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Discrete Gait Generation for the Compass-Type Biped Robot Modeled by Discrete Mechanics

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Abstract—In this study, we consider a discrete gait generation method for the compass-type biped robot modeled by discrete mechanics. First, we derive the discrete compass-type biped robot model. Next, we define an optimal control problem on discrete gait generation and explain a sequential quadratic programming approach to the problem. A simulation is then illustrated to confirm the effectiveness of our method.

1. Introduction

Recently, a lot of work on humanoid robots have been done. Especially, the compass-type biped robot is one of simplest models of humanoid robots and has been mainly researched so far. In most studies on humanoid robots, the continuous-time mathematical model is dealt with, and Poincáre section approach is usually used to generate stable walking or gait [4, 5]. However, a humanoid robot is known to be one of most difficult mechanical systems to control, and many problems are still left unsolved.

On the other hand, *discrete mechanics* has been focused on as a new discretizing tool for mechanical systems over the last decade [1, 2, 3]. By using discrete mechanics, we can directly obtain a discrete-time model of a mechanical system, and the discrete-time model has various advantages in terms of numerical errors and physical characteristics. We have researched discrete mechanics from the standpoint of control theory and derived some results [6, 7, 8]. In these studies, we mainly treat the cart-pendulum system as a physical example. However, it is expected that discrete mechanics can be applied to more complicated mechanical system and discrete mechanics approach may add a fresh dimension to control problems for humanoid robots.

In this paper we apply discrete mechanics to the compass-type biped robot and consider a gait generation problem for the discrete compass-type biped robot formulated by discrete mechanics. This paper is organized as follows. First, some basic concepts on discrete mechanics are summed up in Section 2. In Section 3, we next derive the discrete compass-type biped robot based on discrete mechanics and explain two modes of the system. Then, a gait generation problem of the discrete compass-type biped robot is formulated , and a solving method of it from the viewpoint of the sequential quadratic programming is developed in Section 4. In Section 5, we then show a numerical simulation to confirm the effectiveness of our method.

2. Discrete Mechanics

This section sums up basic concepts of discrete mechanics [1, 2, 3]. Let Q be a configuration manifold and $q \in \mathbf{R}$ be a generalized coordinate of Q. We also refer to T_qQ as the tangent space of Q at a point $q \in Q$ and $\dot{q} \in T_qQ$ denotes a generalized velocity. Moreover, we consider a time-invariant Lagrangian as $L(q, \dot{q}) : TQ \to \mathbf{R}$. We first explain about the discretization method. The time variable $t \in \mathbf{R}$ is discretized as $t = kh$ $(k = 0, 1, 2, \cdots)$ by using a sampling interval $h > 0$. We denote q_k as a point of Q at the time step k, that is, a curve on Q in the continuous setting is represented as a sequence of points $q^d := \{q_k\}_{k=1}^N$ in the discrete setting. The transformation method of discrete mechanics is carried out by the replacement:

$$q \approx (1-\alpha)q_k + \alpha q_{k+1}, \;\; \dot{q} \approx \frac{q_{k+1} - q_k}{h}, \tag{1}$$

where q is expressed as a internally dividing point of q_k and q_{k+1} with a ratio α $(0 < \alpha < 1)$ We then define *a discrete Lagrangian*:

$$L^d_\alpha(q_k, q_{k+1}) := hL\left((1-\alpha)q_k + \alpha q_{k+1}, \frac{q_{k+1} - q_k}{h}\right), \tag{2}$$

and *a discrete action sum*:

$$S^d_\alpha(q_0, q_1, \cdots, q_N) = \sum_{k=0}^{N-1} L^d_\alpha(q_k, q_{k+1}). \tag{3}$$

We next summarize the discrete equations of motion. Consider a variation of points on Q as $\delta q_k \in T_{q_k}Q$ $(k = 0, 1, \cdots, N)$ with the fixed condition $\delta q_0 = \delta q_N = 0$. In analogy with the continuous setting, we define a variation of the discrete action sum (3) as

$$\delta S^d_\alpha(q_0, q_1, \cdots, q_N) = \sum_{k=0}^{N-1} \delta L^d_\alpha(q_k, q_{k+1}). \tag{4}$$

The discrete Hamilton's principle states that *only a motion which makes the discrete action sum (3) stationary is realized*. Calculating (4), we have

$$\delta S^d_\alpha = \sum_{k=1}^{N-1} \{D_1 L^d_\alpha(q_k, q_{k+1})\delta q_k + D_2 L^d_\alpha(q_{k-1}, q_k)\}\delta q_k, \tag{5}$$

where D_1 and D_2 denotes the partial differential operators with respect to the first and second arguments, respectively. Consequently, from the discrete Hamilton's principle and (5), we obtain *the discrete Euler-Lagrange equations*:

$$D_1 L_\alpha^d(q_k, q_{k+1}) + D_2 L_\alpha^d(q_{k-1}, q_k) = 0, \quad k = 1, \cdots, N-1. \tag{6}$$

It turns out that (6) is represented as difference equations which contains three points q_{k-1}, q_k, q_{k+1}, and we need q_0, q_1 as initial conditions when we simulate (6).

