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Attitude Stabilization Control of 3D Space Robot Model with Initial Angular Momentum via Model Predictive Control

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Abstract—This study considers attitude control of a 3D space robot of two rigid bodies with initial angular momentum. First, we explain the universal joint model with initial angular momentum. We next apply model predictive control to an attitude control problem of the universal joint model with initial angular momentum, and show a simulation result to confirm reduction of calculation amount. Then, a simulation with a model error of initial angular momentum is performed in order to check robustness of model predictive control.

1. Introduction

It is well known that for a space robot in 3-dimensional outer space, its conversation law of total angular momentum plays a role of nonholonomic constraints, and hence the robot's attitude can be changed by transforming its shape. A lot of reseaches on such a space robot have been done in the fields of analytic mechanics, control theory and robotics [1, 2, 3]. In most researches on control of space robots, it is assumed that space robots do not have initial angular momentum. In realistic situations, for example, when a mother ship gives a space robot out, space robots often have initial angular momentum. Hence we have focused on 3D space robots with initial angular momentum and derived a control strategy based on the near-optimal control method [5]. However, since the model of a space robot with initial angular momentum is quite complicated and the proposed control law is feedforward-type, a huge quantities of calculation amount is needed. Moreover, the control law does not have the characteristic of robustness for the physical parameters of the system. The purpose of this study is to overcome the disadvantages mentioned above by using model predictive control that consists of feedback-type control laws.

2. 3D Space Robot with Initial Angular Momentum

First, the 3D space robot model treated throughout this paper is explained. We consider a space robot that consists of two rigid bodies and exists in 3D space as shown in Fig. 1. Two rigid bodies (Rigid Body 1 and 2) are connected by a universal joint via two links (Link 1 and 2), respectively. We represent coordinates of the inertial space, Rigid Body 1 and 2 by C_0, C_1 and C_2, respectively. We now assume that the origins of C_1 and C_2 correspond to the centroids of Rigid Body 1 and 2, respectively. Let $A_i \in SO(3)$ be the attitude of Rigid Body $i\,(i = 1, 2)$ with respect to the inertial space C_0, and $w_i \in \mathbf{R}^3$ be the angular velocity of Rigid Body i. Note that $\hat{w}_i = A_i^{\mathsf{T}} \dot{A}_i$ holds [1]. We use the notations; m_i: the mass of Rigid Body i ($\epsilon = m_1 m_2/(m_1 + m_2)$), l_i: the length of Link i, $s_i = [0 0 -l_i]^{\mathsf{T}} \in \mathbf{R}^3$: the vector showing the position of the joint with respect to C_0, $I_i \in \mathbf{R}^3$: the inertia tensor of Rigid Body i ($J_i = I_i + \epsilon \hat{s}_i^{\mathsf{T}} \hat{s}_i,\ J_{12} = \epsilon \hat{s}_1^{\mathsf{T}} A_1^{\mathsf{T}} A_2 \hat{s}_2$). Next, we denote the angles of Link 1 and 2 of the universal joint as $\theta_1,\ \theta_2 \in \mathbf{R}$ ($\theta = [\,\theta_1\ \ \theta_2\,]^{\mathsf{T}} \in \mathbf{R}^2$), respectively. Then,

$$A := A_1^{\mathsf{T}} A_2 = \begin{bmatrix} \sin\theta_1 \sin\theta_2 & \cos\theta_1 & -\sin\theta_1 \cos\theta_2 \\ \cos\theta_2 & 0 & \sin\theta_2 \\ \cos\theta_1 \sin\theta_2 & -\sin\theta_1 & -\cos\theta_1 \cos\theta_2 \end{bmatrix}$$

represents the shape of the space robot and $w_2 = A^{\mathsf{T}} w_1 + w = A_2^{\mathsf{T}} A_1 w_1 + w$ holds for the angular velocity of the joint $w \in \mathbf{R}^3$, $\hat{w} = A^{\mathsf{T}} \dot{A}$. Assuming that the space robot has initial angular momentum $P_0 \in \mathbf{R}^3$, we have the conservation law of the total angular momentum as

$$(A_1 J_1 + A_2 J_{12}^{\mathsf{T}}) w_1 + (A_2 J_2 + A_1 J_{12}) w_2 = P_0. \qquad (1)$$

