2022 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2022, Virtual, December 12-15, 2022

Fast Resource Allocation for the NOMA System Using Coherent Ising Machine

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Abstract—Non orthogonal multiple access (NOMA) schemes have been proposed to meet the growing wireless communication capacity of the next generation. To fully utilize the advantages of NOMA schemes, highly complex resource allocation (RA) optimization problems need to be solved in real time. To achieve this, we propose a fast RA method for the NOMA system using the coherent Ising machine (CIM). The CIM is an Ising system that artificially reproduces the Ising model, a physical model of interacting magnetic spins, and searches for the ground state, the lowest energy state of the Ising model, at high speed. Because many optimization problems can be transformed into Ising problems, which are problems for searching the ground state of the Ising model, CIM can be used to solve NOMA RA optimization problems. The interaction between the spins and the external magnetic field on the spins must be set for each spin of the Ising model to solve the Ising problem. Usually, the value of the external magnetic field is considerably larger than that of the interaction; thus, when solving the Ising problem using the CIM, large values of the external magnetic field in the Ising model may cause instabilities in the actual CIM, resulting in decreased performance. Therefore, in this study, we formulate a stability-aware formulation in which the external magnetic field is removed from the Ising model. To evaluate the proposed method, the stability-aware and conventional formulations were compared using simulations regarding the NOMA system data rate. The simulation results show that the stability-aware formulation had a performance similar to that of the conventional formulation.

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1. Introduction

Hardware-based algorithms, such as Ising machines, have been proposed as methods for solving optimization problems. The coherent Ising machine (CIM) [1] is an Ising machine that reproduces Ising spins using an optical laser network and delivers fairly approximate solutions to optimization problems in milliseconds. Many NP-hard problems can be transformed into ground state search problems for the Ising Hamiltonian, the energy model of Ising spin (i.e., the Ising problem) [2]. The CIM is a machine that searches the ground state of the Ising Hamiltonian at high speed. For example, Ref. [3] shows that the CIM can be used to obtain a solution to the 100000-node MAX-CUT problem approximately 1000 times faster than cutting-edge digital computers. Therefore, the CIM can be used to find fast solutions to many NP-hard optimization problems.

To solve the optimization problem using the CIM, it is necessary to derive the interaction and external magnetic field, which are the parameters of the Ising model corresponding to the optimization problem. The value of the external magnetic field is usually larger than that of the interaction and likely to induce inhomogeneity in the amplitude of the optical pulses representing the spins in the CIM. Therefore, when solving optimization problems, their large value may induce instabilities in the actual CIM, leading to a decrease in performance. Therefore, in this study, we investigate the performance by formulating the Ising model to remove the external magnetic field from the model.

Non orthogonal multiple access (NOMA) schemes have been proposed as one of the key technologies for next-generation communication schemes [4]. Unlike conventional orthogonal multiple access (OMA) systems, multiple user signals can be multiplexed on the same channel for communication. To benefit from the NOMA scheme, appropriate channel and power resources should be allocated to all users [5]. In previous studies, machine-learning and heuristic algorithms have been used to optimize the re-

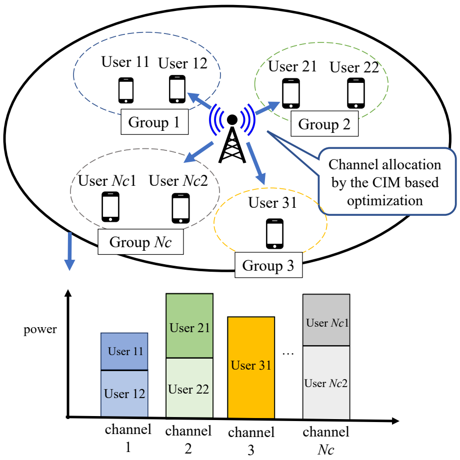

Figure 1: System model

source allocation (RA) problems. However, this type of RA problem has been proven to belong to NP-hard [6]. There is a fundamental trade-off between the optimal solution attain rate and computation time, making the real time optimization required for wireless communication systems difficult.

