2008 International Symposium on Nonlinear Theory and its Applications
NOLTA'08, Budapest, Hungary, September 7-10, 2008

Realization of Bond Graph Models by Wave Digital Structures

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Abstract—Bond graphs are a well established graphical formalism for modeling multidisplinary dynamic systems. This formalism allows to incorporate the energetic and the topological properties of the original system into the resulting model. Wave digital structures show many of those features and so the use of wave digital simulation techniques is proposed. For a simple bond graph, it is demonstrated that there is a one-to-one correspondence between the bond graph elements and the elements of the resulting wave digital structure, which makes it quite easy to derive a simulation algorithm.

1. Introduction

The path from an open problem in physics or engineering to some appropiate numerical results can roughly be described by Fig. 1 [1].

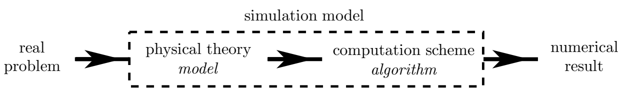

Figure 1: Concept of modeling process

The real system and the numerical results are linked via a simulation model which consists of two main parts. In the first part physical theory is employed for the actual modeling process, while the second refers to the solution of this model on a digital computer.

If a lumped approach is possible, the bond graphs concept is a suitable tool for the modeling process (cf. [1]). It is a graphical description of the dynamic behavior of multidisciplinary physical systems. Systems originating from different physical domains are described in the same manner. The connecting physical concepts are energy and energy exchange.

The resulting bond graph is essentially a labeled, often directed, graph in which the vertices represent idealised descriptions of physical phenomena like storage or dissipation of energy, while the edges, called bonds, represent the power exchange between these phenomena.

The most striking positive features of this concept are the use of power conservation as an ordering principle and the possibility of modular modeling. Thus the bond graph structure reflects the energetic behavior and the topology of the original system and thereby should lead to more reliable models.

However, when it comes to the actual simulation, usually a state space model is derived from the bond graph in order to employ standard numerical algorithms. By doing this, many of the above-mentioned positive features are not represented in the resulting algorithm.

Since a special feature of wave digital structures is their one-to-one correspondence between the elements of the underlying reference circuit and the resulting simulation algorithm, they show exactly the desired properties (cf. [2]). So it seems to make sense to examine how wave digital structures can be employed to simulate bond graph models.

If the bond graphs junction structure is a tree (loop-free) there is an almost one-to-one correspondence between the bond graph elements and the elements of the wave digital structure, which will be demonstrated in the following sections.

2. Ports, interconnections, and elements

In this section some relations between bond graph elements and wave digital elements will be shown. Due to limited space, this will be restricted to elements needed for the example in section 3. In each case, a figure will show the bond graph symbol, an electrical equivalence, and the corresponding wave digital representation.

2.1. Ports

The key concept of both, bond graphs and wave digital structures, is the notion of a port, which is the place where energy exchange takes place. A port consists of two terminals and is assigned with an across variable e (called effort) and a through variable f (called flow). In a bond graph, a port is connected to a straight line, labeled with effort and flow. The indication of the power direction will be omitted in this paper. In the electrical domain efforts and flows represent voltages and currents, respectively (see Fig. 2).

In the wave digital domain, a port is represented by so called wave quantities (waves for short). These are defined by

$$\begin{aligned} a &= e + Rf, \\ b &= e - Rf \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} e &= (a+b)/2, \\ f &= (a-b)/(2R), \end{aligned} \qquad (1)$$

with a port resistance $R > 0$ assigned to each port. The wave a is called the incident wave, b is called the reflected wave. While the causal relationship between efforts and flows is not clear in some cases, reflected waves are always caused by incident waves.

