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Interacting Bifurcations in Switching Systems

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Abstract—We study a class of switching systems whose dynamics are characterized by an inner switching feedback loop and an outer continuous control loop. The loops have two widely separated time scales, performing fast-scale and slow-scale dynamics accordingly. Treating the two time scales separately, the stability of the systems can be studied, with results focusing on fast-scale bifurcation and slow-scale bifurcation. In current-mode controlled switching converters, period-doubling has been identified as fast-scale bifurcation whereas Hopf type bifurcation has been found as slow-scale bifurcation. However, in practice, the fast-scale and the slow-scale dynamics are interacting because the inner loop that is responsible for the fast-scale dynamics is actually controlled by the slow-scale outer feedback. This paper investigates the coexisting fast-scale and slow-scale bifurcations in simple dc/dc converters under peak current-mode control operating in continuous conduction mode. Boundaries of stable region, slow-scale bifurcation region, fast-scale bifurcation region, coexisting fast and slow-scale bifurcation region are identified.

1. Introduction

Electronic systems with multiple control loops are commonly used, with the dynamics of each loop dictated by the ultimate control requirement of the loop. In most practical situations, the transient speeds of the loops differ significantly, and hence, analysis of the entire system can be facilitated by considering the different loops separately one at a time. When a particular loop is studied, the others are assumed to be static as their time scales are sufficiently remote from the loop being considered. This often produces tractable results which can be easily applied in design. Moreover, identification of the dynamical behavior of the system becomes quite accessible, though it has obviously limited scope since only one loop dynamics is considered at a time. Since multiple loops are not considered together, any dynamical behavior pertaining to interaction of two or more loops would not be found.

In this paper we study a class of switching systems whose dynamics are characterized by two feedback loops, as illustrated in Fig. 1. The loops have two widely separated time scales thus performing fast-scale and slow-scale dynamics. Conventional treatment of the two time scales separately provides a simple approach to studying the stability of such systems, with results focusing on either fast-scale dynamics and slow-scale dynamics. Here, we consider the way the loops interact with each other, and identify the complex dynamics that may arise from such interactions. We will use the boost converter under current-mode control as the main illustrative example [1]–[5]. Boundaries of stable region, slow-scale bifurcation region, fast-scale bifurcation region, coexisting fast and slow-scale bifurcation region are identified. Such boundaries of operation provide essential design-oriented information that allows the system parameters to be selected in an informed manner.

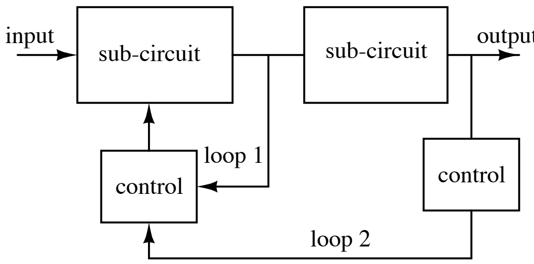
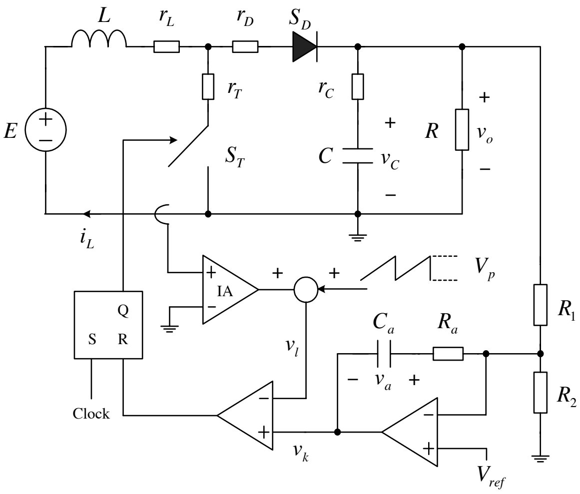


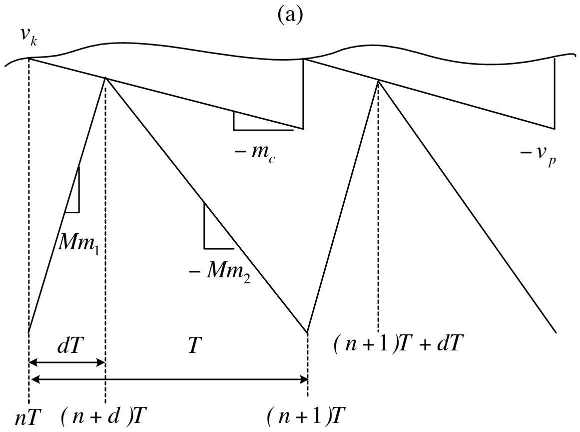
Figure 1: System with two control loops.

