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An efficient observation algorithm that achieves the minimum number of measurements for pairing optimization

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Abstract—Pairing optimization plays a critical role in the latest information and communication technologies, such as non-orthogonal multiple access (NOMA) in wireless communications. Here, all elements in the system should be pair-wise, or pairing is necessary. The problem is how to maximize the total performance of the entire system, which we call total compatibility. Understanding the relationships among elements, called compatibility, is indispensable prior to pairing optimization. This study demonstrates an efficient algorithm that allows grasping all compatibility information via the minimum number of observations. Such an efficient strategy is crucial for dynamically changing environments, notably by mobile wireless communications.

1. Introduction

Combining n (even number) elements into pairs of precisely two elements each to form a $(n/2)$-kind of pairs, which we call pairing, is one of the critical issues in recent information and communication systems and economics. Here we assume that any two elements in the system own a numerical figure which we refer to as *compatibility*. Our interest is to find a pairing that maximizes the *total compatibility*, which is the sum of compatibilities among the paired elements in the system.

The problem of pairing optimization can be seen, for example, in the next-generation communication system called NOMA (Non-Orthogonal Multiple Access), in which multiple terminals share a common frequency band by power domain coordination and signal processing called SIC (Successive Interference Cancellation) [1–8]. When n terminals are communicated in pairs, the throughput differs depending on the location of the terminals.

In NOMA, all terminals should be paired. Here, it is remarkable that the total throughput depends on the pairing. However, maximizing the total throughput of the entire system is extremely difficult because the total number of possible pairings is huge or $(n-1)!!$. Here the notation $(n-1)!!$ means $(n-1)\times(n-3)\times(n-3)\cdots 5\times 3\times 1$.

Therefore, a fast pairing optimization algorithm plays a critical role, and the goal is to develop a pairing optimization algorithm with high efficiency and high performance. In [9], we proposed an algorithm that achieves high performance and efficiency even when more stringent constraints are posed on the above-defined pairing optimization problem. The constraint is that only total compatibility is observable, whereas individual compatibilities among elements cannot be observed directly, which we refer to as the *limited observation constraints*.

Wich such limited observation constraints considered, the algorithm studied in [9] consists of two phases. The first phase is called *observation phase*, by which the individual compatibilities among elements are obtained only via the measurements of total compatibilities of pairings. We have shown that the minimum number of observations to acquire all individual compatibilities is given by $(n-1)(n-2)/2$. The second is called *combining phase* in which the pairing that yields high total compatibility is derived based on information obtained in the observation phase.

Regarding the combining phase, we demonstrate that the pairing optimization problem can be transformed into a traveling salesman problem (TSP) of a three-layer graph structure, which we call Pairing-TSP [9]. Herein, we can benefit from a variety of heuristic strategies in TSP for efficiently obtaining a high total compatibility pairing.

Concerning the observation phase, the minimum number of measurements is found to be $(n-1)(n-2)/2$, as mentioned above. However, how to accomplish such an efficient observation is not known yet. This paper focuses on the observation phase, and demonstrates an observation algorithm that achieves the minimum number of measurements.

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2. Pairing Optimization

Here we assume that the number of elements is an even natural integer n. We define the set of all elements $\mathbb{U}$ as follows;

$$\mathbb{U}(n) = \{i \in \mathbb{Z} \mid 1 \leq i \leq n\}.$$

Then, we define the set of all possible pairs $\mathbb{U}(n)$ as $\mathbb{P}(n)$;

$$\mathbb{P}(n) = \{\{i, j\} \mid i, j \in \mathbb{U}(n), i < j\}.$$

Here, we define *compatibility matrix C* as follows;

$$\begin{aligned} &C \in M_n(\mathbb{R}), \\ &1 \leq i \leq n, C_{i,i} = 0, \\ &\forall \{i, j\} \in \mathbb{P}(n), C_{i,j} = C_{j,i}. \end{aligned}$$