Now, we set $I_u := J_1 + A J_2 A^{\mathsf{T}} + A J_{12}^{\mathsf{T}} + J_{12} A^{\mathsf{T}}$ and parametrize A_1 by using the Cayley-Rodrigues parameter (3). Note that

$$w_1 = U_1(\alpha)\dot{\alpha},\ \ U_1(\alpha) = \frac{2(I - \hat{\alpha})}{1 + \alpha^{\mathsf{T}} \alpha} \qquad (2)$$

holds for the angular velocity w_1 and Cayley-Rodrigues parameter α. Moreover, we refer angular velocities of the universal joint as control inputs, that is, $u_1 := \dot{\theta}_1$, $u_2 := \dot{\theta}_2$, then we have the next:

$$w = \underbrace{\begin{bmatrix} \cos\theta_2 \\ 0 \\ \sin\theta_2 \end{bmatrix}}_{b_1} u_1 + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{b_2} u_2. \qquad (4)$$

Therefore, setting $q := [\,\theta^{\mathsf{T}}\ \ \alpha^{\mathsf{T}}\,]^{\mathsf{T}} \in \mathbf{R}^5$, from (1)–(4) we obtain *the universal joint model with initial angular momentum* as (5), which is represented as a nonlinear affine

[1] $\hat{}$ is the operator that changes a 3-dimensional vector $v = [v_1\, v_2\, v_3]^{\mathsf{T}} \in \mathbf{R}^3$ into a 3×3 skew-symmetric matrix: $\hat{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$.

$$A_1(\alpha) = \frac{1}{1+\|\alpha\|^2}\begin{bmatrix} 1+\alpha_1^2-\alpha_2^2-\alpha_3^2 & 2(\alpha_1\alpha_2-\alpha_3) & 2(\alpha_1\alpha_3+\alpha_2) \\ 2(\alpha_1\alpha_2+\alpha_3) & 1-\alpha_1^2+\alpha_2^2-\alpha_3^2 & 2(\alpha_2\alpha_3-\alpha_1) \\ 2(\alpha_1\alpha_3-\alpha_2) & 2(\alpha_2\alpha_3+\alpha_1) & 1-\alpha_1^2-\alpha_2^2+\alpha_3^2 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\alpha \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ U_1^{-1}I_u^{-1}A_1^{\mathsf T}P_0 \end{bmatrix}}_{f(q)} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -U_1^{-1}I_u^{-1}(AJ_2+J_{12})b_1 & -U_1^{-1}I_u^{-1}(AJ_2+J_{12})b_2 \end{bmatrix}}_{g(q)} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{5}$$

$$J = \frac{1}{2}\int_t^{t+T(t)} (\alpha(\tau)-\alpha_d)^{\mathsf T} Q(\alpha(\tau)-\alpha_d)d\tau + \frac{1}{2}\int_t^{t+T(t)} u(\tau)^{\mathsf T} R u(\tau)d\tau + \frac{1}{2}(\alpha(t+T)-\alpha_d)^{\mathsf T} S(\alpha(t+T)-\alpha_d) \tag{6}$$

control system with 5 states and 2 inputs and does not have any equilibrium points. For the universal joint model (5), we have shown that (5) is strongly locally accessible at any state, and if control inputs are sufficiently large, (5) is small-time locally controllable [5].

