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A Method for Circuit Analysis using Haar Wavelet Transform with Adaptive Resolution

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Abstract– In this paper, we propose a method for the circuit analysis using wavelet transform with adaptive resolutions. Recently, many approaches to the circuit analysis using the wavelet transform have been proposed. However, there are few reports that the multiresolution features of the wavelets are sufficiently applied. The proposed method can choose adaptive resolution automatically using multiresolution analysis and achieve more accurate and efficient calculations.

1. Introduction

Recently, much attention has been paid to the method for circuit analysis using wavelet transform [7]–[16]. The wavelet transform is often used in signal processing because of its orthogonality and multiresolution property. In particular, Barmada et al. have proposed the Fourier-like approach for the circuit analysis using the wavelet transform [13]. In [13], the integral and differential operator matrices are introduced to the analysis, and the differential and integral equations are transformed into the algebraic equations like as using Fourier or Laplace transform. Moreover, the method can treat time varying or nonlinear circuits. Therefore, this method is useful for various circuit analyses.

However, in that method, the use of Daubechies wavelet makes the handling of the operator matrices complicated, especially, in the edges of the interval. Thus, we have proposed the circuit analysis method using Haar wavelet [16]. The Haar wavelet is easy to handle itself, and the operator matrices using the Haar wavelets are easily derived by introducing the block pulse functions [5], [6]. Moreover, the proposed method can treat the nonlinear time varying circuits.

In addition, Haar wavelets have the merit to be able to analyze the trajectory near the singular points where the trajectory moves rapidly with high resolution because of the orthogonality and localization property of the wavelet functions. As circuit analysis methods using this merit, some methods were proposed to pick out the ranges where the trajectory moves rapidly at singular point based on experimental prediction [12], [16]. However, these methods cannot be applied when the behaviors of the circuits can hardly be predicted. To overcome the problems described above, it is necessary to pick out the ranges automatically where the trajectory moves rapidly near singular points. In this paper, we propose a method for the circuit analysis using wavelet transform with adaptive resolutions by automatically picking out the ranges which require higher resolution analysis by using the result of the multiresolution analysis. By the proposed method, it is considered that more accurate and efficient calculation can be achieved even if the behavior of the circuit is hardly predicted. In this paper, we confirm the effectiveness of the above method using a simple example.

2. Haar Wavelet Matrix

Haar functions are defined on interval $[0,1)$ as follows,

$$h_0 = \frac{1}{\sqrt{m}}, \tag{1}$$

$$h_i = \frac{1}{\sqrt{m}} \times \begin{cases} 2^{\frac{j}{2}}, & \frac{k-1}{2^j} \le t < \frac{k-\frac{1}{2}}{2^j}, \\ -2^{\frac{j}{2}}, & \frac{k-\frac{1}{2}}{2^j} \le t < \frac{k}{2^j}, \\ 0, & \text{otherwise in } [0,1), \end{cases} \tag{2}$$

$$i = 0,1,\cdots,m-1, \; m = 2^{\alpha},$$

where α is positive integer, and j and k are nonnegative integers which satisfy $i = 2^j + k$, i.e., $k = 0,1,\cdots,2^j - 1$ for $j = 0,1,2,\cdots$.

$\vec{y}$ is $m \times 1$-dimensional vector whose elements are the discretized expression of $y(t)$ and $\vec{c}$ is $m \times 1$ -dimensional coefficient vector. H is $m \times m$ -dimensional Haar wavelet matrix defined as

$$H = \begin{bmatrix} \vec{h}_0 \\ \vec{h}_1 \\ \vdots \\ \vec{h}_{m-1} \end{bmatrix}, \tag{3}$$

where $\vec{h}_i$ is $1 \times m$ -dimensional Haar wavelet basis vector whose elements are the discretized expression of $h_i(t)$. Using these vectors and matrix, Haar wavelet transform and inverse Haar wavelet transform are described as follows, respectively,

$$\vec{c} = H\vec{y}, \tag{4}$$

$$\vec{y} = H^T\vec{c}\left(= H^{-1}\vec{c}\right). \tag{5}$$

3. Integral and Derivative Operator Matrices using Haar Wavelet

The basic idea of the operator matrix has been firstly introduced by using Walsh function [5]. However, in logical way, the matrices introduced by block pulse function are more fundamental [4], [5]. The block pulse function is the set of m rectangular pulses which have $1/m$ width and are shifted $1/m$ each other.

