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A symmetric two-lane cellular automaton traffic model with a clustering lane-changing rule

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Abstract— In this paper, a symmetric two-lane cellular automaton traffic model with a clustering lane-changing rule is proposed. This model consists of slow and fast vehicles, where all the slow vehicles follow the neighboring slow vehicle to make a slow vehicle cluster. This rule can dissolve the plug that prevents fast vehicles from overtaking. The proposed model is compared numerically with the other existing models, based on three important indexes of real traffic flow. In the proposed model, a low frequency of acceleration can be maintained.

1. Introduction

Traffic flow models can be roughly classified into two types: macro models that are based on fluid dynamics, and micro models that regard traffic flows as an aggregation of individual vehicle behavior. For more than a decade, the micro models, which can be described by coupled differential equations [1–3], coupled map lattices [4–6], and cellular automaton (CA) [7], have attracted great interest in the field of nonlinear physics [8–10]. The CA traffic models are suitable for simulating traffic flows on computers, since they have discrete time and space. Wolfram presented the simplest CA model for single-lane traffic [11]. Nagel and Schreckenberg added acceleration, deceleration, and randomization actions to Wolfram's model [7]; their model is called the Nagel-Schreckenberg (NS) model, and has been modified in consideration of several practical viewpoints [12–17].

The NS model was extended to a symmetric two-lane traffic model [18]. For the two-lane model, each time step has two substeps. The first substep considers only the lane-change actions; for the second substep, all the vehicles move forward. Chowdhury *et al.* introduced two types of vehicles, slow and fast (e.g., cars and trucks), into the symmetric two-lane model [19]. Recently, Li *et al.* modified Chowdhury's model based on the fact that in real traffic flows, fast vehicles following slow vehicles tend to change lanes aggressively [20]. In the modified model, the flux of vehicles is increased compared with that in Chowdhury's original model. This is because fast vehicles blocked by the slow vehicles can overtake them aggressively. However, this aggressive overtaking often induces braking in other vehicles, and such braking seems to be a crucial factor in fuel wastage. From the viewpoint of improving traffic flow and reducing fuel consumption, it is important to increase the flux of traffic flows and reduce the number of braking actions. It is naturally wondered how an individual driver controls the vehicle in order to realize the above two aspects.

In this paper, a symmetric two-lane cellular automaton traffic model with a clustering lane-changing rule is proposed. In this model, all slow vehicles always watch the behavior of neighboring vehicles. If a slow vehicle detects a neighboring slow vehicle, then the vehicle follows its neighboring slow vehicle. To put it briefly, all slow vehicles have the conscious mind making a slow vehicle cluster. If all slow vehicles belong to a cluster, there is no plug formed by slow vehicles, allowing the fast vehicle to overtake the slow vehicles. Most symmetric two-lane CA models have their own lane-changing rules, but commonly employ the NS model for the forward motion rule [8–10, 19–21]. In our model, the forward motion rule is also the same as in NS model, and the modified rule of the Chowdhury's model is used as the lane-changing rule. We compare three models, Chowdury's model, Li's model, and our model, based on three important indexes of real traffic flow. These indexes are the flux of vehicles, the frequency of acceleration, and the frequency of lane-changing. The flux of vehicles is one of the most important indexes for describing traffic jams; the frequency of acceleration is closely related to fuel consumption; the lane-changing frequency could correlate with traffic accidents.

2. Cellular automaton model

Let us review the NS model, which provides a simple description of traffic flow [7]. A single-lane circuit with a periodic boundary is divided into L sites, where N vehicles

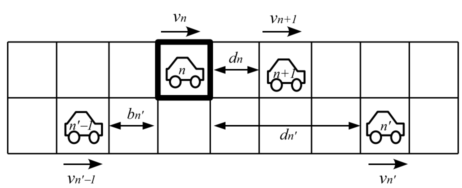


Figure 1: Illustration of two-lane models

are placed. The velocity and position of the nth vehicle are denoted by $v_n \in \{0, 1, ..., v_{\max}\}$ and $x_n \in \{0, 1, ..., L-1\}$ respectively. At each time step $t \to t+1$, v_n and x_n are updated by the following three steps:

Acceleration :

$$v_n = \begin{cases} \min(v_{\max}, v_n + 1) & \text{if } v_n < d_n \\ d_n & \text{if } v_n \geq d_n \end{cases} \quad (1a)$$