To solve the RA problem for the NOMA system at high speed, we first focus on the high speed of the CIM and apply it to a fast optimization of the NOMA optimization problem. Subsequently, considering the stability of the actual CIM, we apply the CIM without an external magnetic field to the RA problem in the NOMA system. The performance in terms of system data rate is evaluated for the CIM with and without an external magnetic field.

2. System Model and Problem Formulation

We assume a downlink NOMA system in which the base station (BS) transmits to users in a circular cell (Figure 1). At the BS, based on the channel state information, channel and power optimizations are performed using the CIM equipped in the BS. Note that performance varies greatly depending on which users are multiplexed and assigned to which channels. Let the set of users in the NOMA system be $\mathbf{U} = \{1, 2, \ldots, u, \ldots, N_u\}$ and the set of subchannels be $\mathbf{C} = \{1, 2, \ldots, c, \ldots, N_c\}$. We define the carrier-to-noise ratio (CNR) for user u using channel c as follows:

$$\Gamma_u^c = \frac{|h_u^c|^2}{\sigma_{z_c}^2}, \tag{1}$$

where h_u^c is the response between the BS and the user when user u transmits using channel c and σ_{z_c} is the variance of additive white Gaussian noise. Let us assume that $\Gamma_1^c > \ldots > \Gamma_u^c > \ldots > \Gamma_{M_c}^c$ holds when up to M_c users are multiplexed on channel c. Because the NOMA system allocates more power to users with lower CNR [5], the relationship $P_1^c < \ldots < P_u^c < \ldots < P_{M_c}^c$ holds, where P_u^c is the power allocation factor for user u in channel c. The SIC performed at the receiver allows each user to easily identify and remove the signals of users assigned to higher power. According to these principles, the data rate that user u can achieve on channel c is as follows:

$$R_u^c = B_c \log_2\left(1 + \frac{P_u^c \Gamma_u^c}{1 + \sum_{i=1}^{u-1} P_i^c \Gamma_u^c}\right), \tag{2}$$

where B_c is the bandwidth of channel c. In this study, we consider the case where up to two users are multiplexed on a single channel; that is, $M_c = 2$. The data rate achieved by a user assigned to channel c is as follows:

$$R_{ijk} = \begin{cases} B_c \log_2\left(1 + P_{ijk}\Gamma_i^j\right), & \text{if } \Gamma_i^j > \Gamma_k^j, \\ B_c \log_2\left(1 + \frac{P_{ijk}\Gamma_i^j}{1+P_{ijk}\Gamma_i^j}\right), & \text{otherwise}, \end{cases} \tag{3}$$

where R_{ijk} and P_{ijk} are the achievable data rate and the allocated power by user $i \in \mathbf{U}$ when communicating with user $k \in \mathbf{U}$ on channel $j \in \mathbf{C}$, respectively. We aim to maximize the total data rate of the NOMA system. The objective function is formulated as follows:

$$\max_{x} \sum_{i=1}^{N_u} \sum_{j=1}^{N_c} \sum_{\substack{k=1 \\ k\neq i}}^{N_u} \left(R_{ik}^j + R_{ki}^j\right) x_{ij} x_{kj} \tag{4}$$

$$s.t. \sum_{j=1}^{N_c} x_{ij} \leq 1, \quad \text{for } \forall i, \tag{4.a}$$

$$\sum_{i=1}^{N_u} x_{ij} \leq 2, \quad \text{for } \forall j, \tag{4.b}$$

where $x_{ij} \in (0,\ 1)$ represents the channel assignment variable: $x_{ij} = 1$ if user i allocated channel j and $x_{ij} = 0$ otherwise. Eqs. (4.a) and (4.b) indicate that users communicate using only one channel and up to two users are multiplexed onto one channel, respectively. Additionally, power allocation is optimized following the method described in [5].