3. Discrete-time Compass-Type Biped Robot

This section derives a discrete-time model of a compass-type biped robot by using discrete mechanics. In this paper we consider a compass-type biped robot as shown in Fig. 1. Let θ and ϕ be the angles of Leg 1 and 2, respectively. We also use the notations: m: the mass of the legs, M: the mass of the waist, I: the inertia moment of the legs, a: the length between the waist and the center of gravity, b: the length between the center of gravity and the toe of the leg, l $(= a + b)$: the length between the waist and the toe of the leg. The Lagrangian of this system is given by

$$\begin{aligned} L^c(\theta, \phi, \dot\theta, \dot\phi) = {} & \frac{1}{2}(I + ma^2 + ml^2 + Ml^2)\,\dot\theta^2 \\ & + \frac{1}{2}(I + mb^2)\dot\phi^2 - mbl\cos(\theta - \phi)\,\dot\theta\,\dot\phi \\ & - (ma + mg + Ml)g\cos\phi + mgb\cos\phi. \end{aligned} \tag{7}$$

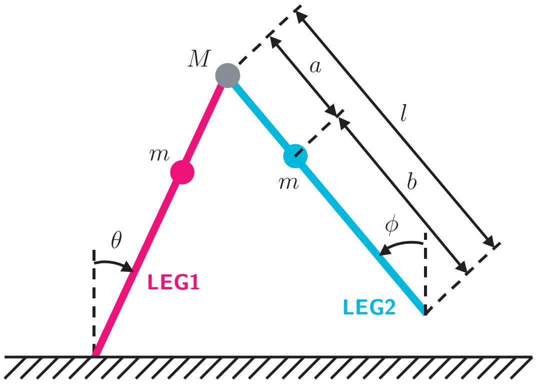

Fig. 1 : Compass-Type Biped Robot

Based on the problem setting above, we now derive *the discrete compass-type biped robot (DCBR)* via discrete mechanics. In general, a model of a compass-type biped robot consists of two modes: *the swing phase* and *the impact phase*. As shown in Fig. 2, it is noted that the swing phase and the impact phase occur alternately and the swing leg and the supporting leg switch positions with each other with respect to each collision. We use the notations: h: a sampling time, α: a division ratio in discrete mechanics, $k = 1, 2, \cdots, N$: a time step, $i = 1, 2, \cdots, L$: an order of swing phases, $\theta_k^{(i)}$, $\phi_k^{(i)}$: the angles of Leg 1 and 2 at the time step k in the i-th swing phase, respectively, $u_k^{(i)}$: the control input at the time step k in the i-th swing phase as a discrete torque for the swing leg.

