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A dendrite model based on asynchronous bifurcation processor

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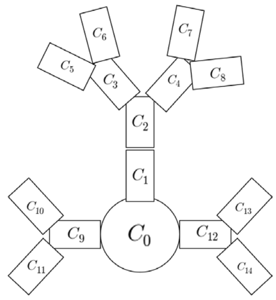
Abstract—In this paper, a multi-compartment dendrite model based on an asynchronous bifurcation processor is presented. It is shown that the presented model can exhibit propagation phenomena of membrane potentials that are typically observed in dendrites of neurons.

1. Introduction

A neuron typically consists of a dendrite (input cable), a soma (cell body), and an axon (output cable), where the dendrite sometimes has a complicated physical structure such as the one in Fig. 1 [1]. It has been suggested that the physical structure of the dendrite plays a certain role in signal processing of a neuron. One of the major modeling method of a dendrite is to discretize a dendrite into a set of small compartments and to model the dendrite by a coupled system of the compartments, where such a dendrite model is often called a *multi-compartment dendrite model* [2]. Note that each compartment model is designed to reproduce the nonlinear dynamics of a membrane potential of the corresponding part of the dendrite. Concerning the membrane potential model (i.e., a single compartment model in the context of this paper), many mathematical and electronic hardware models have been presented (see the references in [3]). These membrane potential models can be classified into the following four classes based on the continuousness and discontinuousness of state variables and times.

(a)



(b)

Figure 1: (a) Complicated structure of dendrite of Purkinje cell [1]. (b) Multi-compartment model of dendrite [2].

- Class CTCS (continuous time and continuous state): A nonlinear differential equation model of a membrane potential, which has a continuous time and continuous states. Such a model can be implemented by an analog nonlinear circuit.
- Class DTCS (discrete time and continuous state): A nonlinear difference equation model of a membrane potential, which has a discrete time and continuous states. Such a model can be implemented by a switched capacitor circuit or by a Poincare map.
- Class DTDC (discrete time and discrete state): A numerical integration model of a membrane potential, which has a discrete time and discrete states. Such a model can be implemented by a digital processor. A cellular automaton model of a membrane potential also belongs to this class, which has a discrete time and discrete states. Such a model can be implemented by a traditional synchronous sequential logic circuit.
- Class CTDS (continuous time and discrete state): An asynchronous cellular automaton model of a membrane potential, which has a continuous (state transition) time and discrete states. Such a model can be implemented by an asynchronous sequential logic circuit.

Our group has been developing several neural system models belonging to the class CTDS (see [3][4] and references therein) and is referring to class CTDS systems designed to exhibit nonlinear phenomena (especially, bifurcation phenomena) as *asynchronous bifurcation processors*. In this paper, a multi-compartment dendrite model based on the asynchronous bifurcation processor is presented. It is shown that the presented model can reproduce typical phenomena observed in dendrites. Note that our group has presented a multi-compartment dendrite model coupled via firing spikes [4], whereas this paper present a multi-compartment dendrite model coupled via membrane potential (i.e., coupling via gap junction).

2. Multi-compartment dendrite model based on ABP

2.1. Structure of the whole dendrite model

Fig. 2 shows a sketch of the presented multi-compartment dendrite model based on the asynchronous bifurcation processor. Major components of the dendrite model are as follows.

- The whole multi-compartment dendrite model has $Q > 0$ asynchronous cellular automaton membrane potential models.
- Each i-th compartment can accept the following stimulation input (not necessarily).

$$I_i(t) = \begin{cases} 1 & \text{if } t \in \{t_i^{(1)}, t_i^{(2)}, \cdots\}, \\ 0 & \text{otherwise,} \end{cases}$$

  where $t_i^{(n)}$ is the n-th spike timing (or rising edges) of the stimulation input $I_i(t)$.