3. Coherent Ising Machine

The CIM is an Ising machine that artificially reproduces the behavior of Ising spins. Let us assume a two-dimensional spin group of $N \times M$. The Ising Hamiltonian representing the energy of the Ising model is as follows:

$$E(\sigma) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{N} \sum_{l=1}^{M} J_{ijkl}\sigma_{ij}\sigma_{kl} + \sum_{i=1}^{N} \sum_{j=1}^{M} \lambda_{ij}\sigma_{ij}, \tag{5}$$

where $\sigma_{ij} \in (-1, +1)$ is the orientation of the (i, j)th spin, J_{ijkl} is the interaction between the (i, j)th and (k, l)th spins, and λ_{ij} is the external magnetic field on the (i, j)th spin. Here, the Ising spins converge to the most energy stable state (i.e., the state that minimizes Eq. (5)). Therefore, by determining J_{ijkl} and λ_{ij} so that the optimal solution to the optimization problem is set to the ground state of Eq. (5), the optimization problem can be solved using the CIM.

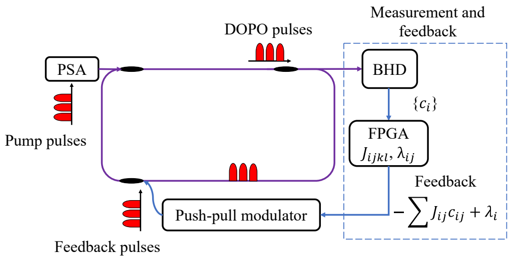

Figure 2: Coherent Ising machine [1]

Figure 2 shows the CIM with measurement and feedback. In the measurement-feedback CIM, the in-phase amplitude of degenerate optical parametric oscillator (DOPO) pulses $c_{ij} \in (0, \pi)$ correspond to the spin orientation $\sigma_{ij} \in (-1, +1)$. In operation, the DOPO pulses separated by the coupler are measured in-phase amplitude c_{ij} by the balanced homodyne detector(BHD). Subsequently, based on the measured amplitude, the interaction J_{ijkl} and external magnetic field λ_{ij} are calculated using the field programmable gate array (FPGA) module and the calculated pulse finally feeds back to the original pulse. This way, the full coupling of spins can be reproduced, and larger-scale spins can be realized.

4. RA Optimization Problem in NOMA with the CIM

4.1. Applying the CIM to NOMA

To solve optimization problems using the CIM, it is necessary to derive J_{ijkl} and λ_{ij} corresponding to the problem. However, it is difficult to map a binary variable optimization problem, such as Eq. (4), directly to Eq. (5). Therefore, we first apply the Hopfield-Tank neural network (HTNN) [7] to Eq. (4). The energy function to be minimized by the HTNN is as follows:

$$E(x) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{N}\sum_{l=1}^{M} w_{ijkl}x_{ij}x_{kl} + \sum_{i=1}^{N}\sum_{j=1}^{M}\theta_{ij}x_{ij}, \quad (6)$$

where $x_{ij} \in (0, 1)$ is the state of the (i, j)th neuron, w_{ijkl} is the coupling weight of the (i, j)th and (k, l)th neurons, and θ_{ij} is the firing threshold of the (i, j)th neuron. From Eqs. (5) and (6), the energy function of the HTNN has a structure similar to that of the Ising Hamiltonian. Therefore, we first derive w_{ijkl} and θ_{ij} by comparing Eq. (6) with Eq. (4) to apply the NOMA RA problem to the HTNN. Subsequently, by converting the output of the HTNN from $(0, 1)$ to $(-1, +1)$, we can derive the J_{ijkl} and λ_{ij}.