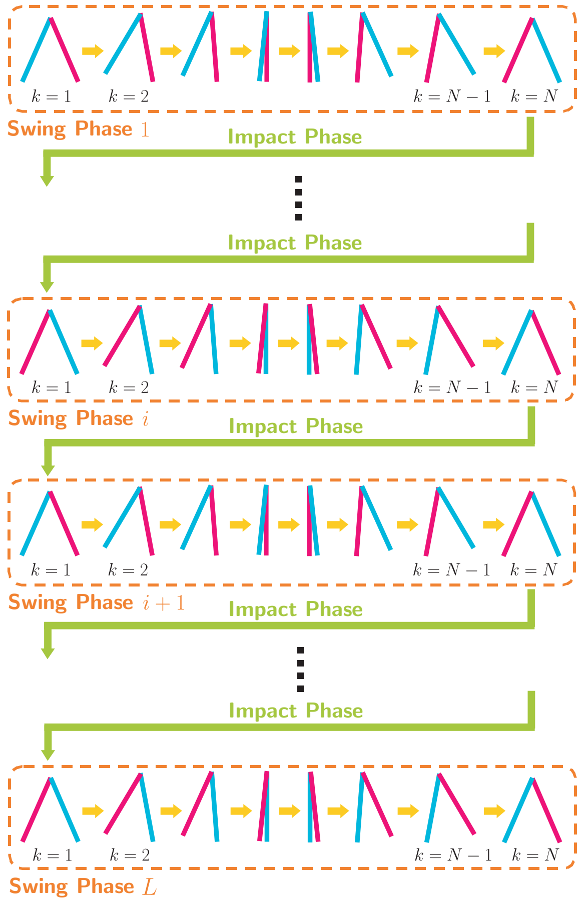

Fig. 2 : Gait Generation for DCBR

First, we derive the swing phase model of the DCBR for the case where Leg 1 is the swing leg and Leg 2 is the supporting leg as shown in Fig. 1. For the case where Leg 1 is the supporting leg and Leg 2 is the swing leg, we can easily obtain the model by changing $\theta_k^{(i)}$ for $\phi_k^{(i)}$. Calculate the discrete Lagrangian L_α^d from (7) as (2) and substitute it into the discrete Euler-Lagrange equations (6). Moreover, adding the control input to the left-hand side of the discrete Euler-Lagrange equations, we obtain the swing phase model as

$$f_1(\theta_{k-1}^{(i)}, \theta_k^{(i)}, \theta_{k+1}^{(i)}, \phi_{k-1}^{(i)}, \phi_k^{(i)}, \phi_{k+1}^{(i)}, u_k^{(i)}) = 0, \tag{8}$$

$$f_2(\theta_{k-1}^{(i)}, \theta_k^{(i)}, \theta_{k+1}^{(i)}, \phi_{k-1}^{(i)}, \phi_k^{(i)}, \phi_{k+1}^{(i)}, u_k^{(i)}) = 0, \tag{9}$$

where functions f_1 and f_2 are defined as (10) and (11), respectively.

Next, we consider the impact phase model of the DCBR. In this paper we assume that the swing leg has a completely-elastic collision with the ground surface. Calculating the condition that discrete momentums before and after a collision are equivalent, that is,

$$D_2 L_\alpha^d(\theta_{N-1}^{(i)}, \theta_N^{(i)}, \phi_{N-1}^{(i)}, \phi_N^{(i)}) = D_1 L_\alpha^d(\theta_1^{(i+1)}, \theta_2^{(i+1)}, \phi_1^{(i+1)}, \phi_2^{(i+1)}),$$

$$D_4 L_\alpha^d(\theta_{N-1}^{(i)}, \theta_N^{(i)}, \phi_{N-1}^{(i)}, \phi_N^{(i)}) = D_3 L_\alpha^d(\theta_1^{(i+1)}, \theta_2^{(i+1)}, \phi_1^{(i+1)}, \phi_2^{(i+1)}),$$

