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Nonlinear Output Tracking Control of a Synchronous Generator using Input-Output Feedback Linearization

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Abstract– The objective of the present work is to apply the concept of exact input-output linearization to design a nonlinear Lyapunov controller for a reduced-order three-dimensional model of a synchronous generator. The design approach structure is compared to a classical input-output linearizing controller. An application of the obtained nonlinear control law to synchronous generator stabilization gives very interesting results. Numerical simulations are presented to illustrate the efficiency of the proposed scheme.

1. Introduction

The stability of synchronous generators is one of the major large problems in power system control [3, 7, 8]. A very number of works have shown that the application of nonlinear control methods gives good performances for synchronous generators, particularly the input-output linearizing control method [2, 3, 4]. The basic idea of this paper is to combine this technique and the Lyapunov stability theory in order to obtain a control structure which achieves asymptotic output tracking in the generator angle. The design approach structure is then compared to a polynomial input-output controller [1, 6].
This paper is organized as follows: In Section 2 the problem is posed with the system modelisation. In Section3, we present a polynomial input-output linearizing control. The Lyapunov Input-Output Controller is presented in Section 4 and Section 5 is dedicated to digital simulation results of the obtained Lyapunov control where comments are formulated. The conclusion is given in Section 6.

2. Modelisation of the synchronous generator

2.1. The system model

The state equations describing the dynamics of the generator are given by [8]:

$$\begin{cases}\dot{\xi}_1 = \omega_b(\xi_2-\omega_s)\\ \dot{\xi}_2 = M_{ad}+\ell_1\xi_3\sin(\xi_1)+\ell_2\sin(2\xi_1)+\ell_3 P_{mech}\\ \dot{\xi}_3 = \ell_4(\xi_2-\omega_s)\sin(\xi_1)+\ell_5\xi_3+\ell_6 u_f\end{cases} \tag{1}$$

The state variables $\xi_1=\delta$, $\xi_2=\omega$ and $\xi_3=i_f$ represent respectively the generator angle, the angular rotor speed and the field winding current. Finally, u_f is the control input which is the field winding voltage. The model parameters of the studied synchronous generator d_2, $\ell_{i=1,\dots,6}$ and the nominal values are given in the Appendix1.

2.2. Synthesis of the polynomial model

For further considerations it is convenient to consider around an operating point of the generator (ξ_n, u_n), the following variable changes:

$$\begin{cases}x = \xi - \xi_n\\ u = u_f - u_n\end{cases} \tag{2}$$

The polynomial development truncated to the third order of the generator model (1) around an operating point entails the following equation:

$$\dot{x} = f_1 x + f_2 x^{[2]} + f_3 x^{[3]} + g_0 u \tag{3}$$

where

$x^{[i]} \in R^{n^i}$: Designates the Kronecker power of vector x,

$$\begin{cases} f_1(1,2)=w_b, f_1(2,1)=a_1, f_1(2,2)=a_2, f_1(2,3)=a_3\\ f_1(3,2)=b_1, f_1(3,3)=b_2, f_1(i,j)=0 .i,j=1,\dots,3\\ f_2(2,1)=a_4, f_2(2,2)=a_5, f_2(2,3)=a_6\\ f_2(3,2)=b_3, f_2(i,j)=0 .i,j=1,\dots,9\\ f_3(2,1)=a_7, f_3(2,2)=a_8, f_3(2,3)=a_9\\ f_3(3,2)=b_4, f_3(i,j)=0.\ i,j=1,\dots,27\end{cases}, \quad g_0=\begin{bmatrix}0\\0\\ \ell_6\end{bmatrix}$$

and the parameters $a_{j=1,\dots,9}$ et $b_{k=1,\dots,4}$ are given in the Appendix1. Although the Input-Output nonlinear controller of the described system were quite satisfactory, we believe that the most proper approach to controller design for synchronous generators is an application of the stability theory of nonlinear systems [5].