The integral operator matrix of the block pulse function matrix B is defined as the following equation [5], [6].

$$\int_0^i B(\tau)d\tau \equiv Q_B \cdot B(t), \tag{6}$$

$$Q_{B(m\times m)} = \frac{1}{m}\left[\frac{1}{2}I_{(m\times m)} + \sum_{i=1}^{\infty} P^i_{(m\times m)}\right], \tag{7}$$

where $B(t)$ is $m\times m$-dimensional matrix whose elements are the discretized expression of the block pulse functions $b_i(t)$, $i = 0,1,\cdots,m-1$ and

$$P^i_{(m\times m)} = \left[\begin{array}{c|c} 0 & I_{(m-i)\times(m-i)} \\ \hline 0_{(i\times i)} & 0 \end{array}\right] \text{ for } i < m,$$

$$P^i_{(m\times m)} = 0_{(m\times m)} \text{ for } i \geq m.$$

And the inverse matrix $Q^{-1}_{B_{(m\times m)}}$ is calculated as follows [5]:

$$Q^{-1}_{B_{(m\times m)}} = 4m\left[\frac{1}{2}I_{(m\times m)} + \sum_{i=1}^{m-1}(-1)^i P^i_{(m\times m)}\right]. \tag{8}$$

As the Haar wavelet matrix H is the set of the orthogonal functions, the integral matrix of H is given as follows:

$$Q_H = HQ_B^T H^{-1} = HQ_B^T H^T. \tag{9}$$

Similarly, the derivative matrix of H can be written as

$$Q_H^{-1} = H\left(Q_B^T\right)^{-1} H^{-1} = H\left(Q_B^T\right)^{-1} H^T. \tag{10}$$

4. Haar Wavelet Expression of Branch Characteristics of Nonlinear Time Invariant Circuit Elements

As the Haar wavelet is defined on interval $[0,1)$, the generic interval $\left[t_{\min}, t_{\max}\right)$ can be rescaled by a new variable τ on $[0,1)$, where $t = \left(t_{\max} - t_{\min}\right)\tau + t_{\min}$. In this paper, $t_{\min} = 0$ without loss of generality, then capacitance c[F] and inductance l[H] are scaled to $C = c/t_{\max}$ and $L = l/t_{\max}$, respectively.

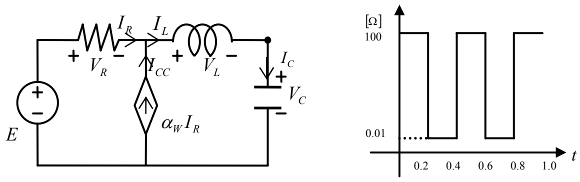
Next, we show the Haar wavelet expression of branch characteristics of nonlinear time varying circuit elements for the expression in wavelet domain. See details in [16].

Capacitor:

$$V = C_w^{-1}\left(C_0V_0 + Q_H I\right), \tag{11}$$

$$C_w = Hdiag\left[C\left(i_0,t_0\right), C\left(i_1,t_1\right), \cdots, C\left(i_{m-1},t_{m-1}\right)\right]H^T$$

Inductor:



(a) (b)

Fig. 1: (a) Example circuit in wavelet domain. (b) Characteristics of time varying resistor.

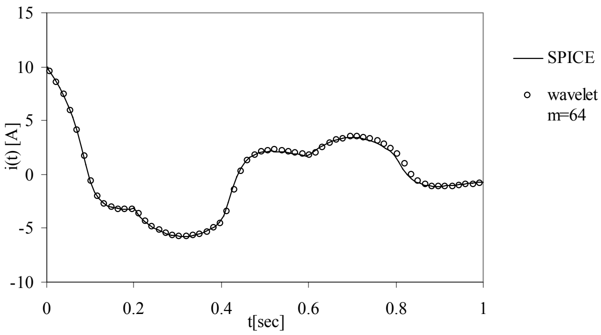


Fig. 2: Calculation result for $m = 64$ [16].

$$I = L_w^{-1}\left(L_0I_0 + Q_H V\right) \tag{12}$$

$$L_w = Hdiag\left[L\left(i_0,t_0\right), L\left(i_1,t_1\right), \cdots, L\left(i_{m-1},t_{m-1}\right)\right]H^T$$

Resistor:

$$V = R_w I \tag{13}$$

$$R_w = Hdiag\left[R\left(i_0,t_0\right), R\left(i_1,t_1\right), \cdots, R\left(i_{m-1},t_{m-1}\right)\right]H^T$$

