2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Property of the chaotic propagating pulse wave in a ring of coupled bistable oscillators

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Abstract– In this paper, some properties of the chaotic propagating pulse wave in a ring of six coupled bistable oscillators are investigated. When coupling factor α becomes large beyond a certain critical point, the standing pulse wave converts to a propagating pulse wave. Further, as α is increased, the propagating pulse wave behaves chaotically. We find some interesting properties of chaotic propagating pulse wave such as random change of propagation direction, stepwise change of pulse position wrt time, and probability density of the time-length and distance, etc.

1. Introduction

The pulse wave propagation phenomena in coupled oscillator systems are very popular in recent years [1]. We have investigated the pulse wave in a ring of coupled bistable oscillator systems in [2], [3]. Generally speaking, for small coupling factor, there is a standing pulse wave which stays in one place. When the coupling factor becomes large beyond a certain critical value, the standing pulse wave converts to the propagating pulse wave. Further, for larger coupling factor, the propagating pulse wave becomes chaotic. It changes its propagation direction at random. In this paper, we investigate the properties of the chaotic propagating pulse wave such as probability density of the time-length and distance, etc.

2. Chaotic propagating pulse wave in a ring of six coupled bistable oscillators

In our previous paper, we investigate transition mechanism from a standing pulse wave to a propagating pulse wave in terms of coupling factor α in a ring of six coupled bistable oscillators [2][3].

The equation we investigate is as follows:

$$\begin{aligned} \dot{x}_i &= y_i \\ \dot{y}_i &= -\varepsilon(1-\beta x_i^2 + x_i^4) y_i \\ &\qquad -(1-\alpha)x_i + \alpha(x_{i-1} + 2x_i + x_{i+1}) \qquad (1) \end{aligned}$$

$$, i = 1,2,\cdots,n, \qquad x_0 = x_N, x_1 = x_{N+1}$$

, where N is number of oscillators. The x_i denotes the normalized output voltage of the i-th oscillator, y_i denotes its derivative. The parameter ε (> 0) shows the degree of nonlinearity. The parameter α (0 ≦ α < 1) is a coupling factor; namely α = 1 means maximum coupling, and α = 0 means no coupling. The parameter β controls amplitude of oscillation. Each isolated oscillator has two steady- states, namely, no oscillation and periodic oscillation depending on the initial condition. In this paper, parameters β and ε are fixed as β = 3.18 and ε = 0.36.

It has been already clarified that the transition from the standing pulse wave to the propagating pulse wave is a bifurcation from the periodic solution to the almost periodic solution, and that the bifurcation originates in a complex combination of the pitchfork and the heteroclinic bifurcations [3]. When coupling factor α is increased, it is noted that the propagation speed increases and beyond a certain critical value of α, the propagating wave become chaotic. The variation of Lyapunov exponents is presented in term of α in **Figure 1**.

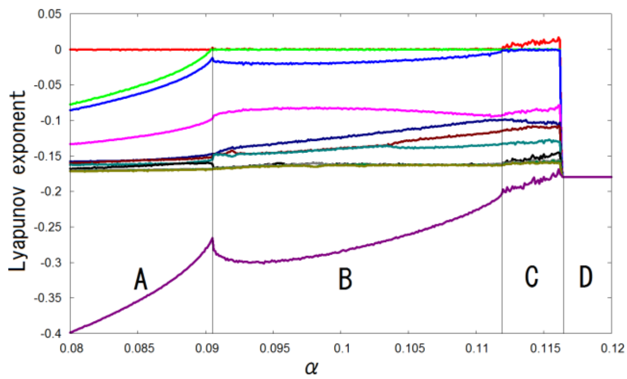
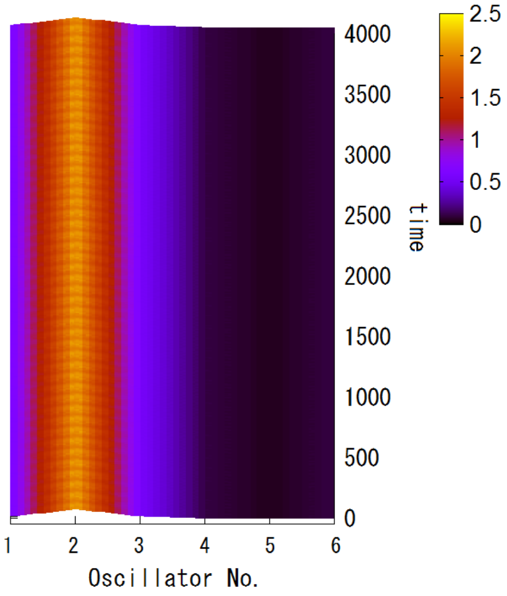


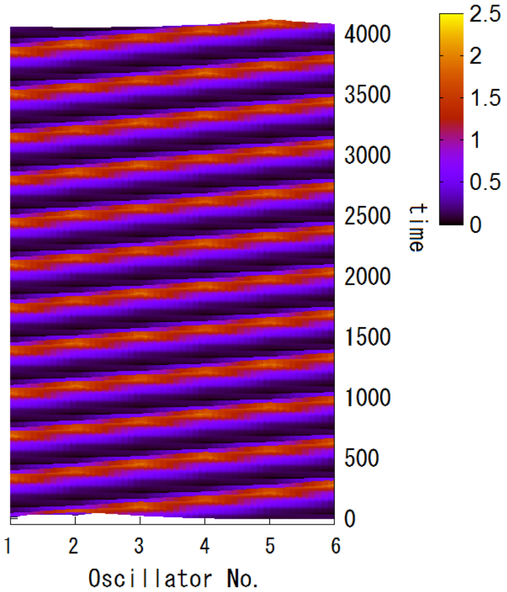
Fig. 1 Transition of 12 Lyapunov exponents of a ring of six coupled bistable oscillators in terms of α for β = 3.18 and ε = 0.36. red: LE1, green: LE2, blue: LE3. In region B, LE1 and LE2 overap.