3. Synthesis of the Polynomial Input-Output Controller (PIOC)

Consider the single-input single-output nonlinear affine input system represented by:

$$\begin{cases}\dot{x} = f(x) + g(x)u\\ y = h(x)\end{cases} \tag{4}$$

where x is the state vector $\in R^n$. $f(.)$ and $g(.)$ are n-dimensional vectors of real variables $x_1,\dots,x_n$. $h(.)$ is a scalar function of vector x. The model of nonlinear state given by equation (3) can be easily transformed into a polynomial model expressed by the following equation:

$$\begin{cases} \dot{x} = \sum_{i=1} f_i\, x^{[i]} + \sum_{j=0} \left(g_j \otimes x^{[i]} \right) u \\ y = \sum_{k=1} h_k\, x^{[k]} \end{cases} \tag{5}$$

Let the relative degree r [5] of system (5) be $r=n$, hence the system is exact input-output linearizable and it will define the state transformation $\Phi(x)$ as follows:

$$\Phi(x) = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{n-1} \\ z_n \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f\, h(x) \\ \vdots \\ L_f^{n-1}\, h(x) \\ L_f^n\, h(x) \end{bmatrix} \tag{6}$$

Using z as state variable, system (4) becomes:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \vdots \\ \dot{z}_{n-1} = z_n \\ \dot{z}_n = L_f^n\, h(\Phi^{-1}(z)) + L_g\, L_f^{n-1}\, h(\Phi^{-1}(z))\, u \\ \quad = v \end{cases} \tag{7}$$

where v is a pole placement control given by $v = -K^T \Phi(x)$ and K should be a Hurwitz vector.

With the reference to developed in [6], it is easy to express the diffeomorphism $\Phi(x)$ in this polynomial form:

$$\Phi(x) = \sum_{k=1} \Phi_k\, x^{[k]} \tag{8}$$

Then, the polynomial input-output linearizing control will be expressed by:

$$U_{PIOC} = \sum_{i=1} \alpha_i\, x^{[i]} + \sum_{j=1} \beta_j\, x^{[j]} \tag{9}$$

with:

$$\begin{cases} \alpha_1 = (J_0^{-1})\, L_1 \\ \vdots \\ \alpha_i = J_0^{-1} [L_i - \sum_{p=1}^{i-1} (J_{i-p} \otimes \alpha_p)] \quad i \geq 1 \end{cases} \qquad \begin{cases} \beta_1 = (J_0^{-1})\, K\, \Phi_1 \\ \vdots \\ \beta_j = J_0^{-1} [K\, \Phi_j - \sum_{p=1}^{j-1} (J_{j-p} \otimes \mu_p)] \quad j \geq 1 \end{cases}$$

where the different expressions of terms L_i and J_j are given by:

$$\begin{cases} L_f^n\, h(x) = \sum_{i=1} L_i\, x^{[i]} \\ L_g\, L_f^{n-1}\, h(x) = \sum_{j=0} J_j\, x^{[j]} \end{cases}$$

4. Synthesis of a Lyapunov Input-Output Controller (LIOC)

In this section we present a technique of combining exact input-output linearization and the Lyapunov stability theory to design a control structure which achieves asymptotic output tracking in a desired trajectory y_d. The aim is to design a Lyapunov controller that mimics a predetermined input-output linearizing controller known for its effective performance in output tracking [5, 8].

Using z as state variable, from system (7) one can write:

$$\dot{z}_n = D(z) + C(z)\, u \tag{10}$$

Since our goal is to let $z_1 = y_d$, we define a tracking error

$$\begin{cases} e_1 = z_1 - y_d \\ e_2 = z_2 - \dot{y}_d \\ \vdots \\ \vdots \\ e_r = z_n - y_d^{n-1} \end{cases} \tag{11}$$

and we choose an input-output linearizing controller such that:

$$U_{IO} = \frac{1}{C(z)} [-D(z) + y_d^n + K^T e] \tag{12}$$

Let's consider the analytic expression of a Lyapunov controller as:

$$U_{LIOC} = \Lambda^T e(t)\ , \qquad \Lambda^T = [\Lambda_1\ \Lambda_2\ \ldots\ \Lambda_{n-1}\ \Lambda_n] \tag{13}$$

where Λ is adjusted such that in the limit we have:

$$\hat{U}_{LIOC} = \hat{\Lambda}^T\, e(t) = U_{IO} \tag{14}$$

the asterisk in $\hat{\Lambda}$ denotes the optimal gain vector.

