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A unified cluster synchronization criterion for impulsive delayed dynamical networks with hybrid coupling

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Abstract—In this paper, the cluster synchronization problem is addressed for a class of impulsive delayed dynamical networks with hybrid coupling. A more general delayed coupling term including different transmission delay and self-feedback delay is considered. Based on the average impulsive interval approach, a novel unified globally exponential cluster synchronization criterion is derived, which is valid for delayed dynamical networks with synchronizing impulses or desynchronizing impulses simultaneously. It is shown that the derived unified criterion are related to impulse strengths, average impulsive interval, and the coupling structure of the networks.

1. Introduction

Over the past decade, basic properties of complex dynamical networks have drawn a great deal of attention from diverse fields of science and engineering [1, 2], especially with respect to synchronization which is a kind of typical collective behavior. From the literature, there exist two common phenomena in many dynamical networks: delay effects and impulsive effects [3–8]. Due to the finite speeds of transmission and spreading as well as traffic congestion, time delay is inevitably encountered in dynamical networks [3, 4, 6–8]. On the other hand, the states of nodes in realistic networks are often subject to instantaneous perturbations and experience abrupt changes at certain instants, which may be caused by switching phenomenon, frequency change, or other sudden noise; that is, they exhibit impulsive effects [5–8]. Recently, impulsive dynamical networks have drawn increasing attention for their various applications in information science, economic systems, automated control systems, etc., [6–8]. Since time delays and impulses can heavily affect the dynamical behaviors of the networks, it is necessary to study both effects of time delays and impulses on synchronization of dynamical networks.

In practice, due to the specific goals, many technological, social and biological networks can be divided into clusters (communities), and nodes in the same cluster usually have the same function or property [1, 9]. This interesting and significant phenomenon can be described as cluster synchronization. By general definition, cluster synchronization is the phenomenon that the nodes in a dynamical network split into clusters, such that the nodes belonging to the same cluster are are completely synchronized (i.e., all the nodes in the same cluster approach to a uniform dynamical behavior), but those in different clusters are not [9]. Owing to its significance in biological sciences and communication engineering, cluster synchronization of complex dynamical network has recently received notable attention, and many excellent results have been obtained [10–12]. However, few results on cluster synchronization of impulsive delayed dynamical networks have been reported.

In general, there exist two types of time delays in dynamical networks. One is internal delay occurring inside the dynamical node [4, 6–8, 11]. The other is coupling delay caused by the exchange of information between dynamical nodes [3, 4, 7, 11]. Therefore, both the internal delay and coupling delay should be taken into account to describe real-world networks. Recently, much efforts have been devoted to synchronization in delayed dynamical networks [3, 4, 6–8, 11]. Unfortunately, most of the existing researches focus on the delayed coupling term given by $\Gamma_1\big(x_j(t-\sigma) - x_i(t-\sigma)\big)$ [3, 11] (i.e., the node's own state and neighbors's states are affected by the same delay) or that described by $\Gamma_1\big(x_j(t-\sigma) - x_i(t)\big)$ [4] (i.e., only transmission delay for signal sent from node j to node i exists in the network). In a real-word signal transmission process, however, delay may affect both the node's own state and neighbors's states and self delay may be different from neighboring delay [7]; that is, the coupling term has the form of $\Gamma_1\big(x_j(t-\sigma_1) - x_i(t-\sigma_2)\big)$, which is feedback with nonidentical delay. Hence, the delayed coupling term involving different transmission delay and self-feedback delay would be more close to the realistic situation. Obviously, this type of delayed coupling term takes the aforementioned two types of delayed coupling terms as a special case. Hence, a general model of impulsive delayed dynamical network with the internal delay and the delayed coupling term as $\Gamma_1\big(x_j(t-\sigma_1(t)) - x_i(t-\sigma_2(t))\big)$ will be discussed in this paper.

The purpose of this paper is to investigate the cluster synchronization of a class of impulsive delayed dynamical networks with both the internal delay and coupling delay. A more general delayed coupling term including different transmission delay and self-feedback delay is considered. Based on the average impulsive interval approach, a novel unified globally exponential cluster synchronization criterion is derived, which is simultaneously valid for synchronizing and desynchronizing impulses.

