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Evolutionary prisoner's dilemma games with cooling-off periods on networks

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Abstract—Recently, quite a lot of studies on evolutionary prisoner's dilemma games (PDGs) on networks have been made. In this paper, we investigate evolutionary PDGs with cooling-off periods on networks. In our study, if a player is defected by his opponent player in the PDG, he quits playing a game against the opponent for a certain periods called "cooling-off periods". We demonstrate that players become more cooperative by cooling-off periods because defective players' chances of participating in the PDG are restricted remarkably.

1. Introduction

In social systems occupied by selfish individuals, cooperative behaviors are often seen. In order to explain those behaviors, a good number of studies have been made. In particular, a prisoner's dilemma game (PDG) has been known as a paradigm for studying the emergence of cooperative behaviors.

In the original PDG, two players simultaneously decide whether to cooperate (C) or to defect (D). They both receive payoff R (Rewards) upon mutual cooperation and payoff P (Punishment) upon mutual defection. A defector exploiting a C player gets payoff T (Temptation to defect), and the exploited cooperator receives payoff S (Sucker's value), such that $T > R > P > S$ and $2R > T + S$. In one-time-only PDG, if the players reasonably work to maximize own payoff under above conditions, it is best to defect regardless of the co-player's decision. As a result, they get to defect for each other. However, mutual cooperation would be better result for them than mutual defection. Therefore, the dilemma is caused by the selfishness of the players. For the emergence of cooperation, the other extensions based on the original PDG need to be explored.

Nowak and May showed that the PDG on a simple spatial structure induces emergence and persistence of cooperation even with the co-existence of spatial chaos [1]. Since then, several studies have reported various network effects on the evolution of cooperation in different network models [2, 3, 4, 5, 6]. In addition, introduction of network coevolution has been also researched [7, 8, 9, 10]. In those models, generally, when a player is defected many times in series, the defected player stochastically deletes the link with the opponent and connects with a new player.

In this study, we consider the suspension of playing with the opponent instead of deleting the link. When a player is defected in the evolutionary PDGs on networks, he quits playing a game against the opponent for a certain periods. We call the periods "cooling-off periods". We demonstrate that players become more cooperative by coolong-off because defective players' chances of participating in the PDG are restricted remarkably.

2. Model

2.1. Networks

In evolutionary PDGs on networks, the following networks are often used. In this study, we use these networks.

1. Regular network (RGN)
   RGN is constructed in the way that all nodes are regularly linked. The square lattice and the circular grid network are typical. We use the circular grid network and links each node with the ten nearest neighbors.
2. Random network (RDN)
   RDN is constructed in the way that all nodes are randomly linked. We randomly link each node with r percent of nodes in total.
3. Small-world network (SWN)
   SWN shares the features of both RGN and RDN; average distance is short among nodes and the clustering coefficient is high. We use the Watts-Strogatz model [11]. It is constructed from RGN by reconnecting a fraction w of links.
4. Scale-free network (SFN)
   In SFN, the degree distribution $p(k)$, which determines the probability of finding exactly k neighbors for a player, shares the feature of a power-law. We use the Barabási-Albert model [12]. In this model, starting from m_0 fully connected nodes, a new node with $m(m \leq m_0)$ links is added to the system step by step. The new node is preferentially linked to those nodes that have large degrees already and is linked to the existing node with a probability depending on the degree.

2.2. Cooling-off periods

In the network, if a player x is defected by his opponent player y, x quits playing a game against y for c steps. During the steps, y cannot play with x. Afterward, they restart the game. We call it "cooling-off periods" between x and y. Here we should note that cooling-off periods are not concerned with player y but the link between x and y. If both defect, the link between them enters in the cooling-off periods as well as the either case.

2.3. Payoff

2.3.1. Payoff matrix

We use the following payoff matrix.

$$A = \begin{bmatrix} R & S \\ T & P \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 0 \end{bmatrix}. \quad (1)$$

Although eq.(1) does not strictly satisfy with the condition of PDG, $T > R > P > S$, we use the matrix in this paper because it can be simply described by one parameter b and is also used in other papers [1, 2, 3, 4, 6, 7, 8, 9].