The set of all *compatbility matrix* whose size is $n \times n$ is defined as Ω_n. With using *compatbility matrix C*, the compatibility of $\{i, j\}$ is denoted by $C_{i,j}$ $(= C_{j,i})$. In addition, we define the reward function $f_{\mathbb{R}} : (\mathbb{P}(n), \Omega_n) \to \mathbb{R}$ as follows;

$$\begin{aligned} &C \in \Omega_n, \\ &\{i, j\} \in \mathbb{P}(n), \\ &f_{\mathbb{R}}(\{i, j\}, C) \equiv C_{i,j}. \end{aligned}$$

Then, we define pairing S as follows;

$$\begin{aligned} &\forall p \in S, p \in \mathbb{P}(n), \\ &\bigcup S = \mathbb{U}(n), \\ &A, B \in S, A \neq B \Rightarrow A \cap B = \emptyset. \end{aligned}$$

With above definitions, $C_{\text{sum}}(S, C)$, which is called *the total compatibility of pairing S* hereafter, is defined as follows;

$$C_{\text{sum}}(S, C) = \sum_{p \in S} f_{\mathbb{R}}(p, C).$$

We define the set of all pairings $\mathbb{S}(n) \equiv \{S\}$ when a number of elements is n. Finally, the pairing problem discussed in this study is formulated as follows;

$$\begin{aligned} &\max: C_{\text{sum}}(S, C), \\ &\text{subject to}: S \in \mathbb{S}(n). \end{aligned}$$

3. Proposed Observation Algorithm

3.1. Mathematical property of compatibility matrix

Our previous study shows that the minimum number of observations to acquire the compatibility matrix is $(n-1)(n-2)/2$ [9]. On the other hand, the number of linearly independent elements in the compatibility matrix is $n(n-1)/2$. That is, the degree of freedom of the minimum number of observation is smaller than the degree of the linearly independent elements by a factor of $N-1$. This means that an infinite number of compatibility matrices provides the same total compatibility set.

Hence, we introduce an equivalence class to examine the mathematical structure behind. We define an equivalence class regarding the relation $\sim$ between compatibility matrices C and $\tilde{C}$ by the following;

$$C \sim \tilde{C} \quad \text{iif} \quad \forall S \in \mathbb{S}(n), C_{\text{sum}}(S, C) = C_{\text{sum}}(S, \tilde{C}). \tag{1}$$

The proposed minimum number of observations algorithm finds the following compatibility matrix.

$$\begin{aligned} &\tilde{C}_{i,j} \\ = \; &\begin{cases} 0, \text{ if } 1 \in \{i, j\}, \\ C_{i,j} - C_{1,i} - C_{1,j} + \frac{2}{n-2} \sum_{k=2}^{n} C_{1,k}, \text{ otherwise.} \end{cases} \end{aligned} \tag{2}$$

3.2. Linear Independent Pairing Set

It is important to know which pairings to observe in this study because of the limited observation constraints. To achieve the minimum number of $(n-1)(n-2)/2$ observations, we need to perform observations on a set of $(n-1)(n-2)/2$ linearly independent pairings. Here we define $\mathbb{L}$, which we call *Linearly Independent Pairing Set (LIPS)* as follows;

$$\begin{aligned} &\forall S \in \mathbb{L}, S \in \mathbb{S}(n), \\ &\text{A number of element of } \mathbb{L} \text{ is } \frac{(n-1)(n-2)}{2}, \\ &\{C_{\text{sum}}(S, C) \mid S \in \mathbb{L}\} \text{ are linearly independent.} \end{aligned}$$

We need to observe $(n-1)(n-2)/2$ linearly independent pairings. Indeed, there are multiple LIPS in conducting these observations. However, what should be remarked is that we have to derive all compatibilities in the system after $(n-1)(n-2)/2$ observations are completed.

Here, an inverse matrix of the size $O(n^2) \times O(n^2)$ allows deriving all compatibility from the observed values of LIPS. However, it is too computationally intense or time consuming.

Therefore, the idea of the present study is to pick up a particular type of LIPS, by which the representation matrix from LIPS to compatibility is analytically solved. We call such a LISP by *Stairs-LIPS*. Again, no matrix inversion is necessary via Stairs-LIPS.