$$
\begin{aligned}
f_1 =& -(ma^2+Ml^2+ml^2+I)(\theta_{k+1}^{(i)}-\theta_k^{(i)}) + mbl(1-\alpha)\sin\left((1-\alpha)(\theta_k^{(i)}-\phi_k^{(i)})+\alpha(\theta_{k+1}^{(i)}-\phi_{k+1}^{(i)})\right)(\theta_{k+1}^{(i)}-\theta_k^{(i)})(\phi_{k+1}^{(i)}-\phi_k^{(i)})\\
&+ mbl\cos\left((1-\alpha)(\theta_k^{(i)}-\phi_k^{(i)})+\alpha(\theta_{k+1}^{(i)}-\phi_{k+1}^{(i)})\right)(\phi_{k+1}^{(i)}-\phi_k^{(i)}) + (ma+ml+Ml)gh^2(1-\alpha)\sin\left((1-\alpha)\theta_k^{(i)}+\alpha\theta_{k+1}^{(i)}\right)\\
&+ (ma^2+Ml^2+ml^2+I)(\theta_k^{(i)}-\theta_{k-1}^{(i)}) + mbl\alpha\sin\left((1-\alpha)(\theta_{k-1}^{(i)}-\phi_{k-1}^{(i)})+\alpha(\theta_k-\phi_k)\right)(\theta_k^{(i)}-\theta_{k-1}^{(i)})(\phi_k^{(i)}-\phi_{k-1}^{(i)})\\
&- mbl\cos\left((1-\alpha)(\theta_{k-1}^{(i)}-\phi_{k-1}^{(i)})+\alpha(\theta_k^{(i)}-\phi_k^{(i)})\right)(\phi_k^{(i)}-\phi_{k-1}^{(i)}) + (ma+ml+Ml)gh^2\alpha\sin\left((1-\alpha)\theta_{k-1}^{(i)}+\alpha\theta_k^{(i)}\right) + hu_k^{(i)}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
f_2 =& -(mb^2+I)(\phi_{k+1}^{(i)}-\phi_k^{(i)}) - mbl(1-\alpha)\sin\left((1-\alpha)(\theta_k^{(i)}-\phi_k^{(i)})+\alpha(\theta_{k+1}^{(i)}-\phi_{k+1}^{(i)})\right)(\theta_{k+1}^{(i)}-\theta_k^{(i)})(\phi_{k+1}^{(i)}-\phi_k^{(i)})\\
&+ mbl\cos\left((1-\alpha)(\theta_k^{(i)}-\phi_k^{(i)})+\alpha(\theta_{k+1}^{(i)}-\phi_{k+1}^{(i)})\right)(\phi_{k+1}^{(i)}-\phi_k^{(i)}) + mglh^2(1-\alpha)\sin\left((1-\alpha)\theta_k^{(i)}+\alpha\theta_{k+1}^{(i)}\right)\\
&+ (mb^2+I)(\phi_k^{(i)}-\phi_{k-1}^{(i)}) - mbl\alpha\sin\left((1-\alpha)(\theta_{k-1}^{(i)}-\phi_{k-1}^{(i)})+\alpha(\theta_k^{(i)}-\phi_k^{(i)})\right)(\theta_k^{(i)}-\theta_{k-1}^{(i)})(\phi_k^{(i)}-\phi_{k-1}^{(i)})\\
&- mbl\cos\left((1-\alpha)(\theta_{k-1}^{(i)}-\phi_{k-1}^{(i)})+\alpha(\theta_k^{(i)}-\phi_k^{(i)})\right)(\phi_k^{(i)}-\phi_{k-1}^{(i)}) + mglh^2\alpha\sin\left((1-\alpha)\theta_{k-1}^{(i)}+\alpha\theta_k^{(i)}\right) - hu_k^{(i)}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
h_1 =& (ma^2+ml^2+Ml^2+I)(\theta_N^{(i)}-\theta_{N-1}^{(i)}) + mbl\alpha\sin\left((1-\alpha)(\theta_{N-1}^{(i)}-\phi_{N-1}^{(i)})+\alpha(\theta_N^{(i)}-\phi_N^{(i)})\right)(\theta_N^{(i)}-\theta_{N-1}^{(i)})(\phi_N^{(i)}-\phi_{N-1}^{(i)})\\
&- mbl\cos\left((1-\alpha)(\theta_{N-1}^{(i)}-\phi_{N-1}^{(i)})+\alpha(\theta_N^{(i)}-\phi_N^{(i)})\right)(\phi_N^{(i)}-\phi_{N-1}^{(i)}) + (ma+ml+Ml)gh^2\alpha\sin\left((1-\alpha)\theta_{N-1}^{(i)}+\alpha\theta_N^{(i)}\right)\\
&- (ma^2+ml^2+Ml^2+I)(\theta_2^{(i+1)}-\theta_1^{(i+1)}) + mbl(1-\alpha)\sin\left((1-\alpha)(\theta_1^{(i+1)}-\phi_1^{(i+1)})+\alpha(\theta_2^{(i+1)}-\phi_2^{(i+1)})\right)(\theta_2^{(i+1)}-\theta_1^{(i+1)})(\phi_2^{(i+1)}-\phi_1^{(i+1)})\\
&+ mbl\cos\left((1-\alpha)(\theta_1^{(i+1)}-\phi_1^{(i+1)})+\alpha(\theta_2^{(i+1)}-\theta_2^{(i+1)})\right)(\phi_2^{(i+1)}-\phi_1^{(i+1)}) + (ma+ml+Ml)gh^2(1-\alpha)\sin\left((1-\alpha)\theta_1^{(i+1)}+\alpha\theta_2^{(i+1)}\right)
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
h_2 =& (mb^2+I)(\theta_N^{(i)}-\theta_{N-1}^{(i)}) + mbl\alpha\sin\left((1-\alpha)(\theta_{N-1}^{(i)}-\phi_{N-1}^{(i)})+\alpha(\theta_N^{(i)}-\phi_N^{(i)})\right)(\theta_N^{(i)}-\theta_{N-1}^{(i)})(\phi_N^{(i)}-\phi_{N-1}^{(i)})\\
&- mbl\cos\left((1-\alpha)(\theta_{N-1}^{(i)}-\phi_{N-1}^{(i)})+\alpha(\theta_N^{(i)}-\phi_N^{(i)})\right)(\phi_N^{(i)}-\phi_{N-1}^{(i)}) + mglh^2\alpha\sin\left((1-\alpha)\theta_{N-1}^{(i)}+\alpha\theta_N^{(i)}\right)\\
&- (mb^2+I)(\theta_2^{(i+1)}-\theta_1^{(i+1)}) - mbl(1-\alpha)\sin\left((1-\alpha)(\theta_1^{(i+1)}-\phi_1^{(i+1)})+\alpha(\theta_2^{(i+1)}-\phi_2^{(i+1)})\right)(\theta_2^{(i+1)}-\theta_1^{(i+1)})(\phi_2^{(i+1)}-\phi_1^{(i+1)})\\
&+ mbl\cos\left((1-\alpha)(\theta_1^{(i+1)}-\phi_1^{(i+1)})+\alpha(\theta_2^{(i+1)}-\theta_2^{(i+1)})\right)(\phi_2^{(i+1)}-\phi_1^{(i+1)}) + mglh^2(1-\alpha)\sin\left((1-\alpha)\theta_1^{(i+1)}+\alpha\theta_2^{(i+1)}\right)
\end{aligned} \tag{15}
$$