First, we transform Eqs. (4), (4a) and (4b) into Eqs. (7), (7a) and (7b), respectively, to put them in a form that can be compared with Eq. (6).

$$E_1 = \sum_{i=1}^{N_u}\sum_{j=1}^{N_c}\sum_{k=1}^{N_u}\sum_{l=1}^{N_c} -\delta_{jl}(1-\delta_{ik})\left(R_{ik}^{j} + R_{ki}^{l}\right)x_{ij}x_{kl}, \quad (7)$$

Table 1: Simulation parameters

Parameters	Values
Total Bandwidth B	5.0 MHz
Path loss coefficient α	4.0
Minimal distance between user and BS	50 m
Noise power spectral density	-170 dBm/Hz

$$E_2 = \sum_{i=1}^{N_u}\sum_{j=1}^{N_c}\sum_{k=1}^{N_u}\sum_{l=1}^{N_c} \delta_{ik}\left(1-\delta_{jl}\right)x_{ij}x_{kl} - \sum_{i=1}^{N_u}\sum_{j=1}^{N_c} x_{ij}, \quad (7.a)$$

$$E_3 = \sum_{i=1}^{N_u}\sum_{j=1}^{N_c}\sum_{k=1}^{N_u}\sum_{l=1}^{N_c} \delta_{jl}(1-\delta_{ik})x_{ij}x_{kl} - 3\sum_{i=1}^{N_u}\sum_{j=1}^{N_c} x_{ij}, \quad (7.b)$$

where δ_{ij} is Kronecker's delta: $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise. Thus, the energy function of the NOMA optimization in the HTNN is as follows:

$$E_{NOMA} = \alpha E_1 + \beta E_2 + \gamma E_3, \quad (8)$$

where α, β, and γ are parameters that adjust the scaling of each term. From the above, by comparing Eqs. (8) and (6), w_{ijkl} and θ_{ij} of the HTNN can be calculated as follows:

$$\begin{aligned} w_{ijkl}^{NOMA} =& 2\{\alpha\delta_{jl}(1-\delta_{ik})\left(R_{ijk} + R_{kli}\right) - \beta\delta_{ik}\left(1-\delta_{jl}\right) \\ &- \gamma\delta_{jl}(1-\delta_{ik})\}, \end{aligned} \quad (9)$$

$$\theta_{ij}^{NOMA} = -(\beta + 3\gamma). \quad (10)$$

Finally, using $\sigma_{ij} = 2x_{ij} - 1$ to set the output of the neuron to $(-1, +1)$, J_{ijkl} and λ_{ij} are obtained as follows:

$$J_{ijkl}^{NOMA} = \frac{w_{ijkl}}{2}, \quad \lambda_{ij}^{NOMA} = \theta_{ij} - \sum_{k=1}^{N_u}\sum_{l=1}^{N_c}\frac{w_{ijkl}}{2}. \quad (11)$$

By setting these parameters, the RA problem in the NOMA system can be solved at high speed using the CIM.

4.2. Simulation Results

In this subsection, simulations are performed to evaluate the performance of the proposed method. Specifically, the proposed method is evaluated by comparing the simulated annealing (SA) and the exhaustive search (ES) with respect to the data rate when the number of users changes. To evaluate the proposed method, the simulation models of the CIM shown in [8] are used. The simulation assumes a scenario where users are randomly placed in a 500-m circular cell. Here, the channel gain between user u and BS using channel c can be calculated as follows:

$$\left|h_u^c\right|^2 = (g_u^c)^2\, d_u^{-\alpha}, \quad (12)$$

where g_u^c represents the Rayleigh distribution, $d_u^{-\alpha}$ is the distance between user u and the BS, and α is the path loss coefficient. Other simulation parameters are shown in Table 1. Figure 3 shows that the proposed method can achieve the same performance as the ES. This indicates that the proposed method is superior not only in the optimization speed but also in searching for the best solutions.