2. Problem formulation and preliminaries

In this paper, we consider a general complex delayed dynamical network consisting of N dynamical nodes with m clusters, where each node is an $n-$dimensional dynamical system with time-varying delay. Without lose of generality, the ith cluster is denoted by C_i $(1 \le i \le m)$ and let $C_1 = \{1, 2, \dots, l_1\}$, $C_2 = \{l_1 + 1, l_1 + 2, \dots, l_1 + l_2\}$, ..., $C_m = \{l_1 + l_2 + \cdots + l_{m-1} + 1, l_1 + l_2 + \cdots + l_{m-1} + 2, \dots, l_1 + l_2 + \cdots + l_{m-1} + l_m\}$, where $1 < l_m \le N$ and $\sum_{r=1}^{m} l_r = N$. The dynamical behavior of the complex delayed dynamical network can be described as

$$\begin{aligned}\dot{x}_i(t) = f_r(t, x_i(t), x_i(t-\tau_r(t))) + c_0 \sum_{j=1, j\neq i}^{N} b_{ij}^{(0)} \Gamma_0\Big(x_j(t) \\ -x_i(t)\Big) + c_1 \sum_{j=1, j\neq i}^{N} b_{ij}^{(1)} \Gamma_1\Big(x_j(t-\sigma_1(t)) \\ -x_i(t-\sigma_2(t))\Big), \quad i \in C_r, \end{aligned} \tag{1}$$

where $r \in \Re = \Big\{1, 2, \cdots, m\Big\}$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^\top \in \mathbb{R}^n$ is the state variable of each isolated node, $f_r : [0, +\infty) \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuously vector-valued function governing the evolution of each individual node in the cluster C_r. The time delays $\tau_r(t)$, $\sigma_1(t)$, and $\sigma_2(t)$ may be unknown but are bounded by known constants, i.e., $0 \le \tau_r(t) \le \tau_r$, $0 \le \sigma_1(t) \le \sigma_1$, and $0 \le \sigma_2(t) \le \sigma_2$, in which $\tau_r(t)$ denotes the internal delay occurring inside the individual node in the cluster C_r, $\sigma_1(t)$ represents the transmission delay for signal sent from node j to node i, and $\sigma_2(t)$ is the self-feedback delay. The positive constants c_0 and c_1 are the coupling strengths. $\Gamma_0 = (\gamma_{ij}^0)_{n\times n} > 0$ and $\Gamma_1 = (\gamma_{ij}^1)_{n\times n}$ represent the inner connecting matrices. $B^{(0)} = (b_{ij}^{(0)})_{N\times N}$ and $B^{(1)} = (b_{ij}^{(1)})_{N\times N}$ are the coupling matrices, in which $b_{ij}^{(0)}$ and $b_{ij}^{(1)}$ are defined as follows: if node i receives direct information from node j at time t and $t - \tau_2(t)$, respectively, then $b_{ij}^{(0)} \neq 0$ and $b_{ij}^{(1)} \neq 0$; otherwise, $b_{ij}^{(0)} = 0$ and $b_{ij}^{(1)} = 0$. Additionally, the diagonal elements of matrices $B^{(0)}$ and $B^{(1)}$ are defined by $b_{ii}^{(l)} = -\sum_{j=1, j\neq i} b_{ij}^{(l)}$, $i = 1, 2, \dots, N$, $l = 0, 1$, and thus one has $\sum_{j=1}^{N} b_{ij}^{(l)} = 0$, $i = 1, 2, \dots, N$, $l = 0, 1$. In general, $B^{(0)}$ and $B^{(1)}$ are the asymmetric matrices and may not be identical. This implies that the network is directed. Throughout the paper, we always assume that there exist some positive constants L_r^0 and L_r^τ such that

$$\begin{aligned}&\Big(x(t) - y(t)\Big)^\top \Big(f_r(t, x(t), x(t-\tau_r(t))) - f_r(t, y(t), y(t-\tau_r(t)))\Big) \\ &\le L_r^0 \Big(x(t) - y(t)\Big)^\top \Big(x(t) - y(t)\Big) \\ &\quad + L_r^\tau \Big(x(t-\tau_r(t)) - y(t-\tau_r(t))\Big)^\top \Big(x(t-\tau_r(t)) - y(t-\tau_r(t))\Big), \end{aligned} \tag{2}$$

for any $x(t), y(t) \in \mathbb{R}^n$ and $r \in \Re$.