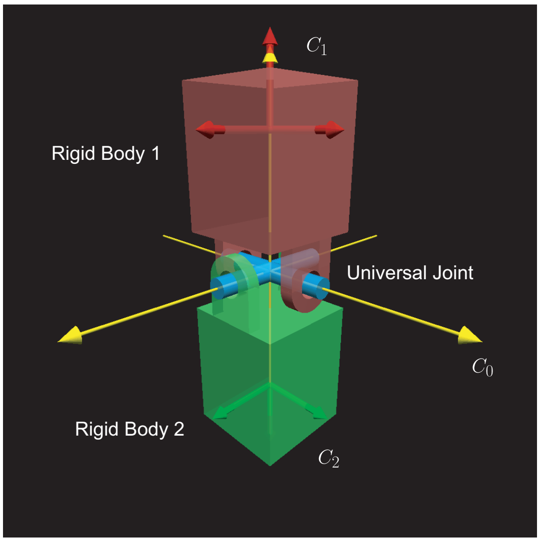


Fig. 1 : Universal Joint Model

3. Attitude Stabilization Control

Since the universal joint model with initial angular momentum (5) does not have any equilibrium points and cannot stand still, we cannot treat normal control problems such as a stabilization problem to the origin. Therefore, this section considers the following control problem. Probelm 1 contains, for example, the situation where we move a solar panel of a space robot to the direction of the sun.

Problem 1: For the universal joint model with initial angular momentum (5), find control inputs such that the attitude of Rigid Body 1 α is stabilized to a desired value α_d. ∎

In this paper, we take the model predictive control approach in order to solve Problem 1. Especially, we use *the C/GMRES method* [4], which is a real-time optimization algorithm. In a simulation, we use the parameters of the universal joint model: $l_1 = l_2 = 1$, $m_1 = m_2 = 1$, $I_1 = I_2 = \mathrm{diag}\{1/2, 1/2, 1\}$, initial angular momentum: $P_0 = [\,0.1 \;\; 0.1 \;\; -0.1\,]^{\mathsf T}$, the initial state: $q_0 = [\,\pi/2 \;\; \pi/2 \;\; 1 \;\; 1 \;\; 1\,]^{\mathsf T}$, the desired attitude: $\alpha_d = [\,0 \;\; 0 \;\; 0\,]^{\mathsf T}$. For the C/GMRES method, we use the evaluation function (6) with the weight matrices $Q = \mathrm{diag}\{4.0, 1.5, 5.0\}$, $R = \mathrm{diag}\{0.01, 0.01\}$, $S = \mathrm{diag}\{0.8, 0.2, 0.4\}$ and the evaluation interval $T(t) = T(1-e^{-at})$, $T = 6.5$, $a = 0.05$. Moreover, we also use the parameters of controller: the division number of the evaluation interval: $N = 50$, the stabilization parameter of the continuation method: $\zeta = 20$, the number of iterations of the GMRES method: $k_{max} = 3$, the sampling time: $\Delta t = 0.05$ [s], the simulation time: 20 [s].

Simulation results are shown in Fig. 2 and 3. Fig. 2 illustrates the time series of θ and α, and Fig. 3 depicts the snapshot of the universal joint model. From these results, it can be confirmed that the attitude of Rigid Body 1 α is stabilized to the desired value $\alpha_d = 0$. The computation time of this simulation is 1.45 [s], and hence we can see that the computation time is drastically reduced in comparison with the case of the near-optimal control method [5].

4. Robustness for Initial Angular Momentum

In parameters of a space robot, the mass, the inertia moment and the length can be easily measured. However, since the value of initial angular momentum changes according to circumstances, we have the difficulty to measure it. So this section verifies the availability of the model predictive control approach in the case where there exists a modeling error in initial angular momentum.