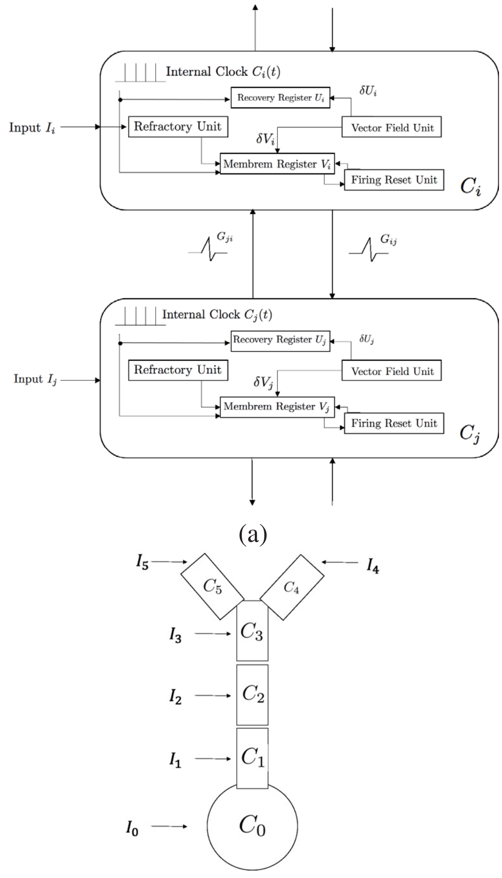
- Each i-th compartment has a membrane register storing a discrete membrane potential $V_i \in \{0, 1, \cdots, N-1\}$, where the integer parameter $N > 0$ determines the resolution of the discrete membrane potential V_i.
- Each i-th compartment has a recovery register storing a discrete recovery variable $U_i \in \{0, 1, \cdots, M-1\}$, where the integer parameter $M > 0$ determines the resolution of the discrete membrane potential U_i.
- The i-th and the j-th compartments are coupled via a discrete conductance $g_{ij} \in \{0, 1, \cdots, L-1\}$, where the integer parameter $L > 0$ determines the resolution of the discrete conductance g_{ij}.
- Each i-th compartment has the following internal clocks.

$$CLK_{Vi}(t) = \begin{cases} 1 & \text{if } t \in \{t_V^{(1)}, t_V^{(2)}, \cdots\}, \\ 0 & \text{otherwise,} \end{cases}$$

$$CLK_{Ui}(t) = \begin{cases} 1 & \text{if } t \in \{t_U^{(1)}, t_U^{(2)}, \cdots\}, \\ 0 & \text{otherwise,} \end{cases}$$

$$CLK_{gi}(t) = \begin{cases} 1 & \text{if } t \in \{t_g^{(1)}, t_g^{(2)}, \cdots\}, \\ 0 & \text{otherwise,} \end{cases}$$

  which trigger asynchronous transitions of the discrete states, where $t_V^{(n)}$, $t_U^{(n)}$, and $t_g^{(n)}$ represent spike timings (or rising edges) of the clocks.



(b)

Figure 2: A sketch of the presented multi-compartment dendrite model based on asynchronous bifurcation processor. (a) Structures of the i-th and j-th compartments for the case of $C_i = CLK_{Vi} = CLK_{Ui} = CLK_{gi}$ and $C_j = CLK_{Vj} = CLK_{Uj} = CLK_{gj}$. (b) Structure of multi-compartment model consisting of $Q = 6$ compartments studied in this paper.

2.2. Nonlinear dynamics and parameter setting of each compartment

Referring to our previous work in [4], in this paper we introduce a single compartment model based on the asynchronous bifurcation processor whose parameter values are specialized to this paper. The discrete states of the compartment transit as follows.

Asynchronous states transitions triggered by clocks

$$V_i(t^+) = V_i(t) + D_V(V_i, U_i) \;\text{ if }\; CLK_V(t) = 1,$$
$$U_i(t^+) = U_i(t) + D_U(V_i, U_i) \;\text{ if }\; CLK_U(t) = 1,$$

where D_V and D_U are discrete functions defined by

$$\begin{array}{ll} D_V(V_i,U_i)=1 & \text{if } (V_i,U_i)\in \mathbf{S}_i^{++}\cup\mathbf{S}_i^{+-},\\ D_V(V_i,U_i)=-1 & \text{if } (V_i,U_i)\in \mathbf{S}_i^{-+}\cup\mathbf{S}_i^{--},\\ D_V(V_i,U_i)=0 & \text{if } (V_i,U_i)\in \mathbf{S}_i^{0},\\ D_U(V_i,U_i)=1 & \text{if } (V_i,U_i)\in \mathbf{S}_i^{++}\cup\mathbf{S}_i^{-+},\\ D_U(V_i,U_i)=-1 & \text{if } (V_i,U_i)\in \mathbf{S}_i^{+-}\cup\mathbf{S}_i^{--},\\ D_U(V_i,U_i)=0 & \text{if } (V_i,U_i)\in \mathbf{S}_i^{0}, \end{array}$$