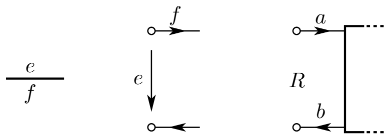

Figure 2: Definition of a port

2.2. Interconnections

Interconnections model the way in which the energy transfer between ports takes place. There exist two basic types of interconnections. The first is the common effort interconnection. For n ports it is described by

$$e_1 = e_2 = \ldots = e_n \quad \text{and} \quad \sum_{\nu=1}^{n} f_\nu = 0. \qquad (2)$$

The corresponding bond graph element is called a **0**-junction. The second one is the common flow interconnection, which is described by

$$f_1 = f_2 = \ldots = f_n \quad \text{and} \quad \sum_{\nu=1}^{n} e_\nu = 0. \qquad (3)$$

In bond graph terminology this is called a **1**-junction.

Note, that (2) and (3) are equivalent to Kirchhoff's current law and voltage law, respectively.

2.2.1. Coupling of two ports

For simplicity only the case of equal port resistances is considered. A common effort interconnection of two ports is then constituted by the equations

$$\begin{aligned} e_1 &= e_2, \\ f_1 &= -f_2 \end{aligned} \quad \text{resp.} \quad \begin{aligned} b_1 &= a_2, \\ b_2 &= a_1. \end{aligned} \qquad (4)$$

Bond graph symbol, electrical circuit equivalence, and wave digital repesentation are shown in Fig. 3. This **0**-junction is contractable and can be replaced by a single bond.

A common flow interconnection of two ports is given by

$$\begin{aligned} e_1 &= -e_2, \\ f_1 &= f_2 \end{aligned} \quad \text{resp.} \quad \begin{aligned} b_1 &= -a_2, \\ b_2 &= -a_1. \end{aligned} \qquad (5)$$

The respective repesentations are shown in Fig. 4.

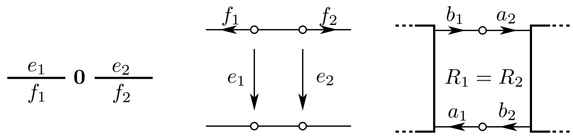

Figure 3: Common effort coupling of ports

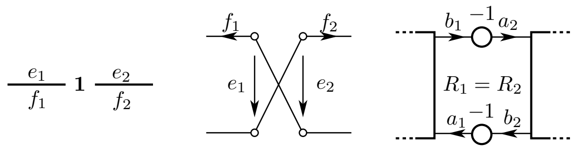

Figure 4: Common flow coupling of ports

2.2.2. Interconnections of three ports

Next, interconnections of three ports with arbitrary port resistances are examined. The corresponding wave digital element to the common effort interconnection is the parallel adaptor depicted in Fig. 5. The parallel adaptor represents the wave flow diagram which maps the incident waves to the reflected waves by

$$b_\nu = -a_\nu + \sum_{\mu=1}^{3} \gamma_\mu a_\mu \quad \text{with}\, \gamma_\mu = \frac{2G_\mu}{G_1 + G_2 + G_3}, \qquad (6)$$

for $\nu = 1, 2, 3$ and with the port conductances $G_\mu = 1/R_\mu$ of the individual ports.

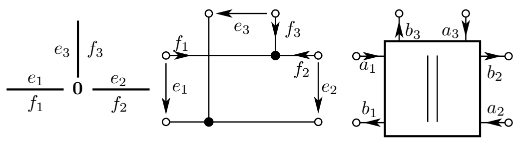

Figure 5: Common effort interconnection

The wave digital element corresponding to the common flow interconnection is the series adaptor shown in Fig. 6 where the reflected waves are calculated as follows

$$b_\nu = a_\nu - \gamma_\nu \sum_{\mu=1}^{3} a_\mu \quad \text{with}\, \gamma_\nu = \frac{2R_\nu}{R_1 + R_2 + R_3}. \qquad (7)$$

More complex interconnections can be modelled by successive interconnection of the three-port interconnections, just described.