5. Circuit Analysis using Adaptive Resolution with Multiresolution Analysis

In this section, we show a new method for automatically picking out the ranges to use the adaptive resolution by analyzing the example circuit shown in Fig. 1. The characteristics of resistor in Fig. 1 (a) is linear and time varying as shown in Fig. 1(b). The characteristics of the inductor is nonlinear such as $\phi\left(i_L\right) = \left(1 + 0.02i_L^{\,2}\right)i_L$. To derive the current through the inductor $i_L(t)$, using the branch characteristics of the circuit elements described in the previous section, the wavelet expression I_L of the current $i_L(t)$ is described as follows,

$$I_L = \left[R_W\left(I_{(m\times m)} + \alpha_w\right)^{-1} + Q_H^{-1}L_w + C_w^{-1}Q_H\right]^{-1} \cdot \left(E - V_0 + Q_H^{-1}L_w I_0\right) \tag{14}$$

Solving this algebraic equation and using the inverse transform by Eq. (5), the approximate solution $\vec{i}_L$ of $i_L(t)$ can be derived. In this paper, we take $t_{\max} = 1$, $t_{\min} = 0$,

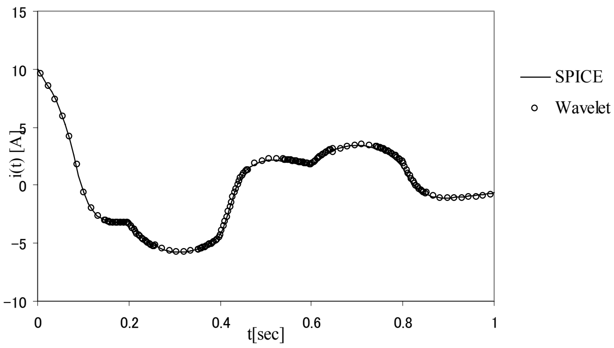


Fig. 3: Calculation result with mixed resolutions [16].

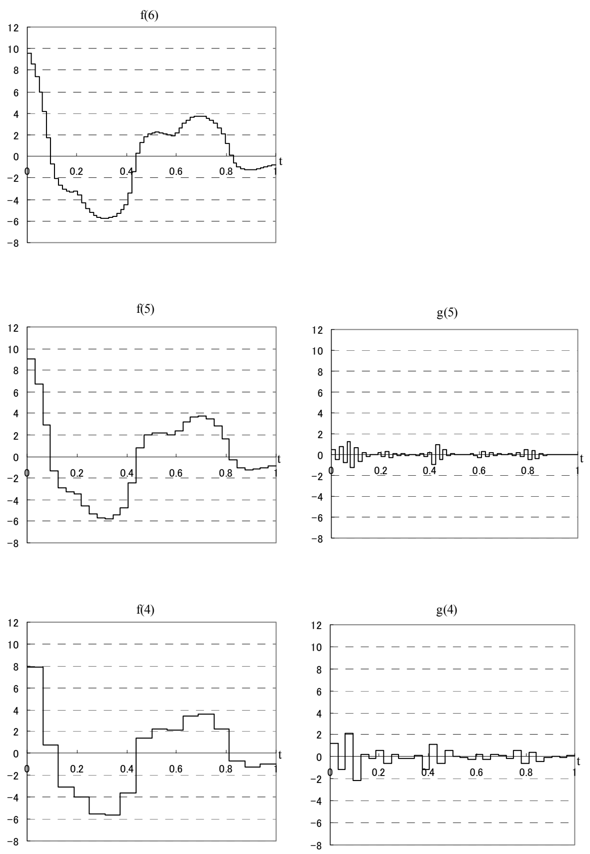


Fig. 4: Multiresolution analysis for $m = 64$.

C=0.006[F], $\alpha = 2$, $e(t) = 20t$ [V], $v_C(0_-) = 50$ [V], $i_L(0_-) = 10$[A], $R_{\max} = 100$ [Ω], $R_{\min} = 0.01$[Ω].

