2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Analysis of a Deterministic Firefly Algorithm

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Abstract—In recent years, many researchers pay attention to swarm intelligence as the application of the optimization problem solver. Firefly algorithm is one of such swarm intelligence algorithms, it is inspired by the flashing and attracting behavior of fireflies. In this article, we consider a deterministic firefly algorithm to analyze the dynamics rigorously. The state update equation of the deterministic firefly algorithm contains two important parameters; β_0 and γ. We analyze the effect of the solution search of these parameters. Based on the analysis result, we propose a modified firefly algorithm to improve the solution search performance. We confirm the solution search performance by using come benchmark functions.

1. INTRODUCTION

Under the given constraints, Optimization Problem is to find a solution that a certain objective function gives the maximum value or the minimum value. The optimization problem has been studied in various fields such as engineering, economics, and et al.

In recent years, many researchers pay attention to swarm intelligence as the application of the optimization problem solver. The swarm intelligence algorithms is between agents to emerge the behavior by local interaction [1]. For example, some methods are based on the behavior of ants colony, slime mold colony, fish flock, and so on. Firefly Algorithm (abbr. FA) is also one of such swarm intelligence algorithms. It is developed based on the characteristics of the blinking of natural firefly by Xin-She Yang etal. in 2007[2].

To analyze the dynamics of FA, we proposed a deterministic FA. Based on the analysis results of the deterministic FA, we propose an improved deterministic FA. We confirm the solution search performance of the proposed FA by using some benchmark functions.

2. FIREFLY ARGORITHM

In the firefly algorithm, there are two important points: the variation in the light intensity and the formulation of the attractiveness. So for optimization problems, a firefly with high/low intensity will attract another firefly with high/low intensity. The distance r_{ij} is the distance between the i-th firefly and the j-th firefly. The light intensity $I(r)$ is in inverse proportion into the square of the distance.

$$I(r) = I_s/r^2 \tag{1}$$

where, I_s is the light intensity at the source. The light intensity I varies with the distance r_{ij} depending upon a fixed light absorption coefficient γ.

$$I = I_0 e^{-\gamma r} \tag{2}$$

where, I_0 is the initial light intensity

Each firefly has its distinctive attractiveness β which implies how strong it attracts other members of the swarm. The attractiveness is varied and it changes depending upon the distance between the i-th firefly and i-th firefly. Since the attractiveness is proportion to the light intensity seen by adjacent fireflies, the attractiveness function is defined as

$$\beta = \beta_0 e^{-\gamma r^2} \tag{3}$$

where, β_0 is the attractiveness at $r = 0$ and γ is a light absorption coefficient.

Cartesian distance between any two fireflies i and j at x_i and x_j, respectively is

$$r_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_i\| = \sqrt{\sum_{d=1}^{D}(\boldsymbol{x}_{i,d} - \boldsymbol{x}_{j,d})^2} \tag{4}$$

The movement of the i-th firefly is attracted to another more attractive (brighter) j-th firefly is determined by

$$\begin{aligned}\boldsymbol{x}_i^{t+1} = \quad & \boldsymbol{x}_i^t + \beta_0 \mathrm{e}^{-\gamma r_{ij}^2}(\boldsymbol{x}_j^t - \boldsymbol{x}_i^t) \\ & +\alpha(rand - 1/2)\end{aligned} \tag{5}$$

where $rand \in [0, 1]$ is a uniform distributed random number[3].

3. DETERMINISTIC FIREFLY ALGORITHM

In order to analyze the dynamics of FA, we consider our proposed deterministic FA[4]. Without loss of generality, we can consider the case of one-dimensional. We assume that the j-th firefly found the optimum solution. The optimum solution locates at the origin. Also, the initial positions of other fireflies are located excepting the origin. We consider the case of $t = 0$ of Eq. (5) as follows.

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t - \beta_0^{\gamma(\boldsymbol{x}_i^t)^2}\boldsymbol{x}_i^t \tag{6}$$

Since the system described by Eq. (6) does not contain stochastic factor, the system can be regarded as a deterministic system. Thus, we call this system a deterministic FA. The dynamics of the i-th firefly of the deterministic FA is described by a one-dimensional return map as shown in Fig. 1. It shows the cases the parameters γ and β_0 are varied. Figure 1 indicates that the characteristic of the one-dimensional return map is depended on the parameter. In the case where γ is large, the most of the domain of the return map into itself. On the other hand, when γ becomes small, the return map is varied depending on γ. In this case, the search range is enlarged. Therefore, the search range is determined by the γ. If β_0 is large, the amount of the movement in the vicinity of the j-th firefly increases. Namely, β_0 controls search range around the found best position.

