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Estimation of Models and Parameter in Generation Processes of Multi-fractal Signals based on the Multi-Agent Systems

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Abstract—This paper deals with the estimation of models and parameters in generation processes of multi-fractal signals (time series) based on the multi-agent systems. In the model for multi-fractal time series, cognitive behaviors of agents are modeled by using the GP for learning. As a result, we find strict multi-fractality in artificial stock prices, and we see the relationship between the realizability (reproducibility) of multi-fractality and the system parameters. Features derived from the multi-fractality based on the wavelet transform are used as the input to the conventional discriminant analysis so as to identify the behaviors of agents. However, generally we need sufficient samples to estimate the Hausdorff dimension $D(h)$. To overcome these difficulties, we extend the method to generate multi-fractal times series based on the Wavelet Transform. As applications, we evaluate the performance of interpolation method of the paper by comparing the result of discriminant analysis of agents' behavior using Genetic Programming (GP) for learning with sufficient samples, and we also discuss the recognition of generation models of real stock prices.

1. Introduction

In recent years, the multi-fractal formalism has been introduced to describe statistically the scaling properties of singular measures [1, 2, 3, 6]. The analysis of origin of multi-fractality is seemed to be much more important than examining the existence of multi-fractalities, while these phenomena suggest us how to find anomalities in time series such as stock prices and surfaces, for example cloud. But there are no discussion about the origin of multi-fractality and related topics such as feature extractions .

In previous works, we investigated the possibility to synthesize (generate) artificial fractal time series even chaotic ones based on multi-agent systems [5, 6]. In the paper, we extend these methods to estimation of modeling of multi-fractal signals.

In the first model for multi-fractal time series, cognitive behaviors of agents are modeled by using the GP for learning. As a result, we find strict multi-fractality in artificial stock prices, and we see the relationship between the realizability (reproducibility) of multi-fractality and the system parameters.

In the second model, considering the process of crystal growth of surface, interactions among agents placed on the two-dimensional lattices reveals as multi-fractal surfaces. By selecting parameters for models, we find multi-fractality in artificial surface data denoted as the height of accumulated resources.

Features derived from the multi-fractality based on the wavelet transform are used as the input to the conventional discriminant analysis so as to identify the behaviors of agents.

2. Multi-Agent based Modeling of Artificial Stock Markets

In conventional works, authors proposed methods to generate multi-fractal time series based on modified wavelet-coefficients by changing the probability distribution of multipliers using the β distributions, and the multiplicative cascade [1]. However, the relation between the characteristics of the multi-fractal time series and mathematical descriptions is not clear. We focus on the artificial multi-agent systems for the stock market where we can observe multi-fractality based on the agents' behavior [5, 6]. In the agent system, by changing the member of agents we can examine the origin of multi-fractality.

The market structure is set up to be as simple as possible in terms of its economic components.

Agents of type 1 and 2

The agents of both type 1 and type 2 are assumed to build forecasts for stock price and dividend of the next period by using arithmetic expressions which are called "forecast equations" including state variables of the market as operands based on the GP procedures. In the parse tree, the non-terminal node is taken from the function sets, such as +,-,/,× and exp, abs ,sqrt. The operands for these functions (operations) are taken from past stock prices themselves, constants and variables. At the end of trading, agents update the accuracy of all matched forecast equations according to the forecast error and using the GP. It is assumed that the agents of type 1 possess their individual forecast rules, but the agents of type 2 only learn from a common pool of forecast

Table 1: Cases depending on the numbers of agents

	Case1	Case2	Case3	Case4	Case5	Case6
N_1	100	100	0	100	0	0
N_2	100	0	100	0	100	0
N_3	100	100	0	100	0	0
N_4	100	0	100	0	100	0
N_5	100	100	100	0	0	100

Agents of type 3 and 4

We assume that agents of type 3 and 4 use the production rules for trading stocks (called forecast rules). These agents forecast only the rise/fall of stock prices based on the forecast rules, and the volume to be traded is assigned at random. The difference of these two types exists in the way to use the common forecast rules. It is assumed that the agents of type 3 possess their individual forecast rules, but the agents of type 4 only learn from a common pool of forecast rules for trading without their own rule bases.

Agents of type 5

We assume agent of type 5 who make decisions which are less than perfectly rational. Agents of type 5 decide at random when they should sell/buy stocks and how much they should trade. Then, agents of type 5 have different characteristics, and behave like speculators.