2.3.2. Calculation of payoff

Each player plays with all linked players (neighbors) except for the links in the cooling-off periods. Then he uses the same strategy for all neighbors. The total payoff P_x of player x can be written as below.

$$P_x = \sum_{y \in \Omega_x} s_x^T A s_y, \quad (2)$$

where s_x is two component vector of his strategy, taking the value $s_x = (1, 0)^T$ for C-player and $s_x = (0, 1)^T$ for D-player, and Ω_x is the set of neighbors except for the links in the cooling-off periods against x.

2.4. Strategy update rule

We use Win-Stay-Lose-Shift as the strategy update rule. Each player compares own payoff with his neighbors'. If his payoff is the maximum, his strategy doesn't change. Otherwise he adopts the strategy of the neighbor with the highest payoff. If there are multiple choices, he randomly chooses one player from the neighbors. All players simultaneously update each strategies.

Even a player with the links in the cooling-off periods can change his strategy. The links are also used according to the rule because they are kept connected in the periods.

2.5. Network update rule

When y defects x, he is in the cooling-off periods. If y defects x again in the immediate game after c steps, he is said to defect x *consecutively*. Then, we update the network as below. Firstly, in the process that player x and y play, if y defected x for $f (f \geq 2)$ times consecutively, x deletes the link with y. Secondly, y finds player z with the highest payoff in his neighbors (including x). Finally, in z's neighbors, y finds the player with the highest payoff except for x and connects with him. If a player has no link, he is isolated and remains so afterword.

2.6. Algorithm

The algorithm in this model is following.

1. Construct a network.
2. Choose each player's strategy.
3. Regard $4 \sim 6$ as one step and repeat steps.
4. Let each player plays with his neighbors who are not in the cooling-off periods.
5. Update their strategies simultaneously.
6. Update network structure.

3. Simulation and discussion

We simulate PDGs on four kinds of networks and construct five patterns for each network, except for RGN (one pattern). These networks consist of 1000 players and are randomly initialized with exactly 50% cooperators and 50% defectors. We set the following parameters: $r = 1$, $w = 0.3$ and $m = m_0 = 3$ and vary $f = 2, 3$, $b = 1.0, 1.1, \ldots, 1.9$ and $c = 0, 1, 3$. Here, $c = 0$ means that we do not consider the cooling-off periods and also run the model under the same condition for comparison. Additionally, we also run the model without network update rule. Then, the parameters vary under the same conditions except for f. For each set of values and network, we carry out 125 runs (=(5 network patterns)×(5 initial strategies patterns)×(5 runs)), except for RGN (25 runs).

As the evaluation indices, we use a degree distribution $p(k)(0 \leq p(k) \leq 1)$ and a cooperation level $\rho_C (0 \leq \rho_C \leq 1)$. They show the change of the network structure and the level of cooperation in the society respectively. We evaluate them in the state that the change of cooperation level is stable in the society (static state). When the change is within 1% for 5 steps in series, we finish the system and take the average value as ρ_C by averaging the values. When the system does not reach the static state within 200 steps, we finish it and take the average value as ρ_C by averaging the values in last 5 steps.

In Figure 1, we show the initial and final time of the degree distribution $p(k)$ in RDN and SFN. We chose each characteristic result from 125 runs in $b = 1.9$, $c = 1$, $f = 2$. In RDN (Figure 1 (a), (b)), $p(k)$ changes greatly between initial and final time, and players with high degree (hub players) emerge. The result in (b) is similar to SFN's feature, power-law. In SFN (Figure 1 (c), (d)), $p(k)$ changes little. Because of page number limitation, we omit the results in RGN and SWN. In final time, their results are similar to SFN. In the case not introducing cooling-off periods, our results in each network are different. This implies that these networks approach to the SFN by introducing the cooling-off periods. Therefore, in SFN, it is thought that $p(k)$ changes little.