we have

$$h_1(\theta_{N-1}^{(i)},\theta_N^{(i)},\theta_1^{(i+1)},\theta_2^{(i+1)},\phi_{N-1}^{(i)},\phi_N^{(i)},\phi_1^{(i+1)},\phi_2^{(i+1)}) = 0, \tag{12}$$

$$h_2(\theta_{N-1}^{(i)},\theta_N^{(i)},\theta_1^{(i+1)},\theta_2^{(i+1)},\phi_{N-1}^{(i)},\phi_N^{(i)},\phi_1^{(i+1)},\phi_2^{(i+1)}) = 0, \tag{13}$$

where functions h_1 and h_2 are defined as (14) and (15), respectively. Moreover, in the impact phase, the swing leg and the supporting leg replace each other, and this can be realized the next equation:

$$\theta_1^{(i+1)} = -\theta_N^{(i)},\ \ \phi_1^{(i+1)} = -\phi_N^{(i)}. \tag{16}$$

Therefore, the impact phase model consists of (8)–(16).

4. Discrete-time Gait Generation for DCBR

In this section we consider a gait generation problem for the DCBR derived in the previous section and we propose the new concept '*discrete gait*.' We here deal with the following problem.

Problem 1: For the discrete compass-type biped robot (8)–(16), find a control input that generates a stable discrete gait. ∎

The purpose of this section is to obtain a control input solving Problem 1. We now formulate Problem 1 as an optimal control problem whose objective function is a sum of the square of a control input. For the DCBR in the i-th swing phase, an optimal control problem can be formulated as follows:

$$\min\ \ J = \sum_{k=1}^{N-1}\{u_k^{(i)}\}^2 \tag{17}$$

s.t. (10), (11)

$$-|(1-\alpha)\theta_k^{(i)}+\alpha\theta_{k+1}^{(i)}|+|(1-\alpha)\phi_k^{(i)}+\alpha\theta_{k+1}^{(i)}|<0 \tag{18}$$

$$\theta_1^{(i)} = -\theta_N^{(i)},\ \ \phi_1^{(i)} = -\phi_N^{(i)}. \tag{19}$$

In the problem above, (18) represents a constraint on the vertical lengths of Leg 1 and 2, and (19) is a boundary condition in order to generate a stable discrete gait. In the impact phase between the i-th and $(i+1)$-th swing phases, we can calculate initial states of the $(i+1)$-th swing phases: $\theta_1^{(i+1)}, \theta_2^{(i+1)}, \phi_1^{(i+1)}, \phi_2^{(i+1)}$ from (12)–(16).

The optimal control problem formulated above can be considered as a finite dimensional constrained nonlinear optimization problem with respect to the $(3N-1)$ variables $\theta_1^{(i)},\cdots,\theta_N^{(i)},\phi_1^{(i)},\cdots,\phi_N^{(i)},u_1^i,\cdots,u_{N-1}^i$. Therefore, we can solve it by using the sequential quadratic programming approach and so on [3, 9].

5. Numerical Simulation

In this section we perform a numerical simulation on gait generation for the DCBR based on the method proposed in the previous section, and check the availability of our method.