3.3. Proposed Observation Algorithm: Stairs-LIPS

The proposed method focuses on an upper triangular matrix with the the diagonal elements being zero. Further, let the columns $2k-1$ and $2k$ of this matrix be called the group G_k for $k \geq 1$. That is:

$$k \geq 2, G_k \equiv \{\{i, j\} \in \mathbb{P}(n) \mid i < j, i \in \{2k-1, 2k\}\}.$$

Here, we denote following values and sets:

$$\begin{aligned} &k \geq 2, x_k \equiv C_{2k-1,2k}, \\ &\mathbb{V} \equiv \left\{\{2i-1, 2i\} \in \mathbb{P}(n) \mid 1 \leq i \leq \frac{n}{2}\right\}. \end{aligned}$$

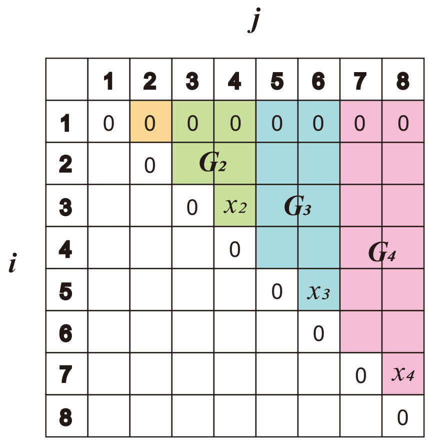

Figure 1: The compatibility matrix $\tilde{C}$ is divided into multiple regions G_k.

Some definition are depicted in Fig.1. Then, we define following mapping e as follows;

$$p_1 \subset S \in \mathbb{S}(n),$$
$$\bigcup p_1 = \bigcup p_2,$$
$$e(S, p_1, p_2) \equiv (S \setminus p_1) \cup p_2.$$

With using mapping e, we define pairing $S_{i,j}$ ($2 \le i < j \le n$) as follows;

$$d_{1,1}(i,j) = \{\{1,2\},\{j,\mathrm{pair}(j)\}\},$$
$$d_{1,2}(i,j) = \{\{2,j\},\{1,\mathrm{pair}(j)\}\},$$
$$d_{2,1}(i,j) = \{\{3,4\},\{i,\mathrm{pair}(i)\}\},$$
$$d_{2,2}(i,j) = \{\{3,i\},\{4,\mathrm{pair}(i)\}\},$$
$$d_{3,1}(i,j) = \{\{1,2\},\{i,\mathrm{pair}(i)\},\{j,\mathrm{pair}(j)\}\},$$
$$d_{3,2}(i,j) = \{\{1,\mathrm{pair}(j)\},\{2,\mathrm{pair}(i)\},\{i,j\}\},$$
$$S_{i,j} = \begin{cases} \mathbb{V}, \text{if } \{i,j\} = \{3,4\}, \\ e(S_{3,4}, d_{1,1}(i,j), d_{1,2}(i,j)), \text{if } i = 2, \\ e(S_{3,4}, d_{2,1}(i,j), d_{2,2}(i,j)), \text{if } i \ge 5 \cap \{i,j\} \in \mathbb{V}, \\ e(S_{3,4}, d_{3,1}(i,j), d_{3,2}(i,j)), \text{otherwise}. \end{cases}$$

Here, we define $\{i, \mathrm{pair}(i)\} \in \mathbb{V}$. Then, the following lemmas hold:

Lemma 1. When $\{i, j\} \in G_k \setminus \mathbb{V}$ and $2 < i < j \le n$, the following equation holds;

$$\tilde{C}_{i,j} = \left(x_{\frac{1+i+\mathrm{pair}(i)}{2}} + x_k\right) - \tilde{C}_{2,\mathrm{pair}(i)} + C_{\mathrm{sum}}(S_{i,j}, \tilde{C}) - C_{\mathrm{sum}}(S_{3,4}, \tilde{C}). \quad (3)$$