Randomization :

$$v_n = \begin{cases} \max(v_n - 1, 0) & \text{if } rand() < p_{\text{NS}} \\ v_n & \text{if } rand() \geq p_{\text{NS}} \end{cases} \quad (1b)$$

Car motion :

$$x_n = x_n + v_n \quad (1c)$$

If the velocity v_n is less than the headway distance of the nth vehicle, i.e., $v_n < d_n := x_{n+1} - x_n - 1$, then v_n is increased by $v_n = \min(v_{\max}, v_n + 1)$, where $v_{\max}$ is the maximum velocity of vehicles. Otherwise, v_n is reduced to d_n. After this update, v_n is decreased by one with the probability $p_{\text{NS}} \in [0, 1]$. The term $rand()$ denotes a uniform random value between 0 and 1. The position x_n is finally updated by $x_n = x_n + v_n$. The updated v_n and x_n are used for the next time step.

2.1. Two-lane models

The NS model was extended to the symmetric two-lane models [8–10, 18–21], as shown in Fig. 1. Let us focus on the nth vehicle framed by the bold square. The $(n+1)$th vehicle runs in front of the nth vehicle in the same lane. n' denotes the index of the vehicle running in front of the nth vehicle in the other lane. All vehicles are classified into fast and slow vehicles: the maximum velocity of the fast (slow) vehicles is given by $v_{\max} = v^{\text{f}}_{\max}$ $(= v^{\text{s}}_{\max})$. For simplicity, if the nth vehicle is fast (slow), then it is represented by $T_n = 1$ $(T_n = 0)$. The velocity of the nth vehicle is represented by $v_n \in \{0, 1, ..., v_{\max}(T_n)\}$, where

$$v_{\max}(T_n) = \begin{cases} v^{\text{s}}_{\max} & \text{if} \quad T_n = 0 \\ v^{\text{f}}_{\max} & \text{if} \quad T_n = 1 \end{cases}. \quad (2)$$

For the nth vehicle, the headway and backward distance in the same lane and in the other lane are defined by

$$d_n := x_{n+1} - x_n - 1, \; d_{n'} := x_{n'} - x_n - 1,$$
$$b_{n'} := x_n - x_{n'-1} - 1.$$

For the two-lane models, each time step has two substeps. The first substep considers only the lane-change action. If a vehicle satisfies all of incentive, safety, and random criteria, then the vehicle changes the lane. Otherwise, the vehicle does not change lanes. This change action is performed for all the vehicles in parallel. The incentive criterion provides an incentive for changing lanes. The safety criterion judges whether the vehicles can change lanes safely. For the second substep, all vehicles move forward according to the NS model algorithm (1).

For Chowdhury's model [19], the incentive criterion is given by

$$d_n < \min(v_n + 1, v_{\max}(T_n)) \quad \text{and} \quad d_n < d_{n'}. \quad (3)$$

Criterion (3) shows that the headway distance in the current lane, d_n, is not sufficient for vehicle acceleration and the headway distance in the target lane, $d_{n'}$, is longer than that in the current lane. In this situation, vehicle n has an incentive to change lanes for overtaking. The safety criterion,

$$b_{n'} > v^{\text{f}}_{\max}, \quad (4)$$

indicates that the backward distance in the target lane, $b_{n'}$, is enough for the safe lane-change. The random criterion,

$$rand() < p_1, \quad (5)$$

shows that vehicle n changes lanes with the probability $p_1 \in [0, 1]$. After the first substep, Chowdhury's model uses algorithm (1) for the second substep.

Li's model employs criterion (3), the modified safety criterion, and the modified random criterion [20]. The modified safety criterion,

$$\begin{cases} b_{n'} \geq 2 \quad \text{and} \quad v_n \geq v_{n'-1} & \text{if} \quad T_n = 1 \quad \text{and} \quad T_{n+1} = 0 \\ \text{Criterion (4)} & \text{otherwise} \end{cases}, \quad (6)$$