Substituting $\hat{U}_{LIOC}$ in (10) yields the following equation:

$$\dot{z}_n = D(z) + C(z)\, U_{LIOC} \tag{15}$$

this can be written as:

$$\dot{z}_n = y_d^n + K^T e - C(z)(U_{IO} - U_{LIOC}) \tag{16}$$

hence one has:

$$\dot{e}_n = y_d^n - \dot{z}_n = -K^T e + C(z)(\hat{\Lambda} - \Lambda)^T e \tag{17}$$

If we define

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ C(z) \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ -k_1 & -k_2 & \cdots & \cdots & -k_n \end{bmatrix}$$

then, we obtain the following closed loop system

$$\dot{e} = Ae + B(\hat{\Lambda} - \Lambda)^T e \tag{18}$$

Note that since A is Hurwitz stable, then for any positive definite matrix Q there exists a positive definite matrix P such that the following Lyapunov equation is satisfied:

$$A^T P + PA = -Q \tag{19}$$

Given system (17), let's choose a Lyapunov candidate function:

$$V(e, \Lambda) = e^T Pe + \frac{1}{\Gamma} (\hat{\Lambda} - \Lambda)^T (\hat{\Lambda} - \Lambda) \tag{20}$$

The Lyapunov controller $U_{LIOC} = \Lambda^T e(t)$ is so well defined by choosing Λ such that:

$$\Lambda = \Gamma \int_0^t e(t)^T PB\, e(t)\, dt\ , \ \Lambda \rangle 0 \tag{21}$$

If we truncate the expression (21) to the third order we have:

$$\begin{cases} \Lambda_1 = \Gamma\, \omega_b\, \ell_6 \int_0^t [(a_3 + a_6 x_1 + a_9 x_1^2)(P_{13} e_1^2 + P_{23} e_1 e_2 + P_{33} e_1 e_3)]\, dt \\ \Lambda_2 = \Gamma\, \omega_b\, \ell_6 \int_0^t [(a_3 + a_6 x_1 + a_9 x_1^2)(P_{23} e_2^2 + P_{13} e_1 e_2 + P_{33} e_2 e_3)]\, dt \\ \Lambda_3 = \Gamma\, \omega_b\, \ell_6 \int_0^t [(a_3 + a_6 x_1 + a_9 x_1^2)(P_{33} e_3^2 + P_{23} e_3 e_2 + P_{13} e_1 e_3)]\, dt \end{cases} \tag{22}$$

5. Application to filed voltage control of a Synchronous Generator

The design of a lyapunov control aims to force an output signal y of the controlled system to track a smooth reference trajectory y_d, the considered output variable to be tracked is δ: the generator load angle. This section is dedicated to study the validity of the two considered controller. A comparative study is then formulated in order to involue the efficiency of the considered approaches.

5.1 Synthesis of the PIOC design

Consider the polynomial model describing the dynamics of a generator (3). The relative degree is $r=3$.Therefore we choose the following state transformation

$$\Phi(x)=\begin{bmatrix} x_1 \\ \omega_b\, x_2 \\ \omega_b\,(a_1x_1+a_2x_2+a_3x_3+a_4x_1^2+a_5x_1x_2 \\ +a_6x_1x_3+a_7x_1^3+a_8x_1^2x_2+a_9x_1^2x_3) \end{bmatrix} \qquad (23)$$

The expression of the diffeomorphism described by equation (23) is then developed under a polynomial form truncated to the third order which yields:

$$\Phi(x)=\Phi_1\, x+\Phi_2\, x^{[2]}+\Phi_3\, x^{[3]} \qquad (24)$$

The control law is also expressed in the following form:

$$U_{PIOC}=\alpha_1 x+\alpha_2 x^{[2]}+\alpha_3 x^{[3]}\; + \;\beta_1 x+\beta_2 x^{[2]}+\beta_3 x^{[3]} \qquad (25)$$

with

$$\begin{cases} \alpha_1(1,1)=-0.04,\ \alpha_1(1,2)=0.64, \alpha_1(1,3)=-0.78. \\ \alpha_2(1,1)=0.16,\ \alpha_2(1,2)=0.18,\ \alpha_2(1,3)=0.1, \alpha_2(1,4)=-0.4, \alpha_2(1,6)=-0.59, \alpha_2(1,4)=0.27. \\ \alpha_3(1,1)=-0.18, \alpha_3(1,2)=-0.118,\ \alpha_3(1,4)=-0.2, \alpha_3(1,6)=-0.8, \alpha_3(1,2)=0.17. \\ \alpha_2(1,i)=0\ i=1,\dots,9,\ \alpha_3(1,i)=0\ i=1,\dots,27. \\ \\ \beta_1(1,1)=0.365,\ \beta_1(1,2)=-0.264, \beta_1(1,3)=0.166. \\ \beta_2(1,1)=-0.434,\ \beta_2(1,2)=0.1674. \\ \beta_3(1,1)=0.65,\ \beta_3(1,2)=0.175. \\ \beta_2(1,i)=0\ i=1,\dots,9,\ \beta_3(1,i)=0\ i=1,\dots,27. \end{cases}$$

Figure 1 represent the schematic implementation of the obtained PIOC.