As shown in Fig. 1, the absolute value of the slope of the one-dimensional return map at the origin is controlled of β_0. So we consider the slope of the one-dimensional return map.

$$\frac{\mathrm{d}x_i^{n+1}}{\mathrm{d}x_i^n} = 1 + (2\gamma(x_i^n)^2 - 1)\beta_0 \mathrm{e}^{-\gamma(x_i^n)^2} \tag{7}$$

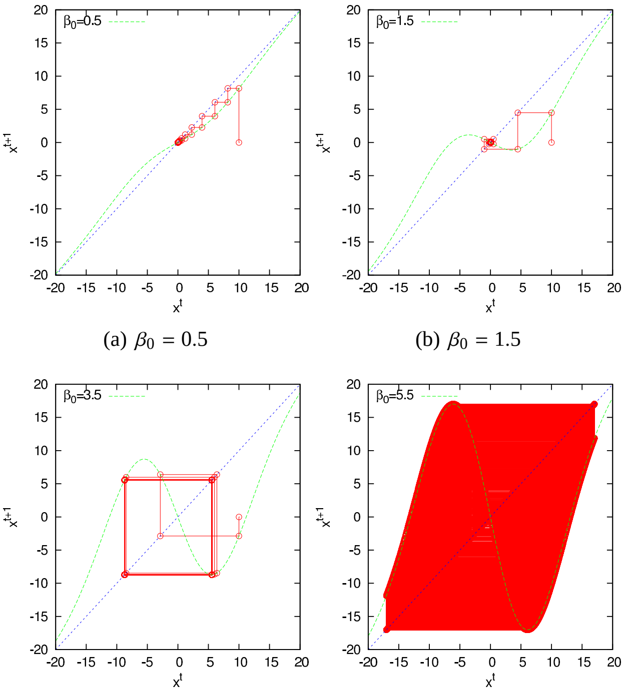
In this case, we assume the optimal position is the origin. If the absolute value of the slope of the origin is greater than 1, the dynamics around the origin is expanded. On the other hand, if the absolute slope around the origin is less than 1, the trajectory converges to the origin. From Eq. (7), the slope of the origin is $\frac{d(x_i^{n+1})}{dx_i^n}|_{x_i=0} = 1 - \beta_0$. Therefore, if $\beta_0 > 2$, the map is expandable. Also, if $0 < \beta_0 < 2$, the map is regarded as a contraction map.

Figure 2 illustrates the time evolution of the search position on the return map depending on the parameters. Fig. 2(a) that the parameter is $\beta_0 = 0.5$, the search point is monotonously attenuated, and all fireflies converge to the origin which corresponds to the optimal position. Fig. 2(b) that the parameter is $\beta_0 = 1.5$, the search point converges to the origin with oscillation. Fig. 2(c) that the parameter is $\beta_0 = 3.5$, the search point converges to the two periodic points. In this case, the firefly to search for only two points. Fig. 2(d) that the parameter is $\beta_0 = 5.5$, the time-series of the search point exhibits non-periodic motion. In order to improve the performance of exploration, we propose the novel deterministic FA that the initial attractiveness parameter β_0 is varied. The initial attractiveness controls the search range.

4. THE PROPOSED METHOD

From the analysis results of one-dimensional deterministic FA, β_0 determines how to explore around the location of the best location. The parameter γ determines the scope of the exploration of each firefly. The state update equation of the deterministic FA is described as

$$\boldsymbol{x}_i^{n+1} = \boldsymbol{x}_i^n + \beta_0 \mathrm{e}^{-\gamma r_{ij}^2}(\boldsymbol{x}_j^n - \boldsymbol{x}_i^n) \tag{8}$$