First, we set parameters as follows; parameters on gait generation: $N = 10$, $L = 3$, parameters on the DCBR: $m = 2.0$ [kg], $M = 10.0$ [kg], $I = 1.0$ [kgm^2], $a = 0.5$ [m], $b = 0.5$ [m], $l = 1.0$ [m], $\alpha = 1/2$, initial states of the DCBR: $\theta_1^1 = \pi/12$, $\phi_1^1 = \pi/12$. In order to solve the optimal control problem (17)–(19), we use the sequential quadratic programming method [9].

Fig. 3–5 show the simulation results. In Fig 3, time series plots of the leg 1 and 2 (θ and ϕ) are illustrated. Fig. 4 shows the phase space of θ - ϕ. In Fig. 5, a snapshot of a discrete gait is depicted. From these results, it can be confirmed that our approach can generate a stable gait for the DCBR.

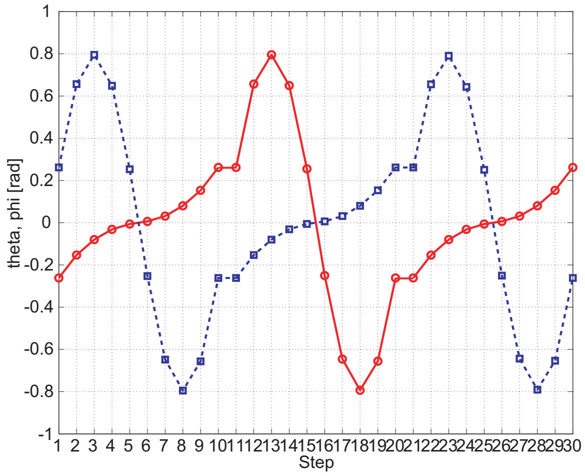

Fig. 3 : Simulation Result - Time Series Plots of θ and ϕ (Red : θ, Blue : ϕ)

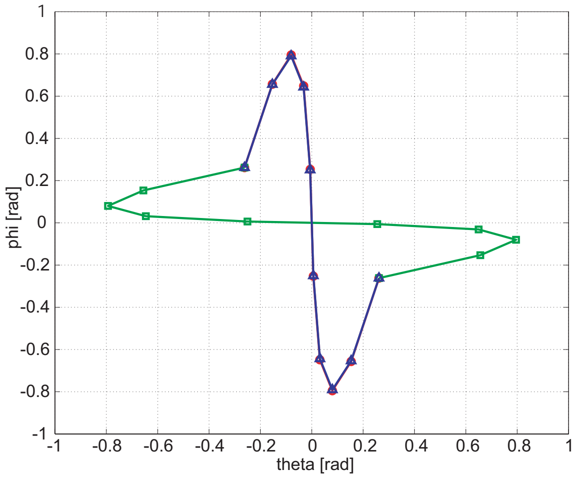

Fig. 4 : Simulation Result - Phase Space of θ - ϕ (Red : 1st Step, Green : 2nd Step, Blue : 3rd Step)

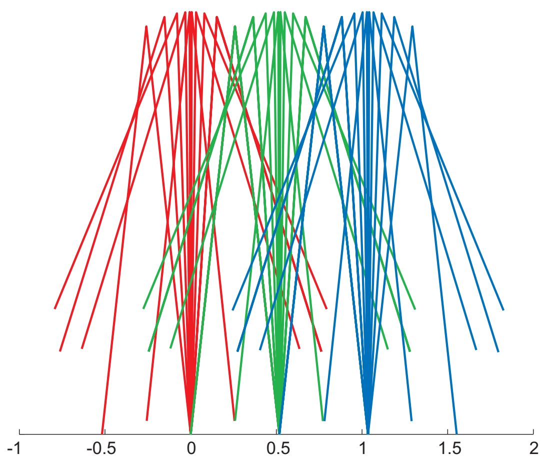

Fig. 5 : Simulation Result - Snapshot of Gait (Red : 1st Step, Green : 2nd Step, Blue : 3rd Step)

6. Conclusion

In this paper we have considered a discrete gait generation problem for the discrete compass-type biped robot (DCBR) and developed a solving method for the problem from the viewpoint of the reduction to a finite dimensional optimization problem and the sequential quadratic programming method. Simulation results have indicated the effectiveness of our approach.

Our future work are as follows: discrete gait generation of the DCBR in various environments such as slopes and stairs, a transformation method of discrete-time inputs into continuous-time inputs and control of the normal compass-type biped robot, experimental validation for the normal compass-type biped robot.

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