Proof. By calculating the difference between $C_{\mathrm{sum}}(S_{3,4}, \tilde{C})$ and $C_{\mathrm{sum}}(S_{i,j}, \tilde{C})$;

$$\begin{aligned} & C_{\mathrm{sum}}(S_{i,j}, \tilde{C}) - C_{\mathrm{sum}}(S_{3,4}, \tilde{C}) \\ = \; & (\tilde{C}_{1,\mathrm{pair}(j)} + \tilde{C}_{2,\mathrm{pair}(i)} + \tilde{C}_{i,j}) \\ & -(\tilde{C}_{1,2} + \tilde{C}_{i,\mathrm{pair}(i)} + \tilde{C}_{j,\mathrm{pair}(j)}) \\ = \; & (\tilde{C}_{2,\mathrm{pair}(i)} + \tilde{C}_{i,j}) - \left(x_{\frac{1+i+\mathrm{pair}(i)}{2}} + x_k\right). \end{aligned} \quad (4)$$

Then, the lemma 1 holds. □

Lemma 2. When $\{2, j\} \in G_k$ and $2 < j \le n$, the following equation holds;

$$\tilde{C}_{2,j} = x_k + C_{\mathrm{sum}}(S_{2,j}, \tilde{C}) - C_{\mathrm{sum}}(S_{3,4}, \tilde{C}). \quad (5)$$

Proof. By calculating the difference between $C_{\mathrm{sum}}(S_{3,4}, \tilde{C})$ and $C_{\mathrm{sum}}(S_{2,j}, \tilde{C})$;

$$\begin{aligned} & C_{\mathrm{sum}}(S_{3,4}, \tilde{C}) - C_{\mathrm{sum}}(S_{2,j}, \tilde{C}) \\ = \; & (\tilde{C}_{1,2} + \tilde{C}_{j,\mathrm{pair}(j)}) - (\tilde{C}_{2,j} + \tilde{C}_{1,\mathrm{pair}(j)}) \\ = \; & \tilde{C}_{j,\mathrm{pair}(j)} - \tilde{C}_{2,j} \\ = \; & x_k - \tilde{C}_{2,j}. \end{aligned} \quad (6)$$

Then, the lemma 2 holds. □

Lemma 3. When $\{i, j\} \in G_k \setminus \mathbb{V}$ and $2 < i < j \le n$, following equation holds;

$$\tilde{C}_{i,j} = x_k - C_{\mathrm{sum}}(S_{2,\mathrm{pair}(i)}, \tilde{C}) + C_{\mathrm{sum}}(S_{i,j}, \tilde{C}). \quad (7)$$

Proof. By lemma 2, following equation holds;

$$\begin{aligned} & \tilde{C}_{2,\mathrm{pair}(i)} \\ = \; & x_{\frac{1+i+\mathrm{pair}(i)}{2}} + C_{\mathrm{sum}}(S_{2,\mathrm{pair}(i)}, \tilde{C}) - C_{\mathrm{sum}}(S_{3,4}, \tilde{C}). \end{aligned} \quad (8)$$

By Eqs. (3) and (8), the lemma 3 holds. □

Lemma 4. When $k \ge 3$, following equation holds;

$$\begin{aligned} & x_k \\ = \; & x_2 + C_{\mathrm{sum}}(S_{2,3}, \tilde{C}) + C_{\mathrm{sum}}(S_{2,4}, \tilde{C}) \\ & -C_{\mathrm{sum}}(S_{3,4}, \tilde{C}) + C_{\mathrm{sum}}(S_{2k-1,2k}, \tilde{C}) \\ & -C_{\mathrm{sum}}(S_{3,2k-1}, \tilde{C}) - C_{\mathrm{sum}}(S_{4,2k}, \tilde{C}). \end{aligned} \quad (9)$$

Proof. By calculating the difference between $C_{\mathrm{sum}}(S_{3,4}, \tilde{C})$ and $C_{\mathrm{sum}}(S_{2k-1,2k}, \tilde{C})$;

$$\begin{aligned} & C_{\mathrm{sum}}(S_{3,4}, \tilde{C}) - C_{\mathrm{sum}}(S_{2k-1,2k}, \tilde{C}) \\ = \; & (x_k + x_2) - (\tilde{C}_{3,2k-1} + \tilde{C}_{4,2k}). \end{aligned} \quad (10)$$