In a simulation, we use the same parameters of the universal joint model except initial angular momentum as the ones shown in Section 3. We set the measured initial angular momentum: $\tilde P_0 = [\,0.1 \;\; 0.1 \;\; -0.1\,]^{\mathsf T}$ and the real initial angular momentum: $P_0 = [\,0.07 \;\; 0.07 \;\; -0.01\,]^{\mathsf T}$. We also use the weight matrices of the evaluation function (6) as $Q = \mathrm{diag}\{2.0, 1.0, 3.0\}$, $R = \mathrm{diag}\{0.01, 0.01\}$, $S = \mathrm{diag}\{0.85, 0.2, 0.4\}$. Moreover, we also use the same parameters of controller as the ones shown in Section 3.

Fig.4 shows the time series of θ and α, and Fig. 5 illustrates the snapshot of the universal joint model. From these results, it turns out that the attitude of Rigid Body 1 α is stabilized to the desired value $\alpha_d = 0$ despite model error, and hence the controller obtained by the model predictive control approach has robustness for initial angular momentum.

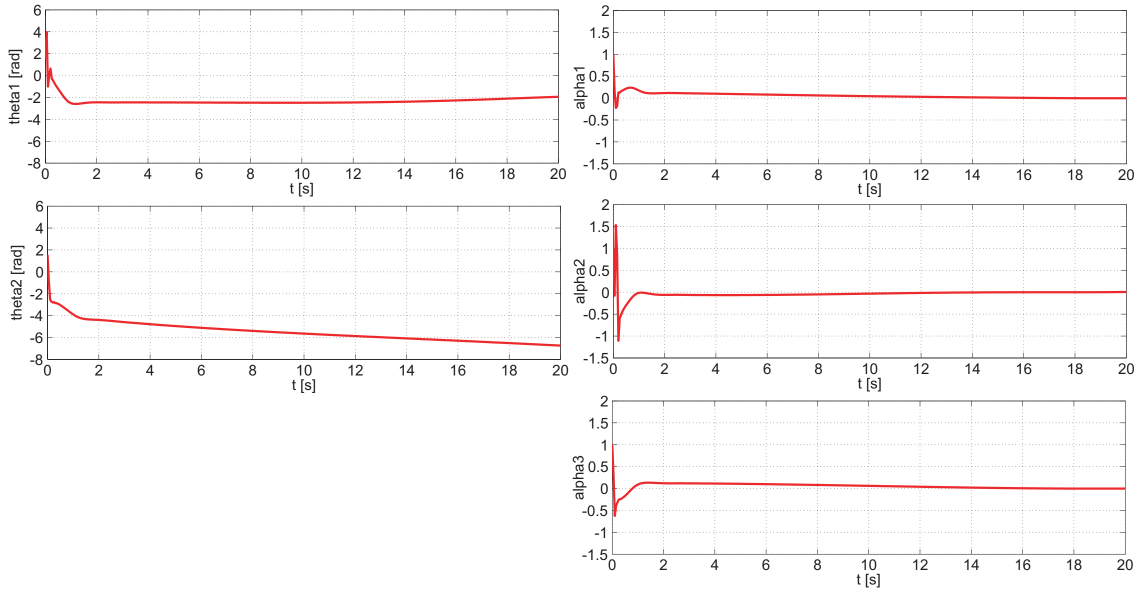
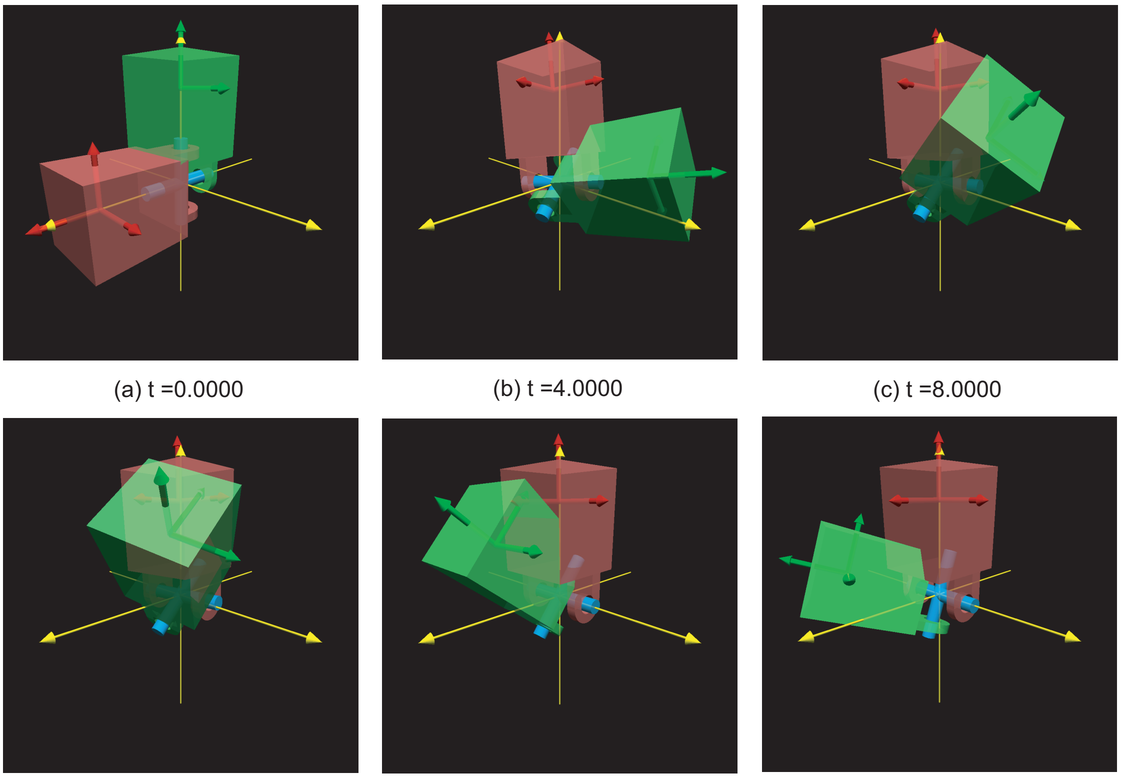