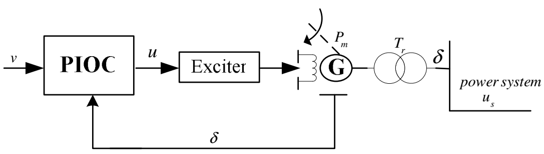

Figure 1. Implementation of obtained PIOC

5.2 Synthesis of a LIOC design

The polynomial model describing the dynamics of a generator (3) is represented by the following dynamical form:

$$\begin{cases} \dot{x}=\begin{bmatrix} F_1(x) \\ F_2(x) \\ F_3(x) \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \ell_6 \end{bmatrix} u \\ y=x_1 \end{cases} \qquad (26)$$

Thus in the new coordinates, system (24) is expressed by the following equations

$$\begin{cases} \dot{z}_1=z_2 \\ \dot{z}_2=z_3 \\ \dot{z}_3=D(z)+C(z)\ U_{IO} \end{cases} \qquad (25)$$

where:

$$\begin{cases} D(z)=L_F^3\, h(x) \\ C(z)=L_G\, L_F^2\, h(x) \end{cases}$$

In this case we define a three dimensional tracking error:

$$\begin{cases} e_1=z_1-y_d \\ e_2=z_2-\dot{y}_d \\ e_3=z_3-y_d^2 \end{cases} \qquad (26)$$

and we choose an input-output linearizing controller such that:

$$U_{IO}=\frac{1}{C(z)}[-D(z)+y_d^3+K^Te],\ K^T=[k_1\ k_2\ k_3] \qquad (27)$$

where $C(z)$ and $D(z)$ are given in the Appendix2. The resulting closed loop system becomes linear and one has:

$$\dot{e}=Ae \qquad (28)$$

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

By referent to the procedure described in the section 4, we suggest here to construct a Lyapunov controller $U_{LIOC}=\Lambda^T e(t)$, such that in the limit conditions we have $\hat{U}_{LIOC}=U_{IO}$.

Recording to equation (22) this yields the following expression of the control covector Λ:

$$\begin{cases} \Lambda_1=27\int_0^t[(-0.8-0.6x_1-0.4x_1^2)(0.33e_1^2+0.6e_1e_2+0.43e_1e_3)]dt \\ \Lambda_2=27\int_0^t[(-0.8-0.6x_1-0.4x_1^2)(0.6e_2^2+0.33e_1e_2+0.43e_2e_3)]dt \\ \Lambda_3=27\int_0^t[(-0.8-0.6x_1-0.4x_1^2)(0.43e_3^2+0.6e_3e_2+0.33e_1e_3)]dt \end{cases}$$

In figure 2 we represent the implementation schema of synthesized Lyapunov controller.

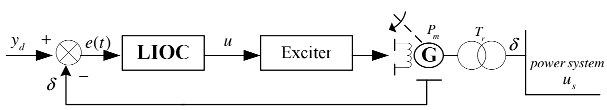

Figure 2. Implementation of obtained LIOC

5.3 Simulation Results

In this section, we will show the result of a numerical simulation corresponding to the implementation of the two controllers presented in the foregoing section. The desired trajectory of the generator angle is designed such that it satisfies technical requiments as described in [8]. It appears in figure 3 that the performance of the LIOC controller is better than that obtained by the PIOC one. In fact, the tracking error obtained for the PIOC is very important when compared with error resulting from LIOC implementation. The load angle dynamic, however, is satisfactory in the two cases. In figure 4 we represent the control signals obtained for the LIOC and PIOC. It is obvious in this figure that the two controllers present the same allure, however the dynamic of the LIOC is faster which explains the perfect concordance between the desired trajectory and the tracking one in this case.

