons are shown from t = 1400 ms to t = 1600 ms. (B) The corresponding orbit in two-dimensional space (excitatory *v.s.* inhibitory firings). The power spectra of time series of averaged firing frequencies are shown for excitatory (C) and inhibitory neurons (D).

4. Discussions

This paper studied the effect of correlated bidirectional connections on the neural network of lognormally distributed EPSPs. We observed that the correlation induced synchronized oscillations among neurons, which are not usually observed in spontaneous firing activities. Short-term average of population firing frequencies showed periodic fluctuations. Our analysis, which focused on neurons with high firing frequencies, revealed that the neurons generate regular spikes when their firing frequencies are high, whereas they generate irregular spikes when their firing frequencies are low. Our result suggests that the regular firings may induce periodic fluctuations of the population firings, whereas irregular firings may contribute to self-sustained firings of the spontaneous activity.

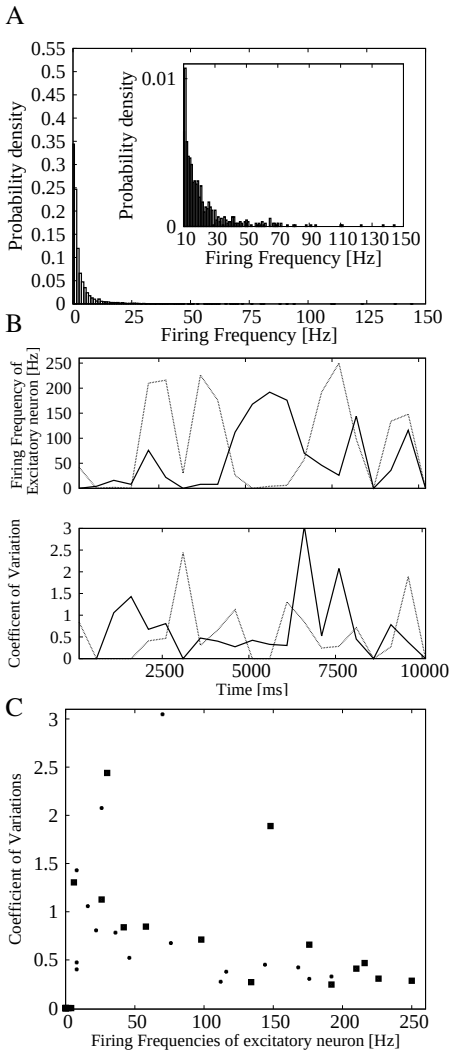

Figure 3: (A) Histogram of the firing frequencies of excitatory neurons. The inserted figure is a magnification of the range from 10 to 150 Hz. (B) Short-term averaged firing frequency (top) and CV (bottom) of two excitatory neurons are shown from t = 100 ms to t = 10100 ms (t = 10 s). (C) Two-dimensional representation of (B), in which firing frequencies are plotted against their corresponding CV. Two makers denote different excitatory neurons (circle and square points correspond to solid or dotted line of (B), respectively).

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References

[1] J. Fiser *et al.*, "Small modulation of ongoing cortical dynamics by sensory input during natural vision," *Nature*, vol. 431, pp.573–578, 2004

[2] T. Hromádka *et al.*, "Sparse Representation of Sounds in the Unanesthetized Auditory Cortex," *PLoS Biology*, vol. 6, e16, 2008

[3] W. R. Softky, and C. Koch, "The Highly Irregular Firing of Cortical Cells Is Inconsistent with Temporal Integration of Random EPSPs," *Neuroscience*, vol.13, pp.334–350, 1993

[4] A. Renart *et al.*, "The Asynchronous State in Cortical Circuits," *Science*, vol. 327, pp. 587–590, 2010

[5] N. Parga and L. F. Abbott, "Network model of spontaneous activity exhibiting synchronous transitions between up and down state," *Frontiers in Neuroscience*, pp. 57–66, 2007

[6] A. Kumar *et al.*, "The High-Conductance State of Cortical Networks," *Neural Computation*, vol. 20, pp. 1–43, 2008

[7] J. Teramae *et al.*, "Optimal spike-based communication in excitable networks with strong-sparse and weak-dense links," *Scientific Reports*, vol. 485, 2012

[8] S. Song *et al.*, "Highly Nonrandom Features of Synaptic Connectivity in Local Cortical Circuits," *PloS Biology*, vol. 3 ,e68, 2005