2. Current-Mode Controlled Boost Converter

The closed-loop current-mode controlled boost converter is shown in Fig. 2 (a). The waveforms of the control voltage and the inductor current analog are illustrated in Fig. 2(b). Each system has an outer voltage loop and an inner current loop. The voltage loop consists of an error amplifier (EA) and a proportional-integral controller, the output of which provides the reference for the inner current loop. The inner current loop consists of a current transformer and a current sensing amplifier (IA). A compensation ramp is added to stabilize the converter if a wide range of output voltage is required [1]. The output of the current loop is then connected to the inputs of the comparator whose output is used to reset a flip-flop latch to give a pulse-width modulated waveform to control switch S_T. The operation can be briefly described as follows. The flip-flop latch is set periodically by the clock signal, turning on the switch S_T. Then, if the inductance L is large enough,



(a)



(b)

Figure 2: (a) Basic boost converter under current-mode control; (b) typical waveforms of the control voltage v_k and the inductor current analog.

the inductor current i_L increases linearly, and is compared with a reference level, which is equal to the output of the error amplifier of the voltage loop minus the compensation ramp signal. When the peak inductor current reaches the reference level, the output of the comparator resets the flip-flop, thereby turning off the switch. When the switch is off, the inductor current decreases almost linearly if the output capacitor is sufficiently large. The cycle repeats when the flip-flop is set again by the clock. Typical waveforms for operation in continuous conduction mode are shown in Fig. 2 (b), where m_c is the slope of the compensation ramp signal, M is the sampling gain of the inductor current, and m_1 and m_2 are the rising and falling slopes of the inductor current with the switch on and off, respectively.

3. Bifurcation Behavior Under Loop Interaction

We begin with a series of typical waveforms which have been obtained from exact cycle-by-cycle computer simulations. The purpose is to provide a quick preview of the kinds of phenomena that would occur in the current-mode controlled boost converter under variation of the input voltage and load resistance. Our focus in this paper is the coexistence of fast-scale and slow-scale bifurcations. Moreover, the unstable operations associated with “saturation” of control signals (typically manifested as saturation of the duty cycle) will not be examined because these operations occur well beyond the boundaries of the first fast-scale or slow-scale bifurcations. The main parameters affecting fast-scale bifurcations are the rising slope of the inductor current m_1, and the compensation slope m_c, whereas those affecting slow-scale bifurcations are the voltage feedback gain g and time constant τ_a [6, 7]. Typical waveforms are shown in Fig. 3. Here, we fix $L = 165$ μH and $m_c = 11.125 \times 10^3$ A/s.

4. Derivation of Boundaries of Operations

In this section, we will take a detailed look into the qualitative behavior of the system, and present the boundaries of stable region, slow-scale unstable region, fast-scale unstable region, and coexisting fast-scale and slow-scale unstable region in terms of selected circuit parameters. Some of these operation boundaries, as shown in Fig. 4, are derived from cycle-by-cycle simulations, analytical solutions derived from discrete-time models [8]–[10], as well as experimental measurements. The three kinds of results agree with each other. Such boundaries of operation provide essential design-oriented information that allows the system parameters to be selected in a systematic manner. Similar studies can be extended to other types of converters.

From the operation boundaries shown in Fig. 4 (as well as in other parameter subspaces not reported here due to page limitation), we may make the following general observations for the particular system under study. It should be emphasized that the above methodology for developing operation boundaries is design-oriented and can be used for any converter system.