In practical, the states of nodes in many realistic networks are often subject to instantaneous perturbations and experience abrupt changes at certain instants due to switching phenomenon, frequency change or other sudden noise, i.e., they exhibit impulsive effects [5–8]. Hence, it is reasonable to assume that at time instants t_k, there are "sudden changes" (or "jumps") in the state of node i such that

$$\Delta x_i\Big|_{t=t_k} \triangleq x_i(t_k^+) - x_i(t_k^-) = d_k x_i(t_k^-), \quad i = 1, 2, \dots, N, \tag{3}$$

where $\{t_1, t_2, t_3, \dots\}$ is an impulsive sequence satisfying $t_{k-1} < t_k$ and $\lim_{k\to\infty} t_k = +\infty$, $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$, $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$, and $d_k \in \mathbb{R}$ represents the strength of impulses. Then, we can obtain the following impulsive delayed dynamical network:

$$\begin{cases} \dot{x}_i(t) = f_r(t, x_i(t), x_i(t-\tau_r(t))) - c_1 b_{ii}^{(1)} \Gamma_1\Big(x_i(t-\sigma_1(t)) \\ \qquad -x_i(t-\sigma_2(t))\Big) + c_0 \sum_{p=1}^{m} \sum_{j\in C_p} b_{ij}^{(0)} \Gamma_0 x_j(t) \\ \qquad + c_1 \sum_{p=1}^{m} \sum_{j\in C_p} b_{ij}^{(1)} \Gamma_1 x_j(t-\sigma_1(t)), \quad t \neq t_k, \\ \Delta x_i = x_i(t_k^+) - x_i(t_k^-) = d_k x_i(t_k^-), \quad t = t_k, \ k \in Z^+, \\ x_i(t_0 + s) = \varphi_i(s), \ s \in [-\sigma, 0], \quad i \in C_r, \ t \ge t_0, \end{cases} \tag{4}$$

where $r \in \Re$, $Z^+ = \{1, 2, \dots\}$ denotes the set of positive integer numbers, and $\sigma = \max\{\sigma_1, \sigma_2, \tau\}$ with $\tau = \max_{r\in\Re} \tau_r$. Without loss of generality, we suppose that $x_i(t)$ is left continuous at $t = t_k$, i.e., $x_i(t_k) = x_i(t_k^-)$. The initial conditions $\varphi_i(s) \in PC\Big([-\sigma, 0], \mathbb{R}^n\Big)$, in which $PC\Big([-\sigma, 0], \mathbb{R}^n\Big)$ denotes the set of all functions of bounded variation and left-continuous on any compact subinterval of $[-\sigma, 0]$.

As preliminaries, the following statements are necessary.

Definition 1. The impulsive delayed dynamical network (4) with N nodes is said to realize cluster synchronization, if N nodes can be divided into m clusters as defined above such that

$$\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j \in C_r, \ r \in \Re, \tag{5}$$

and

$$\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| \neq 0, \quad i \in C_{r_1}, j \in C_{r_2}, \quad r_1 \neq r_2, r_1, r_2 \in \Re. \tag{6}$$

Definition 2. [5] (Average Impulsive Interval) An impulsive sequence $\zeta = \{t_1, t_2, t_3, \dots\}$ is said to have average impulsive interval T_a if there exist positive integer ς_0 and positive number T_a such that

$$\frac{T-t}{T_a} - \varsigma_0 \le N_\zeta(T, t) \le \frac{T-t}{T_a} + \varsigma_0, \quad \forall T \ge t \ge 0, \tag{7}$$

where $N_\zeta(T, t)$ denotes the number of impulsive times of the impulsive sequence ζ on the time interval (t, T), the constant ς_0 is called the "elasticity number" of the impulsive sequence, which implies that, on the time interval (t, T), the practical number of impulsive times $N_\zeta(T, t)$ may be more or less than $(T - t)/T_a$ by ς_0.