$$\begin{array}{l} \mathbf{S}_i^{++}\equiv\{(V_i,U_i)|U_i<f_V(V_i),U_i\le f_U(V_i)\},\\ \mathbf{S}_i^{-+}\equiv\{(V_i,U_i)|U_i\ge f_V(V_i),U_i<f_U(V_i)\},\\ \mathbf{S}_i^{+-}\equiv\{(V_i,U_i)|U_i\le f_V(V_i),U_i>f_U(V_i)\},\\ \mathbf{S}_i^{--}\equiv\{(V_i,U_i)|U_i>f_V(V_i),U_i\ge f_U(V_i)\},\\ \mathbf{S}_i^{0}\equiv\{(V_i,U_i)|(V_i,U_i)\notin\mathbf{S}_i^{++}\cup\mathbf{S}_i^{+-}\cup\mathbf{S}_i^{-+}\cup\mathbf{S}_i^{--}\}, \end{array}$$

where f_V and f_U are discrete functions defined by

$$\begin{array}{l} f_V(V_i)=\alpha(\lfloor k_1(V_i)^2+k_2V_i+k_3\rfloor),\\ f_U(V_i)=\alpha(\lfloor k_4V_i+k_5\rfloor),\\ k_1=\frac{f_1M}{N^2},\\ k_2=-2k_1\lfloor f_2N\rfloor,\\ k_3=k_1(\lfloor f_2N\rfloor)^2+\lfloor f_3M\rfloor,\\ k_4=\frac{f_4M}{N},\\ k_5=\lfloor f_5M\rfloor. \end{array}$$

Asynchronous state transition triggered by stimulation

$$V_i(t^+)=V_i(t)+1 \;\text{ if }\; I_i(t)=1,$$

where $\lfloor\cdot\rfloor$ is the floor function.

In this paper we propose to use the following parameter values for the multi-compartment model.

$$(N,M,f_1,f_2,f_3,f_4,f_5,\alpha,)$$
$$=(64,64,3.5,0.45,-0.05,1.5,-0.43,10).$$

2.3. Coupling dynamics and parameter setting

In this paper we present the following coupling dynamics from the j-th soma-side compartment to the i-th spine-side compartment.

Asynchronous coupling dynamics triggered by clock

$$V_i(t^+)=V_i(t)+g_{ij}(V_i(t)-V_j(t)) \;\text{ if }\; CLK_{gi}=1,$$

where

$$g_{ij}=\begin{cases}1 & \text{if } V_i-V_j>=20,\\ 1 & \text{if } 20>V_i-V_j>=0,\\ 5 & \text{if } V_i-V_j>-20,\\ 3 & \text{if } -20>=V_i-V_j,\\ 1 & \text{otherwise,}\end{cases}$$

$$g_{ji}=\begin{cases}8 & \text{if } V_j-V_i>=20,\\ 8 & \text{if } 20>V_j-V_i>=0,\\ 5 & \text{if } 0>V_j-V_i>-20,\\ 2 & \text{if } -20>=V_i-V_i,\\ 6 & \text{otherwise.}\end{cases}$$

2.4. Reproductions of typical dendritic phenomena

Figs. 3 and 4 show reproductions of typical dendritic phenomena by the presented model. In Fig. 3(a), stimulations I_4 and I_5 are applied to the 4-th and the 5-th compartments, respectively. In this case, activities of the membrane potentials of the 4-th and the 5-th compartments are evoked, but these activities do not propagate to the somatic compartment. In Fig. 3(b), the same stimulations I_4 and I_5 are applied to the 4-th and the 5-th compartments, respectively, and a very weak noisy spike-train n is applied to each compartment. In this case, activities of the membrane potentials of the 4-th and the 5-th compartments do not propagate to the somatic compartment. In Fig. 3(c), the same stimulations I_4 and I_5 are applied to the 4-th and the 5-th compartments, respectively, and a weak noisy spike-train n is applied to each compartment. In this case, activities of the membrane potentials of the 4-th and the 5-th compartments propagate to the somatic compartment, i.e., the presented model exhibits forward propagations of membrane potentials. In Fig. 4(c), a stimulation I_0 is applied to the 0-th somatic compartment. In this case, activities of the membrane potentials of the 0-th somatic compartment propagates to spine-side compartments, i.e., the presented model exhibits backward propagations of membrane potentials.