To investigate the origin of multi-fractality in the artificial stock prices, the composition or the ratio of agents of each type may play an important role. Then we prepare several cases of combination of the number of agents for each type. Table 1 summarizes these cases depending on the number of agents where N_i denote the number of agents of type i. These cases are used for the scheme of simulation studies.

3. Multi-fractality formalism

We use the multi-fractality formalism according to the definition by Arneodo, Bacry and Muzy (Wavelet Transform Modulus Maxima:WTMM)[2, 3]. We assume that wavelet transform is applied to the time series $x(t)$ by generating wavelet coefficients x_n^m.

$$x(t) = \sum_n \sum_m x_n^m \psi_n^m(t), \tag{1}$$

$$x_n^m = \int_{-\infty}^{\infty} x(t)\psi_n^m(t)dt, \tag{2}$$

where $\psi_n^m(t)$is the transformation of basic wavelet function $\psi(t)$ obtained as .

$$\psi_n^m(t) = 2^{-m/2}\psi(2^{-m}t - n), \tag{3}$$

where m and n are scale parameter and space parameter, respectively. In the applications, we use basic wavelet functions given by Daubechies. Then, we take the q-th moments of x_n^m as

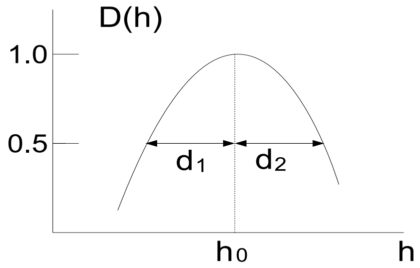

Figure 1: Overview of discriminant analysis of cases based on multi-fractality (definition of discriminant variables).

$$S_m(q) \geq \sum_{n \in N(m)} |2^{-m/2} x_n^m|^q. \tag{4}$$

$$N(m) = N/(2^m). \tag{5}$$

The variable $S_m(q)$ is analogous to the partition function. Then, the following value is obtained by the transform.

$$\log_2 S_m(q) \geq inf_q[m(qh - D(h))], \tag{6}$$

where $D(h)$ is the Hausdorff dimension under the condition where $H(x)$ is h. Then, inversely, we have

$$D(h) = inf_q[qh - \tau(q)]. \tag{7}$$

4. Discriminant analysis of agents' behavior using $D(h)$

We use the Multivaraite Discriminant Analysis (MDA) to estimate agents' behaviors, namely, Case 1~Case 6.. based on the shapes of $D(h)$ calculated from observations. To characterize $D(h)$, we use the value of $h = h_0$ where $D(h) = 1$ and two values of h specified by d_1, d_2 where $D(h_0 - d_1) = D(h_0 + d_2) = 0.5$. Fig.1 shows the definition of three variables, called discriminant variables.

The discriminant functions to classify the observation is organized as follows. We generate sufficient numbers of artificial stock prices corresponding to Case 1~Case 6 (called categories), and then the discriminant functions $f_i(x)$ corresponding to category i is so estimated that for a certain $D(h)$ having $x = (h_0, d_1, d_2)$ belonging to category i the function $f_i(x)$ has larger values than another $f_j(x)$, $j \neq i$.

In the classification stage, we substitute variable $x = (h_0, d_1, d_2)$ obtained from $D(h)$ into every $f_i(x)$, and determine the category where the $f_i(x)$ has largest values.

5. Interpolation for small number of samples

It must be noted that we need at least 10000 samples to obtain reliable results for $D(h)$, therefore a kind of interpolation is necessary for cases with relatively smaller number of samples. In ordinary generation process of artificial

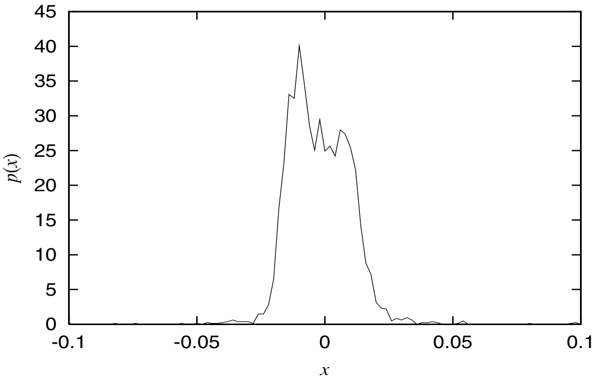

Figure 2: An example of estimated probability density of random number $A_{j,k}$ from time series.