Also, by the lemma 3, the following equation holds;

$$\begin{aligned} & \tilde{C}_{3,2k-1} \\ = \; & x_k - C_{\mathrm{sum}}(S_{2,4}, \tilde{C}) + C_{\mathrm{sum}}(S_{3,2k-1}, \tilde{C}), \end{aligned} \quad (11)$$

$$\begin{aligned} & \tilde{C}_{4,2k} \\ = \; & x_k - C_{\mathrm{sum}}(S_{2,3}, \tilde{C}) + C_{\mathrm{sum}}(S_{4,2k}, \tilde{C}). \end{aligned} \quad (12)$$

By Eqs. (10), (11), and (12), the lemma 4 holds. □

Lemma 5.

$$\begin{aligned} & x_2 \\ = \; & \frac{2}{n-2}\left\{C_{\mathrm{sum}}(S_{3,4}, \tilde{C}) - \sum_{k \ge 3} b_k\right\} - \frac{n-4}{n-2}a, \end{aligned} \quad (13)$$

$$\begin{aligned} & a \equiv C_{\mathrm{sum}}(S_{2,3}, \tilde{C}) + C_{\mathrm{sum}}(S_{2,4}, \tilde{C}) \\ & -C_{\mathrm{sum}}(S_{3,4}, \tilde{C}), \end{aligned} \quad (14)$$

$$\begin{aligned} & b_k \equiv C_{\mathrm{sum}}(S_{2k-1,2k}, \tilde{C}) - C_{\mathrm{sum}}(S_{3,2k-1}, \tilde{C}) \\ & -C_{\mathrm{sum}}(S_{4,2k}, \tilde{C}). \end{aligned} \quad (15)$$

Proof. By calculation of $C_{\text{sum}}(S_{3,4}, \tilde{C})$ with the lemma 4;

$$\begin{aligned} & C_{\text{sum}}(S_{3,4}, \tilde{C}) \\ = & \left(\frac{n}{2} - 1\right)x_2 + \left(\frac{n}{2} - 2\right)a + \sum_{k \geq 3} b_k. \end{aligned} \tag{16}$$

By this equation, lemma 5 holds. □

Theorem 1. All compatibilities can be represented by linear combination of $\{S_{i,j} \mid 2 \leq i < j \leq n\}$.

Proof. With lemmas 2, 3, 4, and 5, all compatibilities can be clearly represented by linear combination of $\{S_{i,j} \mid 2 \leq i < j \leq n\}$. □

With Theorem 1, we can construct compatibility matrix $\tilde{C}$ by observing only the Stairs-LIPS. Here, the number of Stairs-LIPS is $(n-1)(n-2)/2$, which is equal to the necessary and sufficient number of observations. Therefore, Stairs-LIPS satisfies the conditions of the LIPS we look for. That is, it provides the minimum number of observations and yields all compatibiity information without matrix inversion calculations.

4. Conclusion

This study proposed an algorithm that achieves the minimum number of observations for the observation phase in pairing optimization. Although the previous study unveils that the minimum number of observations required to recover the information of the compatibility matrix is $(n-1)(n-2)/2$, how to achieve such efficient observation has not been known. In this study, we define the linearly independent pairings, called LIPS, for the class that allows the minimum number of observations. However, not all LIPS is actually efficient because a complex calculation of an inverse matrix is usually inevitable to derive the compatibility matrix from LIPS. Therefore, we propose Stairs-LIPS, which is a special case of LIPS, by which the transformation to compatibility matrix is obtained by a simple analytical formula, without involving inverse matrix computation. Here, Stairs-LIPS yields a compatibility matrix $\tilde{C}$ with the constraint that $\tilde{C}_{i,j} = 0, (1 \in \{i, j\})$. The explicit formula can express the representation matrix of the transformation. Therefore, Stairs-LIPS can compute the compatibility matrix $\tilde{C}$ from the observed values with small computational complexity, even for large n. This research is expected to improve the efficiency of pairing optimization and may have future applications in communication systems that require fast pairing.

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