Fig. 2 Time Series of θ and α



(a) t =0.0000 (b) t =4.0000 (c) t =8.0000

(d) t =12.0000 (b) t =16.0000 (c) t =20.0000

Fig. 3 Snapshot of Universal Joint Model

5. Conclusion

In this paper, we have considered attitude stabilization control of the universal joint model with initial angular momentum via model predictive control approach. Simulation results have indicated that the attitude of Rigid Body 1 is stabilized to the desired value with a reduced calculation amount compared to our previous work, and robustness with respect to initial angular momentum can be confirmed.

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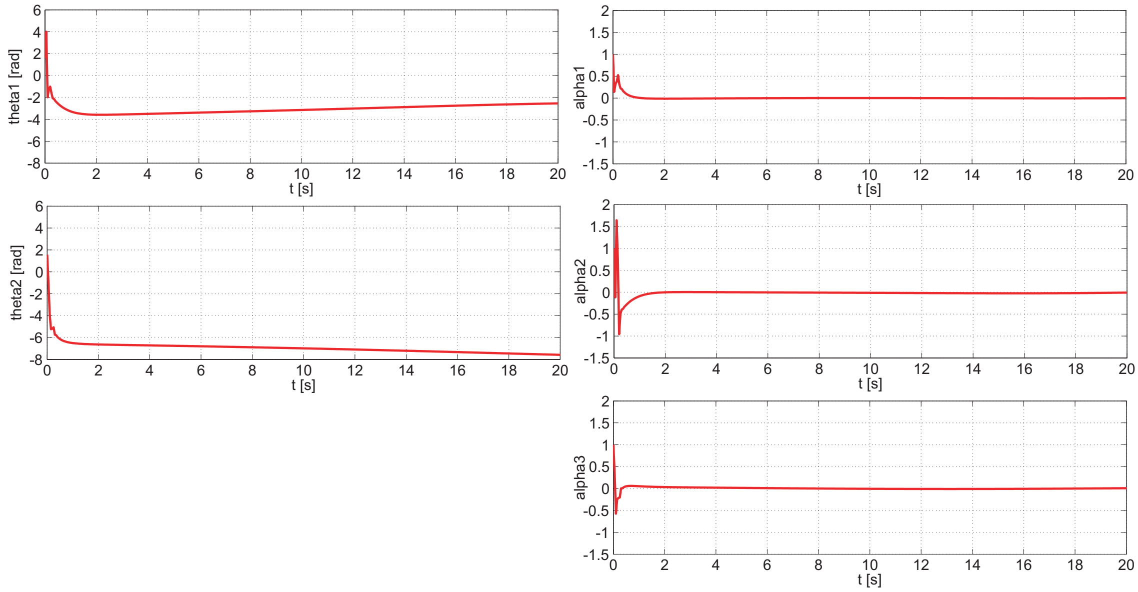
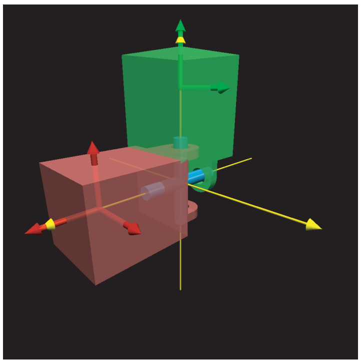
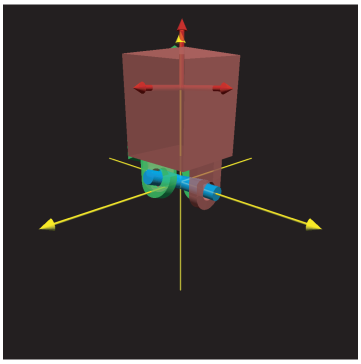


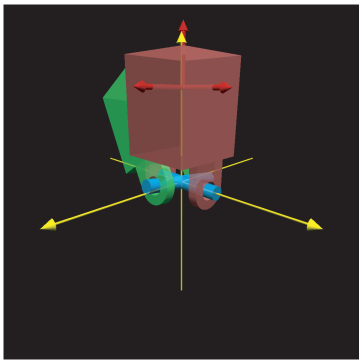
Fig. 4 Time Series of θ and α (with Model Error of Initial Angular Momentum)



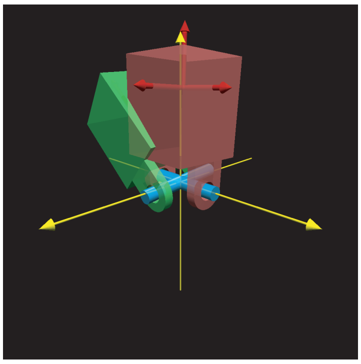
(a) t =0.0000



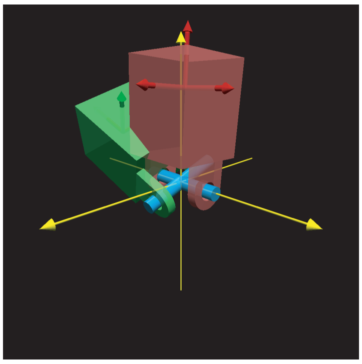
(b) t =4.0000



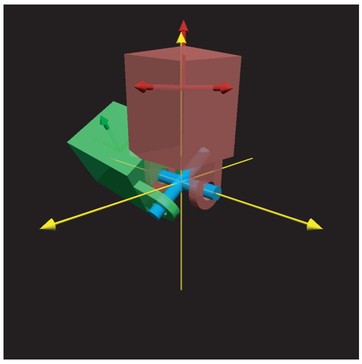
(c) t =8.0000



(d) t =12.0000



(b) t =16.0000



(c) t =20.0000

Fig. 5 Snapshot of Universal Joint Model (with Model Error of Initial Angular Momentum)

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