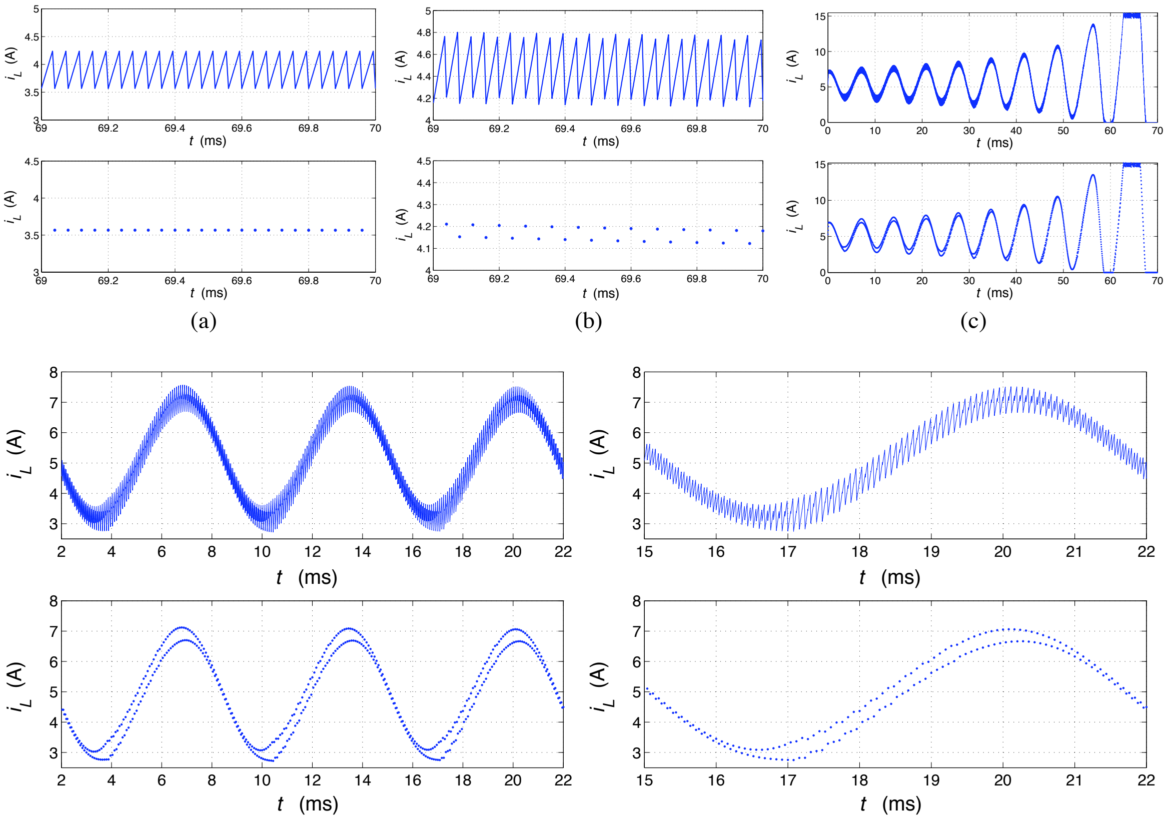
1. Interacting bifurcation occurs easily for relatively large load resistance or small feedback gain.

2. Consistent with the usual understanding, the input voltage has significant effects on both the system’s fast-scale and slow-scale stabilities. The fast-scale, slow-scale and coexisting fast-scale and slow-scale instabilities can be eliminated by increasing the input voltage.

3. Slow-scale or coexisting bifurcation can be eliminated by increasing the load resistance, which effectively changes the system’s natural frequency.

4. The feedback gain can affect both fast-scale and slow-scale stabilities, in contrast to the usual belief that the feedback gain is related only to slow-scale stability.

5. For relatively small feedback gain, coexisting or slow-scale boundaries will shift toward small values of the



(a) (b) (c)

(d1) (d2)

Figure 3: Simulated waveforms for the current-mode controlled boost converter under different input voltages with $L = 165\ \mu\text{H}$, $\tau_a = 0.3196$ ms, $g = 1.0$ and $m_c = 11.125 \times 10^3$ A/s. (a)–(d) are with $R = 30\ \Omega$, and (e) is with $R = 25\ \Omega$. Upper trace: actual waveforms, lower trace: sampled-data waveforms collected at $t = nT$. (a) Stable periodic operation with $E = 3.70$ V; (b) fast-scale bifurcation with $E = 3.40$ V; (c) saturated operation with $E = 3.0$ V; (d1)–(d2) coexisting fast-scale and slow-scale bifurcation with $E = 3.04$ V: (d1) time-domain waveforms of inductance current i_L and (d2) close-up view of (d1)

input voltage. Increasing the feedback time constant enlarges the fast-scale bifurcation area, while the fast-scale boundary region is nearly unaffected for different values of the feedback time constant.

5. Conclusion

In this paper we have studied the coexistence of slow-scale and fast-scale bifurcations in a class of switching systems, and specifically the boost converter under current-mode control. Our study has used exact cycle-by-cycle computer simulations, theoretical analysis based on a nonlinear discrete-time model, as well as experimental measurements. We have shown that current-mode controlled converters can lose stability via fast-scale instability, slow-scale instability, as well as interacting fast-scale and slow-sale instability. This finding clearly points out the mixing effect between the outer feedback loop and the inner current-programming loop which have been studied separately in many of the previous studies. Another important aspect of our study here is the gathering of operation boundaries in terms of practically relevant parameters, resulting in a useful form of information that can assist the design of practical converter systems, especially in avoiding or maintaining a particular kind of operation.