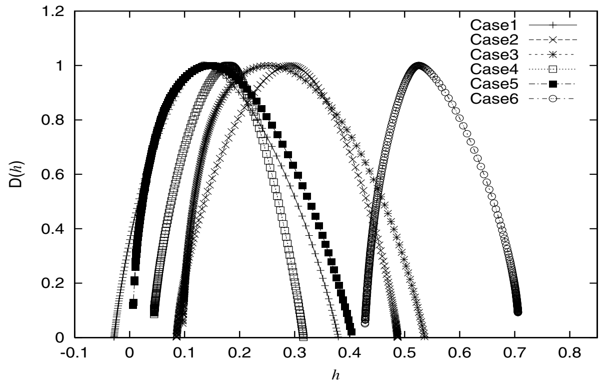

Figure 3: Examples of $D(h)$ for Cases 1~6.

multi-fractal time series, we use following formula originally proposed by Wornell and used for real applications by Riedi et.al [1, 8].

2) starting with $j = 0$ th stage, for the next stage $j = j + 1$ we generate new pairs of coefficients $k = 0, 1, ..., 2^j - 1$ such as.

$$U_{j+1,2k} = 2^{-1/2}(U_{j,k} + W_{j,k}), W_{j,k} = A_{j,k}U_{j,k}, \quad (8)$$

$$U_{j+1,2k+1} = 2^{-1/2}(U_{j,k} - W_{j,k}), \quad (9)$$

where $A_{j,k}$ are random numbers obeying β distribution whose region is $[-1, 1]$.

3) iterate procedure until we obtain sufficient numbers of samples.

By inverting above relation, we have.

$$U_{j,k} = 2^{-1/2}(U_{j+1,2k} + U_{j+1,2k+1}), \quad (10)$$

$$W_{j,k} = 2^{-1/2}(U_{j+1,2k} - U_{j+1,2k+1}). \quad (11)$$

Using these relations, we can estimate $U_{j,k}$ and $A_{j,k}$ in upper stage up to $U_{0,0}$, and finally obtain the data for estimating properties of random numbers $A_{j,k}$.

After identifying the property of random numbers $A_{j,k}$, We can further generate time series until we obtain sufficient samples of data. We have similar relations for the interpolation of surface data.

Fig.2 shows a numerical example of probability density function of random numbers $A_{j,k}$ obtained from an artificial stock price. As is seen the shape of the function is so-called two-top and relatively complicated.

6. Simulation results

Since our main interest exists in finding the features of multi-fractal signals domain, then we skip the process to compare and validate the results of feature extraction using obtainable results by other conventional methods.

Artificial stock prices

We can see various characteristics of multi-fractality for each cases of composition of agents in artificial stock market through $\tau(q), D(h)$. We assume following parameters for simulation studies.

number of individuals: agents of type 1,3 have 50 own individuals, agents of type 2 and 4 have 50 shared individuals

size of array of individual: length of individuals is selected to be 30

record of stock prices:100000 samples

Fig.3 shows an example of $D(h)$ for Cases 1~6. Then, it is possible to estimate composition of dominant agents based on the analysis of features obtained from results such as $\tau(q), D(h)$.

In the following, we use conventional Multi-variate Discriminant Analysis (MDA) by employing statistical package to identify classes. The inputs (discriminant variables) for the MDA are composed of following eleven observations. So as to clarify the definition of these variables, the outlines of definitions are given as follows.

(1) value of $h = h_0$ where $D(h)$ reaches its maximum (one).

(2) spread of $D(h)$ by defining d_1, d_2 for which $D(h_0 - d_1) = D(h_0 + d_2) = 0.5$

We generate 20 time series for each case of composition of agents, and then apply MDA by obtaining features $D(h)$ for these cases. Table 2 shows the result of the classification (recognition) by the MDA for Case 1 through 6 as the second task by showing the rate of correct recognition. Vertical and the horizontal column correspond to the original and estimated classes, and the number in each lattice means the rate of recognition. As is seen from the result, we can identify the cases of composition of agents at the rate about 95%. The fact means that the estimation of more accurate composition of agents dominating the market may be possible by using the method proposed by the paper.

Result of interpolation

We also show the classification result for cases where number of usable samples N is restricted. Artificial stock prices are divided into segments having the length N, and then the interpolation is applied to generate sufficient

Table 2: Result of classification by MDA for multi-fractal stock prices.