(a) $\beta_0 = 0.5$ (b) $\beta_0 = 1.5$

(c) $\beta_0 = 3.5$ (d) $\beta_0 = 5.5$

Figure 2: Individual behavior by attractiveness β_0

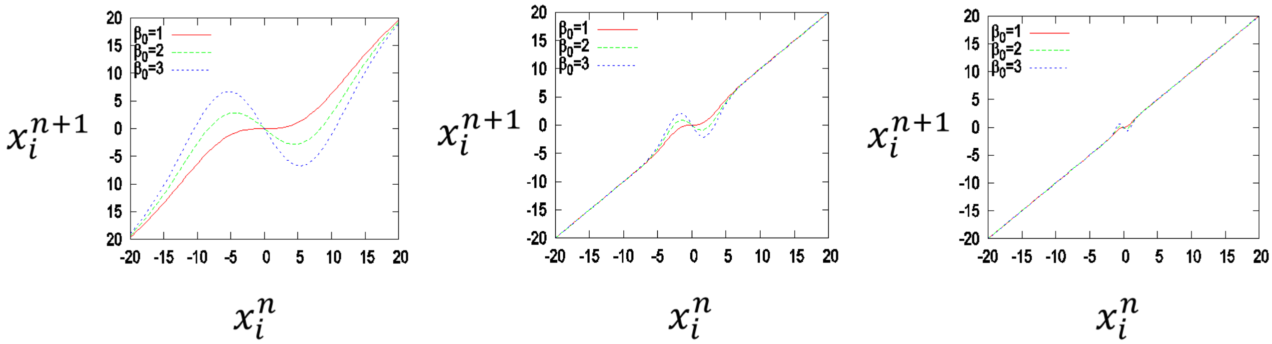
Based on the deterministic FA, we propose a novel deterministic FA with a time variant intension parameter to improve the solution search performance of the deterministic FA. If β_0 is large, the system exhibits non-periodic motion into large search region. In this case, the system is said to be a global search state. On the other hand, if β_0 is small, the system searches the narrow region around the found best position. Such motion is regarded as a local search state. To combine these two states, we change the parameter β_0 to the time variant parameter. At first, the parameter β_0 sets the maximum value β_{max}. The parameter is gradually decrease until the value reaches the minimum value β_{min}.

Figure 2(a) shows the case where all fireflies are concentrated in one place and the solution search process is terminated when β_0 is less than equal to 2. To avoid such solution search termination to improve the search performance, it is need to change the search state if all fireflies are centralized around the found best potion. So, we define a criterion region ϵ, if all fireflies are concentrated in this criterion region, we apply re-arrangement process.

The parameter β_0 and the update equation of the novel FA is described as

$$\begin{cases} \boldsymbol{x}_i^{n+1} = \boldsymbol{x}_i^n + \beta_0 e^{-\gamma r_{ij}}(\boldsymbol{x}_j - \boldsymbol{x}_i) \\ \beta_0^{n+1} = \begin{cases} \beta_{min} & (\beta_0^n - \delta < \beta_{min}) \\ \beta_0^n - \delta & (\text{otherwise}) \end{cases} \end{cases} \tag{9}$$

However, if all fireflies are concentrated in one criterion region ϵ, Namely, $|\boldsymbol{x}_{best}^n - \boldsymbol{x}_i^n| < \epsilon$, The parameter β_0 and the update equation of the novel FA is is changed as follows



(a) $\gamma = 0.01$ (b) $\gamma = 0.1$ (c) $\gamma = 1$

Figure 1: Effect of parameters to search region

Table 1: Simulation conditions

Item	Conditions
β_0	5.5(Deterministic FA) $1 < \beta_0 \leq 6.5$(Proposed FA)
γ	0.001
δ	0.01
Dimension of Evaluation function	10
Number of trials	10
Number of fireflies	10
Maximum iteration	10000
Initial value	[-20:20]

$$\begin{cases} \mathbf{x}_i^{n+1} = \mathbf{Uniform}(l_{min}, l_{max}) \\ \beta_0^{n+1} = \beta_{max} \end{cases} \tag{10}$$

$\mathbf{Uniform}(l_{min}, l_{max})$ is a uniform distributed random number whose region is $[l_{min}, l_{max}]$

5. NUMERICAL SIMULATIONS

In order to confirm the effect of the parameters β_0 and γ, we use two-dimensional Sphere function to the search range and parameter γ are shown in Fig. 3. The horizontal axis represents the light absorption coefficient γ, and the vertical axis represents the search range. From this result, we confirm that the search range is narrowed when γ is increased.

Figure 4 shows the relationship between the search range and the parameter β_0. When β_0 is less than equal to 2, The firefly converges to the found best position. Such motion corresponds to the stagnation of the solution search process. Our proposed method can overcome such situation.