Assumption 1. Suppose that there exist constants a_r for $r \in \Re$ such that the diagonal elements of the coupling matrix $B^{(1)}$ of dynamical network (4) satisfy

$$b^{(1)}_{11} = \ldots = b^{(1)}_{l_1,l_1} = -a_1,$$
$$b^{(1)}_{l_1+1,l_1+1} = \ldots = b^{(1)}_{l_1+l_2,l_1+l_2} = -a_2, \ldots,$$
$$b^{(1)}_{l_1+l_2+\cdots+l_{m-1}+1,l_1+l_2+\cdots+l_{m-1}+1} = \ldots = b^{(1)}_{NN} = -a_m.$$

Assumption 2. Suppose the coupling matrix $B^{(l)}$ $(l = 0, 1)$ of dynamical network (4) has the following block form:

$$\begin{pmatrix} B^{(l)}_{11} & B^{(l)}_{12} & \cdots & B^{(l)}_{1m} \\ B^{(l)}_{21} & B^{(l)}_{22} & \cdots & B^{(l)}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ B^{(l)}_{m1} & B^{(l)}_{m2} & \cdots & B^{(l)}_{mm} \end{pmatrix} \tag{8}$$

where each block $B^{(l)}_{uv}=(b^{(l)}_{ij}) \in \mathbb{R}^{l_u\times l_v}$ $(u, v \in \Re)$ is a zero-row-sum matrix, i.e., $\sum_{j\in C_v} b^{(l)}_{ij} = 0$, $i \in C_u$, and each diagonal block $B^{(l)}_{uu}=(b^{(l)}_{ij}) \in \mathbb{R}^{l_u\times l_u}$ satisfies $b^{(l)}_{ij} \geq 0$ $(i \neq j)$ and $b^{(l)}_{ii}=-\sum_{j\in C_u} b^{(l)}_{ij}$. In addition, $\mathrm{rank}\big(B^{(0)}_{uu}\big) = l_u - 1$, $u \in \Re$.

Remark 1. In general, $b^{(l)}_{ij} > 0$ (or < 0), $i \neq j$, $l = 0, 1$, is viewed as the cooperative (or competitive) relationship between node i and node j [13]. Hence, Assumption 2 implies that nodes belonging to the same cluster only have cooperative relationships, while the nodes in different clusters can have both competitive and cooperative relationships. Additionally, the matrix $B^{(0)}_{uu}$ can be regarded as the Laplacian matrix of a weighted graph with a spanning tree, and $B^{(0)}_{uu}$ has an eigenvalue 0 with multiplicity 1 [14].

In this paper, we are mainly interested in studying the cluster synchronization problem for the impulsive delayed dynamical network (4). For this purpose, we introduce $s_r(t) = \frac{1}{l_r}\sum_{w\in C_r} x_w(t)$, $r \in \Re$, and define error vectors as $e_{ir}(t) = x_i(t) - s_r(t)$, $i \in C_r$ and $r \in \Re$. Based on Assumption 2, one has $\sum\limits_{j\in C_p} b^{(0)}_{ij}\Gamma_0 s_p(t) = \sum\limits_{j\in C_p} b^{(1)}_{ij}\Gamma_1 s_p\big(t-\sigma_1(t)\big) = \mathbf{0}_n$ for i =1, 2, . . . , N and $p = 1, 2, \ldots, m$, where $\mathbf{0}_n$ denotes the $n-$demensional vector of zeros. Hence, we obtain

$$\begin{aligned} \dot{e}_{ir}(t) &= \dot{x}_i(t) - \dot{s}_r(t) = \dot{x}_i(t) - \frac{1}{l_r}\sum_{w\in C_r}\dot{x}_w(t) \\ &= \tilde{f}_r(t, x_i, s_r, x_i^{\tau_r}, s_r^{\tau_r}) + a_r c_1\Gamma_1\Big(e_{ir}(t-\sigma_1(t)) \\ &\quad -e_{ir}(t-\sigma_2(t))\Big) + c_0\sum_{p=1}^{m}\sum_{j\in C_p} b^{(0)}_{ij}\Gamma_0 e_{jp}(t) \\ &\quad +c_1\sum_{p=1}^{m}\sum_{j\in C_p} b^{(1)}_{ij}\Gamma_1 e_{jp}\big(t-\sigma_1(t)\big) + J_r, \quad t \neq t_k, \end{aligned}$$