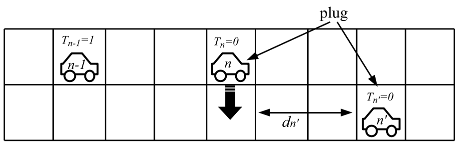
shows that if vehicle n is fast and the vehicle preceding it in the same lane is slow, then vehicle n overtakes the slow vehicle aggressively. Otherwise, vehicle n employs criterion (4) of Chowdhury's model. We notice that Li's model allows fast vehicles to change lanes easily compared with Chowdhury's model. The modified random criterion,

$$\begin{cases} rand() < p_1 & \text{if} \quad T_n = 1 \quad \text{and} \quad T_{n+1} = 0 \\ rand() < p_2 & \text{otherwise} \end{cases} \quad (7)$$

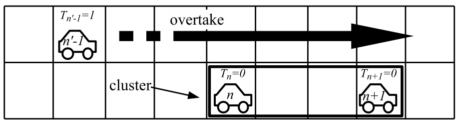
shows that the fast vehicle n changes lanes aggressively with a high probability $p_1 (\gg p_2)$, if vehicle n is fast and the vehicle preceding it in the same lane is slow. After the first substep, Li's model also uses algorithm (1) as the second substep.

2.2. Model with a clustering rule

This paper proposes a symmetric two-lane cellular automaton traffic model with a clustering lane-changing rule. This model employs a new lane-changing rule (i.e., the first substep) based on drivers' kind consideration: the slow vehicles always watch the behavior of neighboring vehicles, and determine their actions not to cause congestion.



(a) before lane-changing



(b) after lane-changing

Figure 2: Sketches of criterion (8).

The proposed model is identical to Chowdhury's one except its incentive criterion: the following criterion is used as our incentive criterion.

$$\{\ T_n + T_{n+1} > 0\ \} \ \text{ and }$$
$$\{\ (T_n = T_{n'} = 0 \ \text{ and } \ d_{n'} > v_n) \ \text{ or } \ \text{Criterion (3)}\ \} \quad (8)$$

This criterion consists of the situations:

(i) at least one of vehicles n, $n+1$ are fast;

(ii-a) both of vehicles n and n' are slow and there is the sufficient headway distance $d_{n'}$ in the target lane;

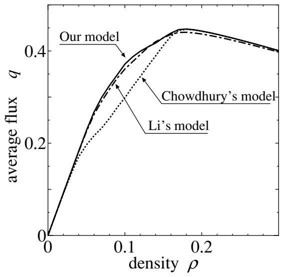
(ii-b) criterion (3) is hold.

The vehicle n has the lane-changing incentive if "(i) and (ii-a)" or "(i) and (ii-b)" is satisfied. For "(i) and (ii-a)" (see Fig. 2(a)), in order to make the formation of a slow vehicle cluster, the slow vehicle n has the incentive to follow the slow vehicle n' in the target lane (see Fig. 2(b)). The cluster has the two features: it allows fast vehicles to overtake several slow-vehicles without lane-changing (see Fig. 2(b)); it does not make the formation of the plug that prevents the fast vehicles from overtaking (see Fig. 2(a)). For "(i) and (ii-b)", like Chowdhury's model, the vehicle n has the incentive to overtake the vehicle preceding it.

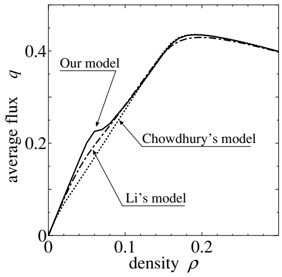
3. Numerical Simulations

The three models under the periodic boundary are investigated by numerical simulations. In particular, we evaluate these models, based on three values: the flux of vehicles, the frequency of acceleration, and the frequency of lane changing. Throughout this paper, the parameters are fixed as $L = 2000$, $v^{\rm f}_{\max} = 5$, $v^{\rm s}_{\max} = 3$, $p_{NS} = 0.3$, $p_1 = 1.0$, $p_2 = 0.05$. The total number of vehicles, N, and the ratio of slow vehicles, R, are controllable parameters[1]. For a parameter set (N, R), 50 initial random configurations are prepared. For each initial configuration, the three values are computed by averaging over 30,000 time steps after the initial behavior for $t = 0 \sim 10,000$.

[1] R := (number of slow vehicles)/(N: number of all vehicles)



(a) $R = 0.05$



(b) $R = 0.30$

Figure 3: Fundamental diagrams for the three models.