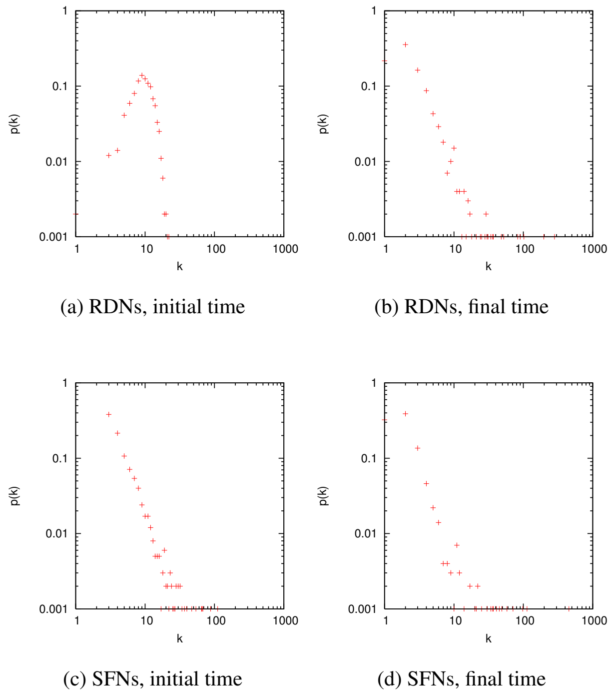

Figure 1: $p(k)$ in initial and final time for $c = 1$, $f = 2$ and $b = 1.9$

In Figure 2, we show the average of ρ_C in 125 runs on each network (25 runs on RGN). The results with $f = 2, 3$ and without network update rule (no update) are shown for each network and c's value. ρ_c also increases rapidly by introducing cooling-off periods. Therefore, in these networks, it is shown that the coevolution model is efficient. However, in SFN, it is not clear that cooling-off periods are effective. In RGN (Figure 2 (b), (c)), when $c = 1$, the case of $f = 2$ is better than the case of $f = 3$. However, when $c = 3$, ρ_C reaches 1 for any b and $f = 2, 3$. If either f or c is fixed, the cooperation level is improved when the other is larger. In RDN (Figure 2 (e), (f)), when b is larger, ρ_C reaches 1 except for $c = 1$ and $f = 2$. In SWN (Figure 2 (h), (i)), ρ_C attains entirely the best result in all the networks. In SFN (Figure 2 (k), (l)), although the case of $c = 1, 2$ are better than $c = 0$, ρ_C does not reach 1 in any b. Thus, in these results, it is shown that players become more cooperative by cooling-off periods. As the reason, it is thought that defectors try to become cooperative because their payoffs are low in the cooling-off periods.

4. Conclusions

By introducing cooling-off periods in evolutionary prisoner's dilemma game on four kinds of the networks, the degree distribution $p(k)$ changed greatly and the cooperation level ρ_C increased except for SFN. Also, when c is larger, ρ_C increases. Thus, our approach has proved very effective.

As further research in the future, we do simulations with c, r and w in a wider range. Also, to evaluate network structure, we investigate the average degree and the average distance among nodes. Additionally, we will propose the new coevolution model introducing the cooling-off periods. Now, we have investigated the model that the link between player x and y stochastically enters in the cooling-off periods when x is defected by y.