Acknowledgment

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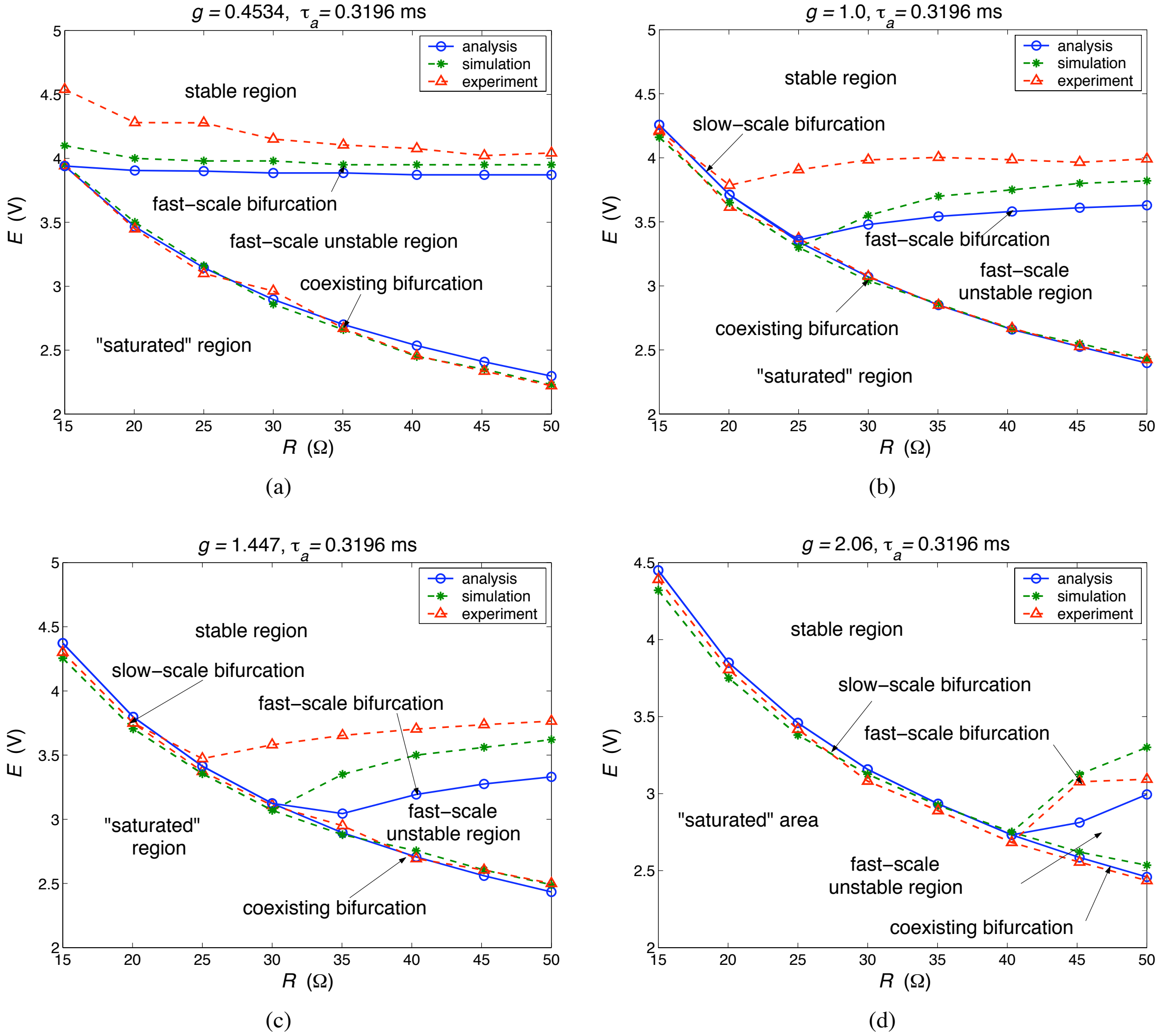

Figure 4: Operating boundaries for the current-mode controlled boost converter plotted in the (R, E) parameter space, with $\tau_a = 0.3196$ ms and different values of feedback gain g.

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