2.3. Some elements

In this section, those elements will be presented which are linear models of two basic physical effects, energy storage and energy dissipation. In most physical domains there exist two kinds of energy storage elements. The first kind is the generalization of the electrical capacitance. In bond graph terminology it is

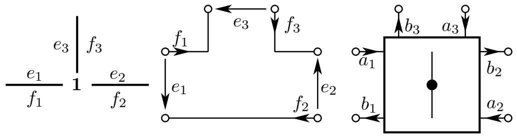

Figure 6: Common flow interconnection

a **C**-element. This element stores energy in the form of the integrated flow, q, which is a generalized charge, and is ruled by the equations

$$q = ce \quad \text{and} \quad f = \dot{q}, \tag{8}$$

where $c > 0$ denotes the value of the capacitance. The bond graph symbol is shown in Fig. 7. To get the corresponding wave digital element, some discretization rule preserving the passivity of the element is employed. Applying the (lossless) trapezoidal rule to the integral

$$e(t) = e(t-T) + \frac{1}{c}\int_{t-T}^{t} f(\tau)\, d\tau \tag{9}$$

with stepsize $T > 0$, and choosing the port resistance $R = T/(2c)$ results in the simple equation

$$b(kT) = a((k-1)T). \tag{10}$$

This is a simple delay in the wave digital domain (Fig. 7). Please note, that the wave digital concept is

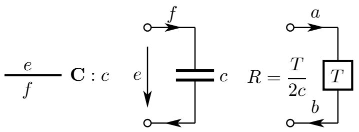

Figure 7: Capacitance element

not restricted to the use of the trapezoidal rule. It has been extended to other passive integration methods of higher accuracy [3].

The second energy storing element is the inertia element, the **I**-element in bond graph terminology. It is dual to the generalized capacitance and is in the electrical domain represented by an inductance. It stores energy in form of the integrated effort, p, and is ruled by the equations

$$p = mf \quad \text{and} \quad e = \dot{p}, \tag{11}$$

with $m > 0$. Applying the trapezoidal rule and choosing the port resistance $R = (2m)/T$ yields

$$b(kT) = -a((k-1)T) \tag{12}$$

in the wave digital domain (Fig. 8).

To model the dissipation of energy, a resistance element is used. The corresponding bond graph **R**-element is shown in Fig. 9. Since there is no definite

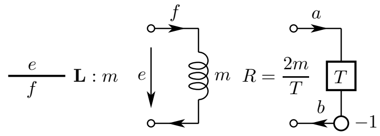

Figure 8: Inertia element

input-output relationship between effort and flow the defining equation is written in the form

$$e - rf = 0, \tag{13}$$

which, with port resistance $R = r$, leads directly to the description of the wave digital element (Fig. 9):

$$b = 0. \tag{14}$$

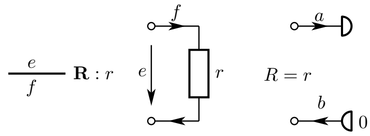

Figure 9: Dissipation element

Since the equivalence of the bond graph and the wave digital realisation of the nonlinear element in the following example shows a lack of generality, it will not be dealt with in this section but in the next.

3. Example: Chua's Circuit

Chua's circuit, as depicted in Fig. 10, a well known example of a chaotic system, will be used to demonstrate the interrelation between the bond graph model and the wave digital structure. Since the focus here is on their structural similarities, no simulation results will be presented (they can be found in [4]).

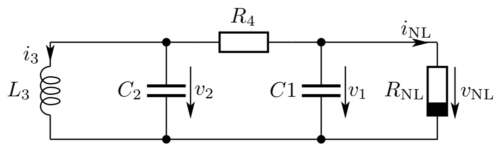

Figure 10: Chua's circuit

The nonlinear (active) element is described by

$$i_{NL} = G_1 v_{NL} + \tfrac{1}{2}(G_2 - G_1)(|v_{NL} + v_0| - |v_{NL} - v_0|) \tag{15}$$

with $G_1 < 0$, $G_2 < 0$, and v_0 as given parameters.