To confirm the search performance of our proposed FA, we carry out some numerical simulations. In order to confirm the search ability, we compare FA, the deterministic FA, and the proposed deterministic FA. The simulation conditions conditions are shown in Table 1. Table 2 shows benchmark functions used in our numerical simulations. Table 3 shows the simulation results. The simulation results indicate that the performance of the proposed FA is better than the deterministic FA. However, the performance of the proposed FA is worse than the conventional FA.

At first, the β_0 is a large value then the proposed method performs global search. After that, β_0 is reduced. The system exhibits the state transition to the local search state. Depending on such state transition, the proposed system can find a good solution than the deterministic FA.

6. CONCLUSIONS

In this article, we confirmed the effect of the parameters β_0 and γ. Based on the analysis results, we proposed the novel a deterministic FA with time variant parameter β_0 The numerical simulation results indicate that the solution search performance of the proposed deterministic FA is better than the deterministic FA. However, the performance is inferior than the conventional FA. To improve the search performance is the most important issue of our future problems.

Acknowledgement

This work was supported by Grant-in-Aid for Scientific Research(C) Number 15K06077.

Table 2: Benchmark Functions

Function	Equation
Shifted Griewank Function	$f_1(\boldsymbol{y}) = 1 + \frac{1}{4000}\sum_{d=1}^{D} \boldsymbol{y}_i^2 - \prod_{d=1}^{D}\cos(\frac{\boldsymbol{y}_i}{\sqrt{i}}),\ \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$
Shifted Rosenbrock Function	$f_2(\boldsymbol{y}) = \sum_{d=1}^{D-1}(100(\boldsymbol{y}_{d+1} - \boldsymbol{y}_d^2)^2 + (\boldsymbol{y}_d - 1)^2),\ \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$
Shifted Rastrigin Function	$f_3(\boldsymbol{y}) = 10N + \sum_{d=1}^{D}((\boldsymbol{y}_d^2 - 10\cos(2\pi \boldsymbol{y}_d))), \boldsymbol{y} = \boldsymbol{x} - \boldsymbol{o}$
Rotated Griewank Function	$f_4(\boldsymbol{x}) = f_1(\boldsymbol{z}), \boldsymbol{z} = M\boldsymbol{x}$
Rotated Rosenbrock Function	$f_4(\boldsymbol{x}) = f_2(\boldsymbol{z}), \boldsymbol{z} = M\boldsymbol{x}$
Rotated Rastrigin Function	$f_6(\boldsymbol{x}) = f_3(\boldsymbol{z}), \boldsymbol{z} = M\boldsymbol{x}$

$\boldsymbol{o}$ is uniform random number, M is $D \times D$ rotating matrix

Table 3: Simulation results

Function	Method	Mean value	Deviation	Best value	Worst value
Shifted Griewank Function	DFA	1135.69	83.09	938.64	1264.60
	The proposed DFA	852.89	136.57	718.29	1094.45
	FA	701.05	0.03	700.10	701.10
Shifted Rosenbrock Function	DFA	10082.96	2255.75	6622.65	14361.05
	The proposed DFA	1779.76	1668.28	732.96	6391.67
	FA	474.42	47.18	408.09	579.64
Shifted Rastrigin Function	DFA	1039.71	28.46	981.54	1079.02
	The proposed DFA	1007.59	49.36	925.08	1089.97
	FA	874.72	19.47	849.28	908.83
Rotated Griewank Function	DFA	705.96	1.80	703.80	710.13
	The proposed DFA	852.89	136.57	718.29	1094.45
	FA	700.90	0.09	700.76	701.020
Rotated Rosenbrock Function	DFA	520.43	49.27	446.94	625.26
	The proposed DFA	403.96	2.79	401.36	411.96
	FA	400.41	0.19	400.12	400.75
Rotated Rastrigin Function	DFA	932.93	11.65	914.94	955.46
	The proposed DFA	864.90	37.72	821.45	934.85
	FA	821.51	16.60	804.16	856.32

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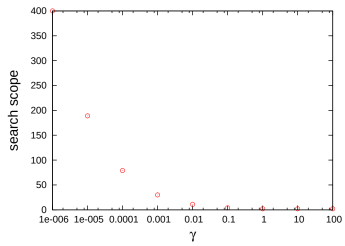

Figure 3: γ and search range

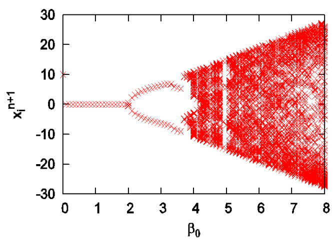

Figure 4: Solution search range in the state update 100-200 times