$$\begin{aligned} \Delta e_{ir}(t_k) &= e_{ir}(t_k^+) - e_{ir}(t_k^-) \\ &= x_i(t_k^+) - x_i(t_k^-) - \frac{1}{l_r}\sum_{w\in C_r}\Big(x_w(t_k^+) - x_w(t_k^-)\Big) \\ &= d_k x_i(t_k^-) - \frac{d_k}{l_r}\sum_{w\in C_r} x_w(t_k^-) = d_k e_{ir}(t_k^-), \ t = t_k, \end{aligned}$$

$$\sum_{i\in C_r} e_{ir}(t) = \sum_{i\in C_r}\Big(x_i(t) - s_r(t)\Big) = \sum_{i\in C_r} x_i(t) - l_r s_r(t) = \mathbf{0}_n,$$

where $\tilde{f}_r\big(t, x_i, s_r, x_i^{\tau_r}, s_r^{\tau_r}\big) = f_r\big(t, x_i(t), x_i(t-\tau_r(t))\big) - f_r\big(t, s_r(t), s_r(t-\tau_r(t))\big)$ and $J_r = f_r\big(t, s_r(t), s_r(t-\tau_r(t))\big) - \frac{1}{l_r}\sum_{w\in C_r} f_r\big(t, x_w(t), x_w(t-\tau_r(t))\big) - \frac{1}{l_r}\sum_{w\in C_r}\Big(c_0\sum_{j=1}^{N} b^{(0)}_{wj}\Gamma_0 x_j(t) + c_1\sum_{j=1}^{N} b^{(1)}_{wj}\Gamma_1 x_j(t-\sigma_1(t))\Big)$. Note that $x_i(t)$ is left continuous at t=t_k, i.e., $x_i(t_k)$=$x_i(t_k^-)$, then the error dynamical system can be characterized by:

$$\begin{cases} \dot{e}_{ir}(t) = \tilde{f}_r(t, x_i, s_r, x_i^{\tau_r}, s_r^{\tau_r}) + a_r c_1\Gamma_1\Big(e_{ir}(t-\sigma_1(t)) \\ \qquad -e_{ir}(t-\sigma_2(t))\Big) + c_0\sum\limits_{p=1}^{m}\sum\limits_{j\in C_p} b^{(0)}_{ij}\Gamma_0 e_{jp}(t) \\ \qquad +c_1\sum\limits_{p=1}^{m}\sum\limits_{j\in C_p} b^{(1)}_{ij}\Gamma_1 e_{jp}\big(t-\sigma_1(t)\big) + J_r, \ t \neq t_k, \\ e_{ir}(t_k^+) = (1 + d_k)e_{ir}(t_k), \ t = t_k, \ k \in Z^+, \ t \geq t_0, \ i \in C_r. \end{cases} \tag{9}$$

where $r \in \Re$. Clearly, if the zero solution of the error system (9) is globally exponentially stable, then globally cluster synchronization of the impulsive delayed dynamical network (6) is achieved according to Definition 1.

Remark 2. When $|(1 + d_k)| > 1$, i.e., the impulsive strengths $d_k > 0$ or $d_k < -2$, the impulses can potentially destroy the synchronization of the impulsive delayed dynamical network (4) because the absolute values of the synchronization errors are enlarged. Hence, the impulses with $|(1 + d_k)| > 1$ are desynchronizing impulses. Conversely, when $|(1 + d_k)| < 1$, i.e., the impulsive strengths $-2 < d_k < 0$, the impulses are synchronizing impulses, since the absolute values of the synchronization errors are reduced. In addition, when $|(1+d_k)| = 1$, i.e., the impulsive strengths $d_k = 0$ or $d_k = -2$, the impulses are neither beneficial nor harmful for the synchronization of the impulsive delayed dynamical network (4), since the absolute values of the synchronization errors are unchanged. This type of impulses are called inactive impulses [5]. Due to the fact that inactive impulses have no effect on the synchronization dynamics of the impulsive delayed dynamical network (4), we will not discuss this trivial case in this paper.