The three models are compared by the flux of vehicles. The flux is the number of vehicles passing a definite point in a unit of time. The flux q is defined as $q = \rho\bar{v}$, where $\bar{v}$ is the average velocity of all vehicles and $\rho = N/L$ is the vehicle density. Thus, the flux q is computed by

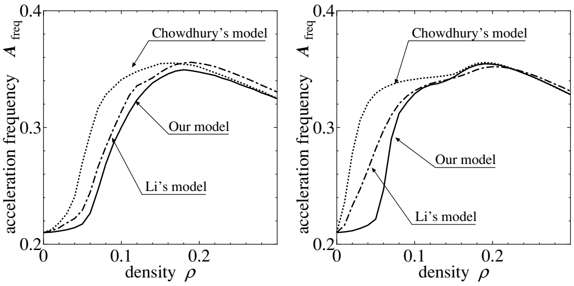
$$q = \rho\,\bar{v} = \frac{1}{L}\sum_{n=1}^{N} v_n.$$

Figure 3 shows the fundamental diagrams, i.e., the $\rho - q$ characteristic, of the three models. It can be seen that for the congested phase ($\rho > 0.2$), the flux of the three models is approximately the same. The reason is that for the congested phase, lane changing is not easy due to a lack of safety space in all the models; hence, differences are not observed among the models. For the free phase ($\rho < 0.2$), the flux of our model on average somewhat exceeds both Chowdhury's and Li's models. These results indicate that plugs are dissolved efficiently by criterion (8).

It is well known that a vehicle consumes large quantities of fuel when its driver presses down on the accelerator. Thus, the acceleration frequency of a vehicle is generally closely related to its fuel consumption. In particular, a high acceleration frequency (i.e., vehicles running with acceleration-deceleration behavior) lowers fuel economy, and a low acceleration frequency (i.e., vehicles running smoothly) improves fuel economy. Thus, from a viewpoint of low fuel consumption, the acceleration frequency should be reduced as much as possible. The acceleration frequency, $A_{\rm freq}$, is estimated by averaging the number of accelerating vehicles at time step t, $A_{\rm acce}(t)$,

$$A_{\rm freq} = \frac{1}{N\tau}\sum_{t=1}^{\tau} A_{\rm acce}(t),$$

where τ is total number of time steps the vehicles runs. $A_{\rm acce}(t)$ is the number of vehicles whose velocity at $t+1$ is greater than that at t. The estimated acceleration frequency $A_{\rm freq}$ against the vehicle density ρ is shown in Fig. 4. The



(a) R = 0.05 (b) R = 0.30

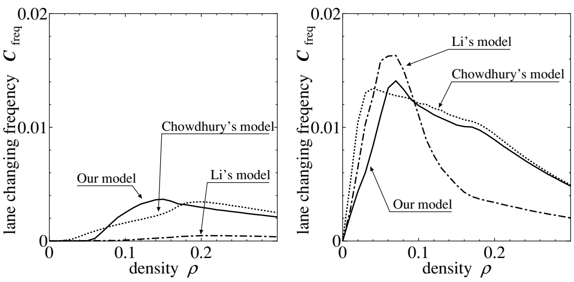
Figure 4: Acceleration frequency of three models.

frequency in our model is clearly lower than that in Chowdhury's and Li's models in the free phase ($\rho < 0.2$). For the free phase, the vehicles in our model run smoothly compared with the other models. These data confirm that our model promotes low fuel consumption.

Frequent lane changing could increase the risk of traffic accidents; thus, a low frequency would be desirable for real traffic. The lane-changing frequency, C_{freq}, is estimated by averaging the number of lane-changing vehicles,

$$C_{\text{freq}} = \frac{1}{N\tau}\sum_{t=1}^{\tau} C_{\text{chan}}(t),$$

where $C_{\text{chan}}(t)$ is the number of vehicles that change lanes at t. The estimated lane-changing frequency of slow and fast vehicles for $R = 0.15$ is shown in Fig. 5. For both slow and fast vehicles, the frequency of our model is almost conformed to that of Chowdhury's model.



(a) Slow vehicles (b) Fast vehicles

Figure 5: Lane-changing frequency of slow and fast vehicles for $R = 0.15$.

4. Conclusions

In this paper, a symmetric two-lane cellular automaton traffic model with a clustering lane-changing rule has been proposed. The proposed model was compared with other models numerically, based on three important indexes of real traffic flow. The numerical simulations have shown that if the drivers of slow vehicles control the vehicle with the conscious mind making a slow vehicle cluster, the low frequency of acceleration can be maintained.

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