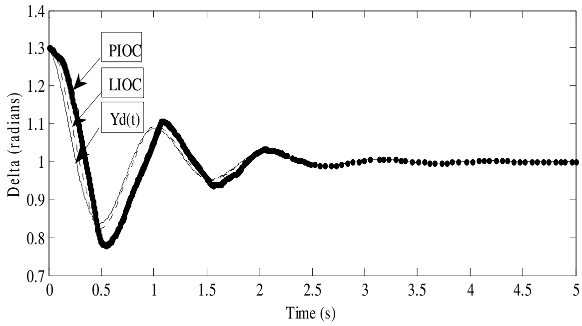

Figure 3. Dynamics of the generator angle tracking trajectory

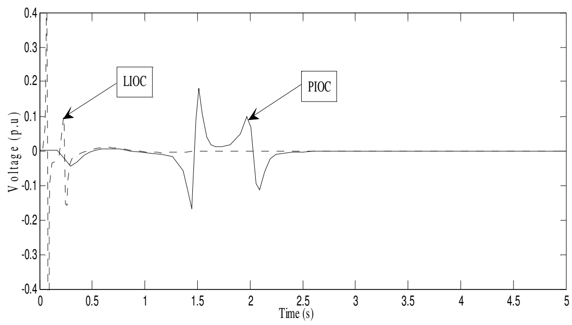

Figure 4. Evolution of nonlinear control laws dynamics

6. Conclusion

In this communication, nonlinear Input-Output controllers schemes have been studied and applied to an improved reduced-order three-dimensional model of a synchronous generator. The obtained nonlinear closed-loop system has very interesting dynamic properties for the case of the considered PIOC and the LIOC controllers. Simulations study shows that the implementation of the studied controllers gives good transient properties of the power system, such properties are essentially characterized by a very reduced tracking error especially for the LIOC. An interesting extension of this work would be the application of the proposed methodology with a synthesis of performant observers of the generator load angle. A stability study of closed-loop system will be essential.

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Appendix

1. Parameters of a synchronous generator model

$$\ell_1 = (-U_s X_{AD}) \big/ (T_M (x_d + x_L)), \ell_2 = 0.5 U_s^2 (x_d - x_q^{'}) / (T_M (x_q + x_L))(x_L + x_d)$$

$$\ell_3 = 1 / T_M, \ell_4 = -(x_L + x_d) / ((x_L + x_d^{'}) \tau_{do}^{'})$$

$$\ell_5 = (x_d - x_d^{'}) U_s \omega_b / ((x_d^{'} + x_L) X_{AD}), \ell_6 = (x_L + x_d) / ((x_d^{'} + x_L) \tau_{do}^{'} r_f)$$

$$d_2 = -\tau_{qo}^{'} (1 - x_q^{'} / x_q) U_S^2 / x_q, a_1 = 2\ell_2 c_2 + c_1 \ell_1 \xi_{n3}, a_2 = d_2 c_1^2, a_3 = s_1 l_1$$

$$a_4 = -2\ell_2 s_2 - s_1 \ell_1 \xi_{n3} \big/ 2, a_5 = -2 d_2 c_1 s_1, a_6 = c_1 \ell_1, a_7 = -4 \big/ 3 c_2 \ell_2 - c_1 \ell_1 \xi_{n3} \big/ 3!, a_8 = s_1^2 - c_1^2$$

$$a_9 = -s_1 \ell_1 \big/ 2, b_1 = \ell_4 s_1, b_2 = \ell_5, b_3 = \ell_4 c_1, b_4 = -\ell_4 s_1 \big/ 2, s_1 = \sin(\xi_{n1}), c_1 = \cos(\xi_{n1})$$

$$c_2 = \cos(2\xi_{n1}), x_d = 2.459\, pu, x_q = 2.354\, pu \;\; x_d^{'} = 0.315\, pu, \tau_{do} = 7.95\, s \;\; X_{AD} = 2.28\, pu$$

$$T_M = 8s \;\; \tau_{qo}^{'} = 0.39\, s \;\; x_q^{'} = 0.476\, pu \;\; x_r = 0.191\, pu, r_f = 0.002\, pu \;\; \tau_{do}^{''} = 0.01\, s, \tau_{qo}^{''} = 0.016\, s$$

2. The Lie derivatives of the process model

$$\begin{cases} L_F\, h(x) = \omega_b x_2 \\ L_F^2\, h(x) = \omega_b F_2(x) \\ L_F^3\, h(x) = F_1(x) \dfrac{\omega_b \partial F_2(x)}{\partial x_1} + F_2(x) \dfrac{\omega_b \partial F_2(x)}{\partial x_2} + F_3(x) \dfrac{\omega_b \partial F_2(x)}{\partial x_3} \end{cases}$$

the following partial derivatives

$$\begin{cases} \dfrac{\partial F_2(x)}{\partial x_1} = a_1 + x_1 (2 a_4 + 2 a_7 x_1 + 2 a_8 x_2 + 2 a_9 x_3) + a_5 x_2 + a_6 x_3 \\ \dfrac{\partial F_2(x)}{\partial x_2} = a_2 + x_1 (a_5 + a_8 x_1) \\ \dfrac{\partial F_2(x)}{\partial x_3} = a_3 + x_1 (a_6 + a_9 x_1) \end{cases}$$