	Case1	Case2	Case3	Case4	Case5	Case6
Case1	0.95	0	0	0	0	0.05
Case2	0	0.95	0	0	0.05	0
Case3	0	0.1	0.9	0	0	0
Case4	0	0	0	0.95	0.05	0
Case5	0	0	0.05	0	0.95	0
Case6	0	0.05	0	0	0	0.95

Table 3: Result of classification by using interpolation for multi-fractal stock prices.

N	Case1	Case2	Case3	Case4	Case5	Case6
4096	0.68	0.68	0.83	0.82	0.70	0.68
2048	0.68	0.67	0.80	0.81	0.70	0.66
1024	0.67	0.65	0.80	0.79	0.64	0.64
512	0.67	0.59	0.79	0.78	0.60	0.60
256	0.43	0.59	0.55	0.52	0.45	0.55
128	0.43	0.53	0.54	0.50	0.45	0.54
64	0.43	0.51	0.52	0.50	0.43	0.50

length of time series using wavelet transform. Table 3 shows the classification result. Vertical and the horizontal column correspond to the original and estimated classes, and the number in each lattice means the rate of recognition. As is seen from the result, if the usable number of samples N is greater than 512, we have about 60% correct classification by using the segments. From the result, it is expected that if we have at least 512 samples of stock prices, the classification and estimation of agents' behavior dominating the market is possible. On the other hand, even though the available number N is large such as 4096, the effect of interpolation is gradual and a kind of limitation reveals.

Real stock prices

Then, we apply the classification scheme based on the interpolation of real stock prices. We use the Nikkei average index observed from July 13, 1994 to November 5, 2002 (2048 samples). We divide the samples into a set of portions of prices with length N, then apply the interpolation and WTMM to get $\hat{D}(h)$ used for classification. As the discriminant functions we use the agents simulation result with sufficient large number of prices. Table 4 shows the result of classification into Case 1~Case 6 depending on the length N. As is seen from the result, as N decreases the diversification of classification becomes large.

7. Conclusion

This paper treated the estimation of models and parameter in generation processes of multi-fractal signals based on the multi-agent systems. For future works, it is necessary to apply the method of the paper to real data such as stock prices.

Table 4: Result of estimation of generation models for Nikkei stock prices when the number of available samples is N.

N	Case1	Case2	Case3	Case4	Case5	Case6
2048	0.00	0.00	0.00	1.00	0.00	0.00
1024	0.00	0.50	0.00	0.50	0.00	0.00
512	0.00	0.50	0.00	0.50	0.00	0.00
256	0.00	0.25	0.00	0.38	0.38	0.00
128	0.00	0.44	0.00	0.313	0.25	0.00
64	0.06	0.38	0.06	0.31	0.16	0.03

References

[1] R. H. Riedi,M. S. Crouse,V. J. Ribeiro and R. G. Baraniuk, "A multifractal wavelet model with application to network traffic," *IEEE Trans. IT*, vol.45, no.3, pp.992–1018, 1999.

[2] J. F. Muzy,E. Barcy and A. Arneodo, "Multifractal formalism for fractal signals: The structure- function approach versus the wavelet-transform modulus-maxima method," *Physical Review E*, vol.47, no.2, pp.875–884, 1993.

[3] E. Barcy, J. Delour and J. F. Muzy,"Modeling financial time series using multifractal random walks," *Physica A*, vol.299, pp.84–92, 2001.

[4] S. H. Chen and C. H. Yeh, "Evolving traders and the business school with genetic programming: A new architecture of the agent-based artificial stock market," *Journal of Economic Dynamic and Control*, vol.25, pp.363–393, 2001.

[5] Y. Ikeda and S. Tokinaga, "Chaoticity and fractality analysis of an artificial stock market by the multi-agent systems based on the co-evolutionary Genetic Programming," *Trans. IEICE*, vol.E87, no.9, pp.2387–2394, 2004. no.9,pp.2387-2394

[6] Y. Ikeda and S. Tokinaga," Multi-fractality Analysis of time series in artificial stock market generated by multi-agent systems based on the Genetic Programming and its applications," *IEICE Trans. Fundamentals*, vol.E90–A, no.9, pp.2212–2222, 2007.

[7] S. Tokinaga and N. Takagi, "Decomposition of surface data into fractal signals based on mean likelihood and Importance Sampling and its applications to feature extraction," *IEEE Trans. Fundamentals*, vol.E88–A, no.7, pp.1946–1956, 2005.

[8] G. W. Wornell, Signal Processing with Fractals: A Wavelet-based Approach, Prentice-Hall, 1996.