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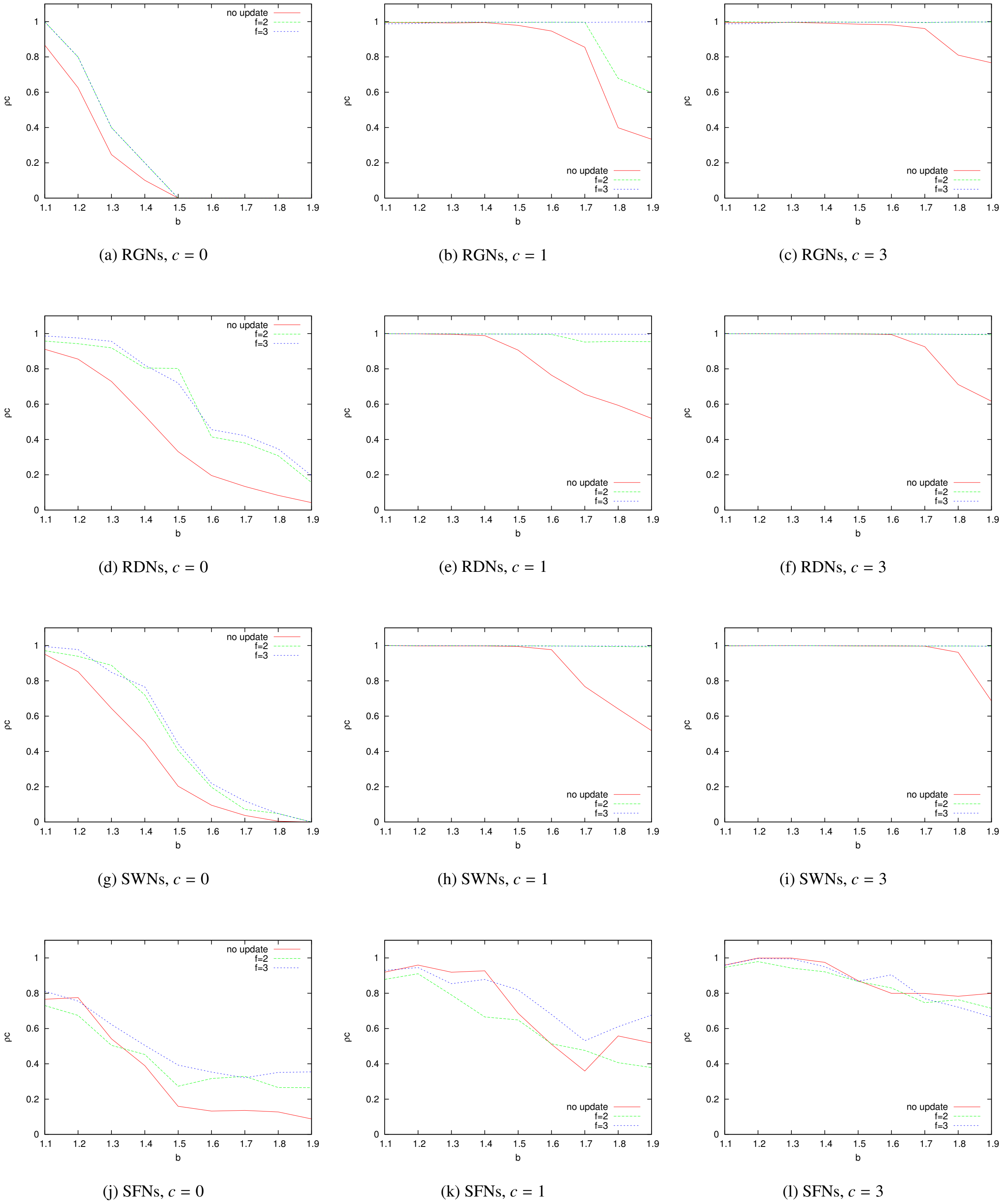

(a) RGNs, $c = 0$ (b) RGNs, $c = 1$ (c) RGNs, $c = 3$

(d) RDNs, $c = 0$ (e) RDNs, $c = 1$ (f) RDNs, $c = 3$

(g) SWNs, $c = 0$ (h) SWNs, $c = 1$ (i) SWNs, $c = 3$

(j) SFNs, $c = 0$ (k) SFNs, $c = 1$ (l) SFNs, $c = 3$

Figure 2: Average of ρ_C as the function of b for different values of c and f on different kind of networks