3. Main results

For convenience, define the matrix $\tilde{B}^{(0)}_r$ as $\tilde{B}^{(0)}_r \triangleq \Big(B^{(0)}_{rr} + B^{(0)\top}_{rr}\Big) - \Xi_r$, where $\Xi_r = \mathrm{diag}\Big(\xi^r_1, \xi^r_2, \cdots, \xi^r_{l_r}\Big)$ with $\xi^r_j = \sum_{u\in C_r} b^{(0)}_{uj}$. Under Assumption 2, it is easy to see that the matrix $\tilde{B}^{(0)}_r (r \in \Re)$ is a symmetrical irreducible matrix with zero-row-sum and nonnegative off-diagonal elements. This means that zero is an eigenvalue of $\tilde{B}^{(0)}_r (r \in \Re)$ with multiplicity 1, and all the other eigenvalues of $\tilde{B}^{(0)}_r (r \in \Re)$ are strictly negative [15]. Hence, eigenvalues of $\tilde{B}^{(0)}_r (r \in \Re)$ can be ordered as $0 = \tilde{\lambda}^r_1 > \tilde{\lambda}^r_2 \geq \cdots \geq \tilde{\lambda}^r_{l_r}$.

Theorem 1. Suppose that Assumptions 1-2 hold, and the impulsive sequence $\zeta = \{t_1, t_2, t_3, \ldots\}$ satisfies (7) with average impulsive interval T_a and elasticity number ς_0. Then the impulsive delayed dynamical network (4) is globally cluster synchronized if there exist positive constants ς_1, ς_2, ς_3,ς_4, d and a constant q_1 such that

(i) $(1 + d_k)^2 \leq d, \quad k \in Z^+$,

(ii) $\Omega_r + \Theta_r \lambda(\Theta_r) I_{l_r} - q_1 I_{l_r} \leq 0, \quad r \in \Re$,

(ii) $\varpi \triangleq \dfrac{\ln d}{T_a} + q_1 + \gamma q_2 < 0$,

where $\Omega_r = \Big(2L_r^0 + a_r c_1 \varsigma_1 \lambda_{\max}(\Gamma_1 \Gamma_1^\top) + a_r c_1 \varsigma_2 \lambda_{\max}(\Gamma_1 \Gamma_1^\top) + (m-1)\varsigma_3 \max\limits_{1 \leq r, q \leq m, r \neq q} \Big(\lambda_{\max}\Big(B_{rp}^{(0)} B_{rp}^{(0)\top}\Big)\Big) \lambda_{\max}\Big(\Gamma_0 \Gamma_0^\top\Big) + (m-1)\varsigma_3^{-1} + m\varsigma_4 \max\limits_{1 \leq r, q \leq m} \Big(\lambda_{\max}\Big(B_{rp}^{(1)} B_{rp}^{(1)\top}\Big)\Big) \lambda_{\max}\Big(\Gamma_1 \Gamma_1^\top\Big)\Big) I_{l_r}$, $q_2 = 2\Big(\max\limits_{1 \leq r \leq m} L_r^\tau\Big) + a_r c_1 \varsigma_1^{-1} + a_r c_1 \varsigma_2^{-1} + m\varsigma_4^{-1}$, $\gamma = \max\{d^{-\varsigma_0}, 1, d^{\varsigma_0}\}$, and $\Theta_r = c_0\Big(\tilde{\lambda}_2^r + \Big(\max\limits_{1 \leq j \leq l_r} \xi_j^r\Big)\Big)$ with

$$\lambda(\Theta_r) = \begin{cases} \lambda_{\max}(\Gamma_0), & \text{if } \Theta_r > 0, \\ 0, & \text{if } \Theta_r = 0, \\ \lambda_{\min}(\Gamma_0), & \text{if } \Theta_r < 0. \end{cases} \tag{10}$$

Remark 3. It can be seen that the condition $0 < d < 1$ or $d > 1$ is not imposed in Theorem 1. This means that the cluster synchronization criterion derived in Theorem 1 not only can be applied to the case with $|1+d_k| < 1$ (synchronizing impulses) but also to the case with $|1 + d_k| > 1$ (desynchronizing impulses). Thus, Theorem 1 gives a unified globally cluster synchronization criterion for the hybrid-coupled impulsive delayed dynamical network (4), which is simultaneously applicable for synchronizing impulses and desynchronizing impulses.

4. Conclusion

This paper deals with the globally cluster synchronization of general delayed dynamical networks with hybrid coupling and impulsive effects. The delayed coupling term considered includes the transmission delay and self-feedback delay; more general than most existing results. By the average impulsive interval approach, a unified globally cluster synchronization criterion is obtained for the proposed hybrid-coupled impulsive delayed dynamical networks, which is simultaneously effective for synchronizing and desynchronizing impulses.

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