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Optimal Backstepping Control for Genesio–Tesi chaotic system Using Genetic Algorithm

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Abstract– This paper has presented chaos synchronization in the Genesio-Tesi using the backstepping approach. Backstepping approach consists of parameters which accept positive values. The parameters are usually chosen optionally. The system response is different for each value. It is necessary to select proper parameters to obtain a good response because the improper selection of the parameters leads to inappropriate responses or even to instability of system. Genetic algorithm can select appropriate and optimal values for the parameters. GA by minimizing the fitness function can find the optimal values for the parameters. This selected fitness function is for minimizing the least square error. Fitness function forces the system error to decline to zero rapidly that causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach its minimum value. This hybrid makes an optimal backstepping controller.

1. Introduction

Chaos is a very interesting nonlinear phenomenon and has applications in many areas. One of the important problems in chaotic systems is synchronization. A robust adaptive PID controller for chaotic systems had been presented in [1]. Linear feedback for controlling chaos and Routh–Hurwitz criteria based on stability analysis has been done in [2]. Chaos suppression of Genesio system is achieved to use adaptive feedback linearization-based controller in [3]. Exponential Synchronization in the Genesio Tesi via a novel feedback control has been presented in [4]. Considerable effort has been also done to design control systems using feedback linearization and backstepping design technique for deterministic as well as uncertain chaotic systems [5-10]. Synchronization in the Genesio Tesi via Backstepping Approach has been presented in [11]. Backstepping design based on synchronization of two Genesio chaotic systems is proposed in [12]. Genetic algorithms(GAs) have been extensively applied to the off-line design of controllers [13].

Until now for controlling synchronization, Genesio-Tesi chaos has been used. In many of these methods, the designed controller has high overshoot and the system is reached stability after a long time. In some other controllers, the system is reached stability in appropriate time, but the system error is too much for a while.

In the backstepping controllers, the parameters of controller are chosen arbitrarily. In these controllers if we change some of these values, the system will be led to instability and it won't respond well. In these controllers, the system's behavior may not be good and they may have high overshoot. In some other controllers, the system has mush setting time and they may have oscillation behavior and show bad errors.

The paper is organized as follows: Section 2 describes Genesio-Tesi chaotic system. In section 3, a backstepping controller for synchronization is designed. Section 4 describes GA and algorithm used here. In section 5 a backstepping controller is designed for step tracking. Section 6 provides the conclusion.

2. Genesio-Tesi Chaotic System

Genesio–Tesi chaotic system can be represented by following set of nonlinear differential equations [14]:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= -ax_3 - bx_2 - cx_1 + dx_1^2 + ku\end{aligned} \qquad (1)$$

where x_1, x_2 and x_3 are state variables, and a, b and c are positive real constants satisfying $ab < c$. For instance, the system is chaotic for the parameters $a = 1.2$, $b = 2.92$ and $c = 6$. Here, a, b and c are linear parameters and d is nonlinear parameter which is taken as one without loss of generality. Constant scalar k is assumed to be known and u is the control input to the model. The initial condition for the states is taken as $x(0) = [0.1; -0.2; 0.2]^T$. Before controlling (u=0) the nonlinear Genesio-Tesi given by equation (1) exhibits varieties of dynamical behavior including chaotic motion - displayed in Figure 1.

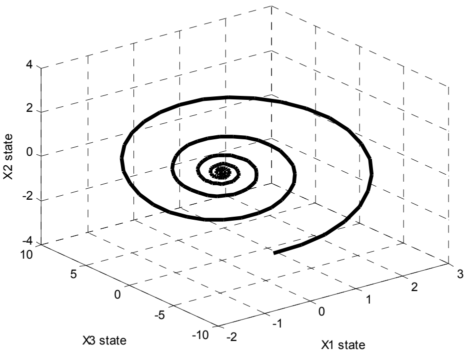

Fig. 1. Phase portrait of system with controller and without Genetic Algorithm.

3. Design Controller with Backstepping

The backstepping is used to bring the states x_1, x_2, x_3 to the desired references via the torque u calculated with four steps.

Step 1.

Consider the first subsystem of equation (1).

$$\dot{x}_1 = x_2 \tag{2}$$

Construct the joint Lyapunov function.

$$V_0(x_1) = \frac{1}{2}x_1^2 \tag{3}$$

Take x_2 as a virtual control input and choose.

$$x_2 = \Phi_0(x_1) = -x_1 \tag{4}$$

Step 2.