The corresponding bond graph model is shown in Fig. 11. The appearance of the two-port **1**-junction is necessary to comply with the sign conventions of Fig. 10. The nonlinear element is represented by the symbol **NL**. In contrast to other publications (e.g. [5]), no internal bond graph model of this element will be given.

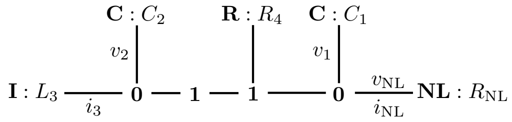

Figure 11: Bond graph of Chua's circuit

3.1. Derivation of the wave digital structure

An essential feature of both, bond graph models and wave digital structures, is that the structure of the interconnections and the elements are separate model parts. In a first step the core bond graph (i.e. only the **0**- and **1**-junctions) is considered. Replacing every junction by the corresponding wave digital element (adaptor) results in the adaptor structure of Fig. 12.

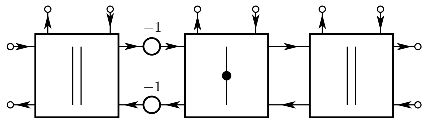

Figure 12: Corresponding adaptor structure

Then the wave digital equivalences of the linear elements according to Sec. 2 are added. The wave digital representation of the nonlinear element is given by

$$b_{\mathrm{NL}} = \varrho_1 a_{\mathrm{NL}} + \tfrac{1}{2}(\varrho_2 - \varrho_1)(|a_{\mathrm{NL}} + a_0| - |a_{\mathrm{NL}} - a_0|), \quad (16)$$

where ϱ_1, ϱ_2, and a_0 are determined by the parameters of (15) and an appropriate port resistance [4].

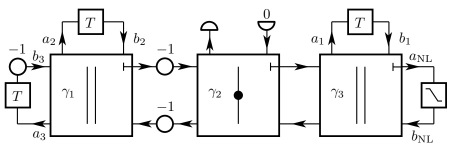

Figure 13: Resulting wave digital structure

Port resistances of ports connecting adaptors are chosen to lead to reflection free ports. This is essential to avoid algebraic loops in the resulting algorithm. Reflection free ports are marked with a ⊢-shaped symbol.

Please note that this wave digital structure is not just another model representation but already the simulation algorithm.

3.2. Modularity

Since bond graph models and wave digital structures are assembled in a modular way, local changes in the modeled system lead only to local changes in the models.

To demonstrate this, the circuit is changed by adding losses to the model of the inductor by introducing the resistance R_3 (Fig. 14).

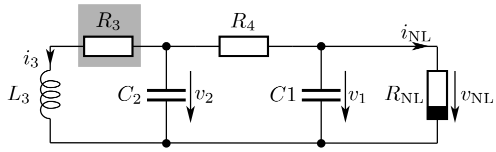

Figure 14: Chua's circuit with lossy inductor

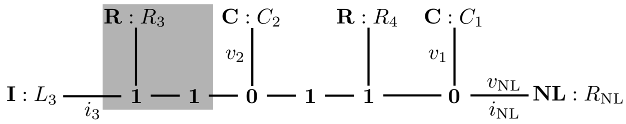

Figure 15: New bondgraph

Looking at Figs. 15 and 16 one can see that only local changes (highlighted in grey) in the bond graph and in the wave digital structure take place. In this example just the adaptor coefficient have to be adjusted.

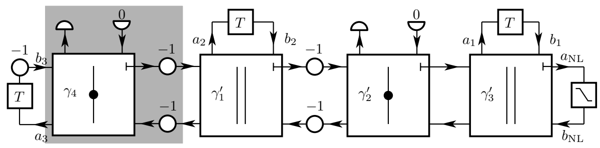

Figure 16: New wave digital structure

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