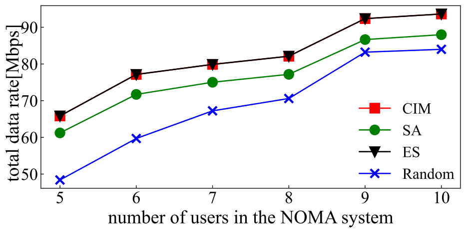

Figure 3: Total data rate versus the number of users

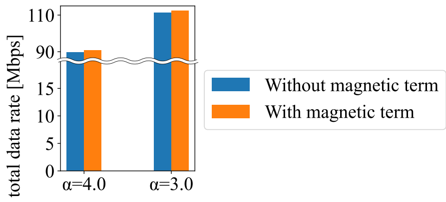

Figure 4: Total data rate with and without a magnetic term

5. RA Optimization Problem in NOMA Using CIM Without an External Magnetic Field

In the last section, we applied the CIM to NOMA by obtaining the interaction between spins J_{ijkl}^{NOMA} and the external magnetic field λ_{ij}^{NOMA}. Generally, the external magnetic field λ_{ij} in the Ising Hamiltonian is considerably larger than the strength of the interaction J_{ijkl}. Its large λ_{ij} is likely to induce amplitude instability in the actual CIM, which may cause a performance decrease. Therefore, in this section, we attempt to overcome these problems by reformulating the parameters of the Ising model. Specifically, we consider increasing the number of spins to reproduce λ_{ij} .

First, the $N \times M$ two-dimensional array of spins is extended to a two-dimensional array of $(N+N_{ex})\times(M+M_{ex})$ spins. Here, N_{ex} and M_{ex} represent the number of additional spins corresponding to N and M, respectively. Second, each additional spin adds a $\lambda_{ij}/(N_{ex}+M_{ex})$ strength of the interaction to the existing spins, which adds an interaction equal to the original λ_{ij}. This allows the spin to be affected by the external magnetic field without the λ_{ij} term in Eq. (5). Note that all additional spins must be in the same orientation to reproduce the λ_{ij}. The Ising Hamiltonian with additional spins is given as follows:

$$E(\sigma) = -\frac{1}{2}\sum_{i=1}^{N+N_{ex}}\sum_{j=1}^{M+M_{ex}}\sum_{k=1}^{N+N_{ex}}\sum_{l=1}^{M+M_{ex}} Q_{ijkl}\sigma_{ij}\sigma_{kl}, \qquad (13)$$

where Q_{ijkl} is the spin interaction, which is as follows:

$$Q_{ijkl} = \begin{cases} J_{ijkl}^{NOMA} & \text{if } i,k \le N \text{ and } j,l \le M \\ \lambda_{ijkl}^{NOMA}/(N_{ex}+M_{ex}) & \text{if } i,k > N \text{ or } j,l > M \\ A_{ijkl} & \text{if } i,k > N \text{ and } j,l > M \end{cases} \qquad (14)$$

where the conditions in Eq. (14) indicate that the (i, j)th spin and the (k,l)th spin are both existing spins, between existing and additional spins, and both additional spins, respectively. A_{ijkl} is the strength of the interaction between the additional spins and is adjusted to keep the additional spins in the same orientation.

Simulation is conducted for the case with and without an external magnetic field to evaluate performance. The evaluation assumes a NOMA system with 10 randomly placed users, and the simulation parameters are shown in Table 1. Additionally, simulations were performed for $\alpha = 3$ and $\alpha = 4$. Figure 4 shows that the performance is almost identical with and without the magnetic fields. These differences will become more pronounced when the actual CIM is used instead of the simulation model.

6. Conclusions

In this study, we focused on the CIM for fast optimization of highly complex NOMA RA problems. The simulation results show that the CIM-based RA optimization is not only in the optimization speed but also in searching for the best solution. Additionally, by formulating considering the stability, the performance of NOMA RA optimization without external magnetic fields is indicated. Future studies will investigate evaluation using the actual CIM.

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