Consider (x_1, x_2) of equation (1):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \end{aligned} \tag{5}$$

Take x_3 as a virtual control input and choose:

$$x_3 = \Phi_1(x_1, x_2) = -(x_1 + x_2)(1 + k_1) \tag{6}$$

And take the Lyapunov function as

$$V_1(x_1, x_2) = V_0 + \frac{1}{2}(x_2 - \Phi_0)^2 \tag{7}$$

Step 3.

Consider all system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_3 - bx_2 - cx_1 + dx_1^2 + ku \end{aligned} \tag{8}$$

Take u as an actual control input and choose:

$$u = \Phi_2(x_1, x_2, x_3) = \frac{1}{k}[\frac{\partial \Phi_1}{\partial x_1}x_2 + \frac{\partial \Phi_1}{\partial x_2}x_3 - (x_1 + x_2) - k_2(x_3 - \Phi_1) - (-cx_1 - bx_2 - ax_3 + dx_1^2)] \tag{9}$$

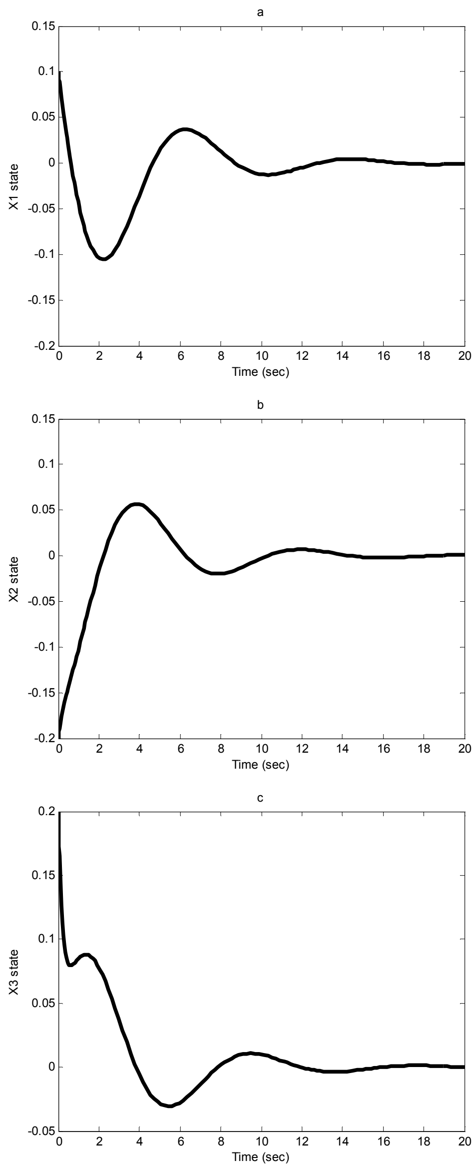

Fig. 2. Response of system with controller and without GA, (a), (b) and (c) state trajectory variation for Genesio system.

And Lyapunov function as:

$$v_2(x_1, x_2, x_3) = v_1 + \frac{1}{2}(x_3 - \Phi_1)^2 \tag{10}$$

Before using the Genetic Algorithm the estimated results are shown in figures 2. Figure 2a–c shows the state trajectory variation for Genesio system.

The backstepping method was expanded for class of nonlinear MIMO systems [15].

4. Genetic Algorithm

In its most general usage, genetic algorithms refer to a family of computational models inspired by evolution. These algorithms start with many initial points in order to cover all search intervals and encode a potential solution to a specific problem on a simple chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implantation of genetic algorithms begins with a population of chromosomes randomly bred. We evaluate each chromosome by using the objective function called Fitness function. In order to apply the genetic reproductive operations called crossover and mutation, we select, randomly, two individuals called parents and we apply the crossover operation, if its probability reaches, between parents by exchanging some of their bits to produce two children. A mutation is the second operator applied on the single children by inverting its bit if the probability reaches. After this stage we obtain two population : a parent population and a children population, the individual who has a goodness solution is preserved [16].

The genetic algorithms are used to search the optimal parameters k_j in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response. The fitness function used is

$$f = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(x_i - x_{di})^2} \tag{11}$$

x_i is system state and x_{di} is favorite mood for x_i. Based the system purpose for placing the states at zero value; $x_{di} = 0$. By the training, can be obtained optimal parameters as $k_1 = 0.708$ and $k_2 = 0.17$.

Table 1. Genetic algorithm parameters

Parameters	Values
Size population	100
Maximum of generation	300
Prob.crossover	75
Prob.mutation	0.001
K_j search interval de	[0.1 10]

After using the Genetic Algorithm the result estimated are showed in figure (3). Figure 3a–c shows the state trajectory variation for Genesio system.