3. Conclusions

In this paper, the multi-compartment dendrite model based on an asynchronous bifurcation processor was presented. It was shown that the presented model can reproduce typical phenomena observed in dendrites. Future problems include (a) more detailed analysis of the presented model, (b) hardware implementation of the presented model, and (c) quantitative comparisons of hardware costs of the presented model and a DSP dendrite model.

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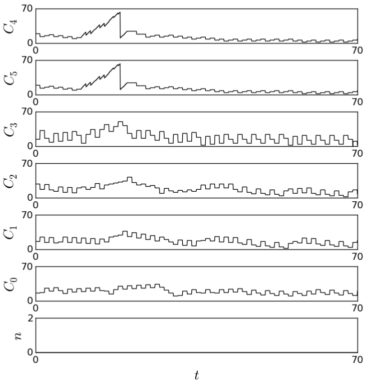
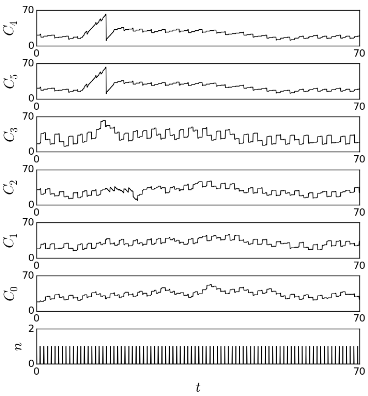


(a)



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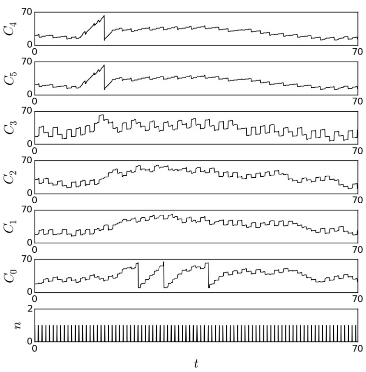
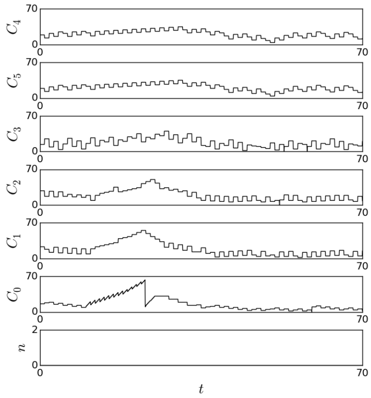
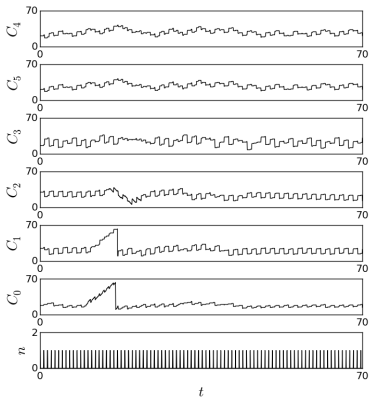


(c)

Figure 3: Reproductions of typical dendritic phenomena by the presented model. Stimulations I_4 and I_5 are applied to the 4-th and 5-th compartments, respectively. (a) Failure of forward propagation. (b) Failure of forward propagation. A very weak noisy spike-train n is applied to each compartment. (c) Forward propagation. A weak noisy spike-train n is applied to each compartment.



(a)



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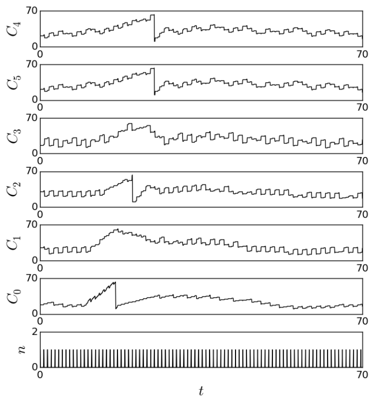

(c)

Figure 4: Reproductions of typical dendritic phenomena by the presented model. A stimulation I_0 is applied to the 0-th somatic compartment. (a) Failure of backward propagation. (b) Failure of backward propagation. A very weak noisy spike-train n is applied to each compartment. (c) Backward propagation. A weak noisy spike-train n is applied to each compartment.