First, we show the result for $m = 64$ without the adaptive resolution in Fig. 2 [16]. If the waveforms have the singular points, more detailed analyses are needed by using the smaller intervals. However, it makes the calculation cost higher, for example, by taking $m = 128$. Therefore, we have proposed the method with the mixed resolutions as shown in Fig. 3 [16]. In the example circuit shown in Fig. 1, we can experimentally imagine that the singular points appear at the points where the resistance of

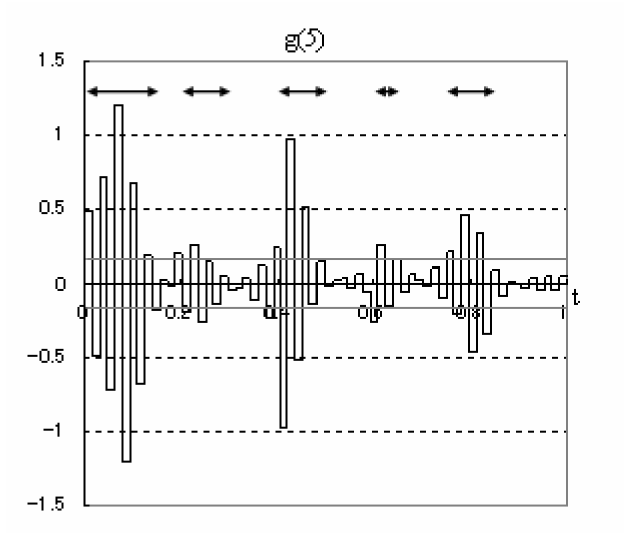


Fig. 5: Multiresolution analysis for $m = 64$ ($g(5)$).

Table 1: Adaptive resolution in picked out intervals.

picked out range	0–0.16	0.20–0.28	0.38–0.50	0.63–0.66	0.76–0.88
total plots p	10	6	8	2	8
sampling interval	1/32	1/16	1/16	1/4	1/16

the time varying resistor switches. Thus, in this case, we analyzed it with higher resolutions around such singular points.

However, we cannot predict the singular points for every circuit in the real world. To overcome this problem, we need to develop the method to pick out automatically the ranges for higher resolution analyses. For this purpose, in this study, we propose the method using the results of the multiresolution analysis.

Figure 4 shows the results of the multiresolution analysis of the waveform shown in Fig. 2. $f(j)$ indicates the amounts of the results for $0,1,\cdots,j-1$-th resolution wavelets and $g(j)$ indicates the j-th resolution ones. Note that $f(6) = f(5) + g(5)$. Because $g(j)$ can be considered as the error between the lower and the higher resolution cases, if $g(j)$ is large then the approximation is not sufficient. On the other hand, if $g(j)$ is small, it is considered that the approximation has already achieved. In the proposed method, we take the threshold ε for $g(j)$, then pick out the ranges for $|g(j)| > \varepsilon$. Figure 5 shows the magnified trace of $g(5)$. In this example, we take $\varepsilon = 0.13$ and the thin lines indicate the threshold $\pm\varepsilon$. In this case, the arrows in the figure describe the picked out ranges. In those ranges, we analyze the circuit again with higher resolution wavelets. To avoid the over sampling, we set the sampling width as $1/2^{\alpha}$ to satisfy $2^{\alpha-1} < 2p \le 2^{\alpha}$ where p is the number of plots in the picked out ranges. Table 1 shows the relationship between the picked out ranges and the sampling width. The result for proposed method is shown in Fig. 6. The total number of plots is 123 and it is less

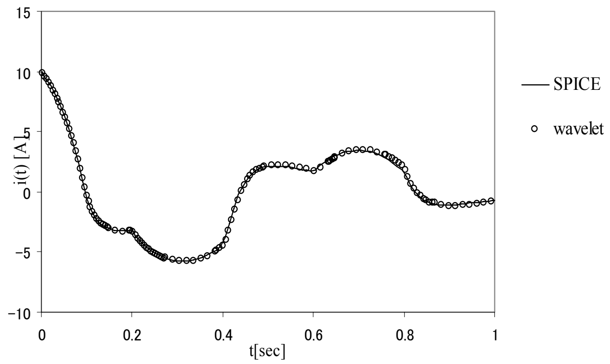


Fig.6: Calculation result with adaptive resolution.

than 128 that are the number of plots for the case of one higher resolution analysis. Moreover, it is less than 168 for mixed resolution case shown in Fig. 3. Applying this process recursively, it is considered that more accurate and efficient calculation can be achieved for circuit analysis.

6. Conclusions

In this paper, we have proposed a method for the circuit analysis using wavelet transform with adaptive resolutions. In particular, the ranges for higher resolution analysis are picked out automatically by using the result of the multiresolution analysis. By the proposed method, it is considered that more accurate and efficient calculation can be achieved even if the behavior of the circuit is hardly predicted.

At this moment, the accuracy of the method is not enough discussed. The accuracy depends on the setting of ε, and the discussion how to set the threshold ε is one of our future problems.

Acknowledgments

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