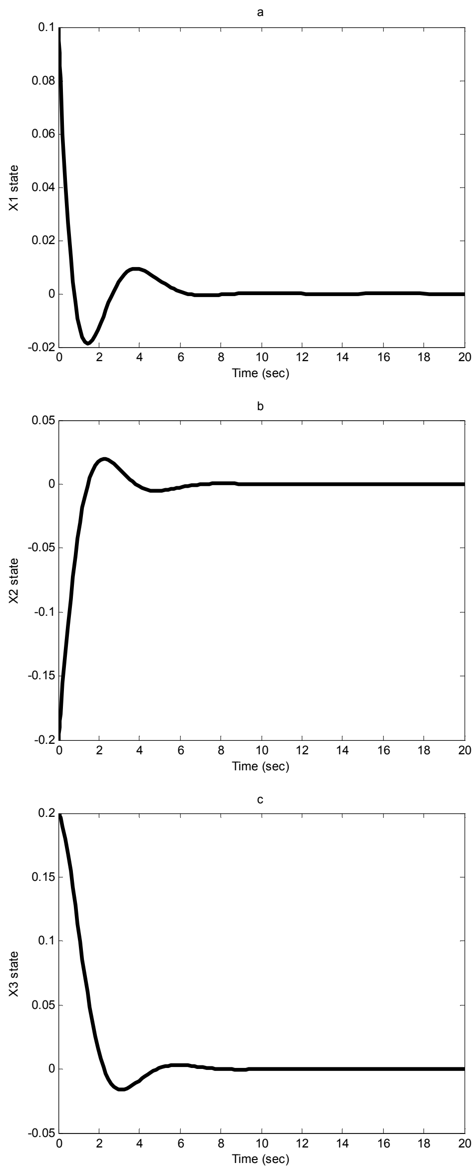

Fig. 3. Response of system with controller and GA, (a), (b) and (c) state trajectory variation for Genesio system

5. Step Response Tracking

Suppose, the x_1 state would be output of the system and it would track the input response. . In this case by using the change of variable $y = 1 - x_1$ equation (1) would be converted to the equation (12)

$$\begin{aligned} \dot{y} &= -x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_3 - bx_2 - c(1-y) + d(1-y)^2 + ku \end{aligned} \tag{12}$$

Take u as an actual control input and choose:

$$\begin{aligned} u = \Phi_2(y, x_2, x_3) = \frac{1}{k}[-\frac{\partial \Phi_1}{\partial y}x_2 + \frac{\partial \Phi_1}{\partial x_2}x_3 - (x_2 - y) \\ -k_2(x_3 - \Phi_1) - (-c(1-y) - bx_2 - ax_3 + d(1-y)^2)] \end{aligned} \tag{13}$$

After using GA for the controller equation (13) obtain $k_1 = 9.965,\ k_2 = 5.114$. The setting time of optimal backstepping controller obtain 4.785s. The system response of step tracking is shown in figure 4.

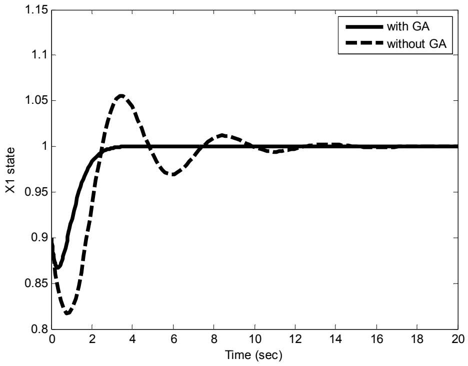

Fig 4. Step input tracking by Genesio-Tesi system

5. Conclusion

This paper has presented a new hybrid backstepping approach with genetic algorithm demonstrated to have more optimal behavior when compared with previous methods. This approach is used for chaos synchronization in the Genesio-Tesi by using backstepping method to controll the Genesio-Tesi chaos. The designed controller consists of parameters which accept positive values. The controlled system presents different behaviors for different values. Improper selection of the parameters causes an improper behavior which may cause serious problems such as instability of system.

Genetic algorithm optimizes the controller to gain optimal and proper values for the parameters. For this reason, GA minimizes the fitness function to find minimum current value for it. On the other hand, fitness function finds minimum value minimizing least square errors.

By this approach, the setting time and overshoot reach their minimum values demonstrated to have more optimal values when compared with previous methods. Also by selecting different fitness functions can have other appropriate results.

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