Namely, in region A(0.08 ≦ α < 0.0905), where the standing pulse wave exists, LE1 = 0 and LE2~LE12 < 0. Therefore, this is a periodic solution. In region B (0.0905 ≦ α < 0.1118) where the non-chaotic propagating pulse wave exists, LE1 = LE2 = 0 and LE3~LE12 < 0. Therefore, this is an almost periodic solution. In region C (0.1118 ≦ α < 0.1162) where the chaotically propagating pulse wave exists, LE1 > 0, LE2 = 0 and LE3~LE12 < 0. Therefore, this is a chaotic attractor. In region D (0.1162 ≦ α) there is no oscillation.

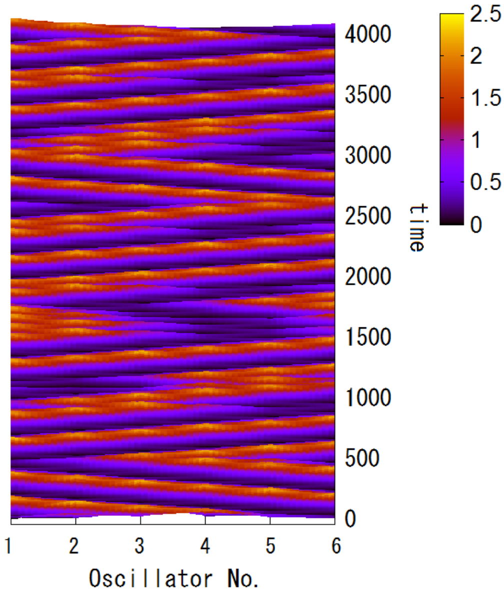
Figures 2 demonstrates 3D representation of typical (a) standing pulse wave, (b) non-chaotic propagating pulse wave, and (c) chaotic propagating pulse wave. Note that the standing pulse wave stays in one position, propagating pulse wave propagates in one direction; namely propagating direction is unchanged, once it is determined. In contrast, the chaotic propagating pulse wave changes its direction occasionally in random manner.



(a)



(b)

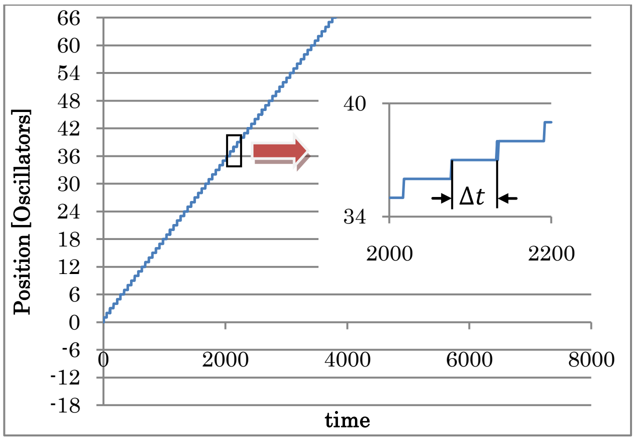


(c)

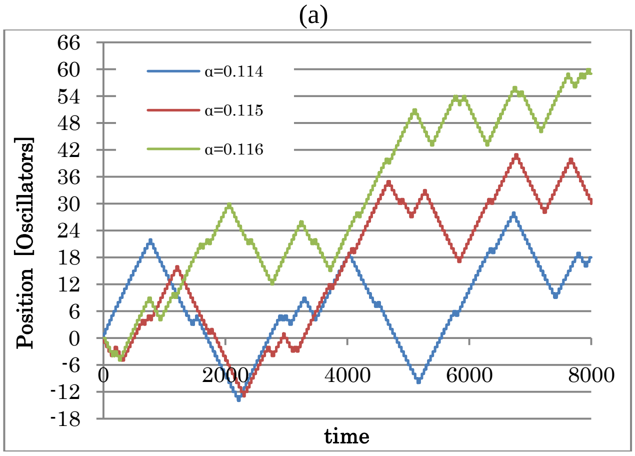
Fig. 2 Three typical waves: (a) standing pulse wave for α = 0.08, (b) non-chaotic propagating pulse wave for α = 0.10 and (c) chaotic propagating pulse wave for α = 0.115. The fixed parameters are ε = 0.36 and β = 3.18. The absolute magnitude of $\sqrt{x_i^2 + y_i^2}$ (i : number of oscillators) is shown in colors.

Figures 3 (a) and (b) present the propagating distance measured by oscillator number in terms of time for (a) α chosen in non-chaotic regime, and for (b) three values of α all chosen in chaotic regime. It is recognized that the propagating direction do not change in non-chaotic regime, but it suddenly changes in random manner in chaotic regime. It seems that the absolute value of propagation speed (= magnitude of the slope) in chaotic regime is constant for fixed values of α.

Figure 4 shows absolute value of propagation speed in terms of α. It is recognized that the propagation speed in chaotic regime is a smooth extension of the non-chaotic regime. That is, the (absolute value of) propagation speed increases with the increase of α.



(a)



(b)

Fig. 3 Propagating distance measured by oscillator number. (a) Non-chaotic propagating pulse wave for $\alpha = 0.100$. (b) Chaotic propagating pulse wave for $\alpha = 0.114$ (blue), 0.115 (red) and 0.116 (green). The fixed parameters are $\varepsilon = 0.36$ and $\beta = 3.18$. Small figure is (a) presents a magnified diagram. Same structure can be seen in (b).

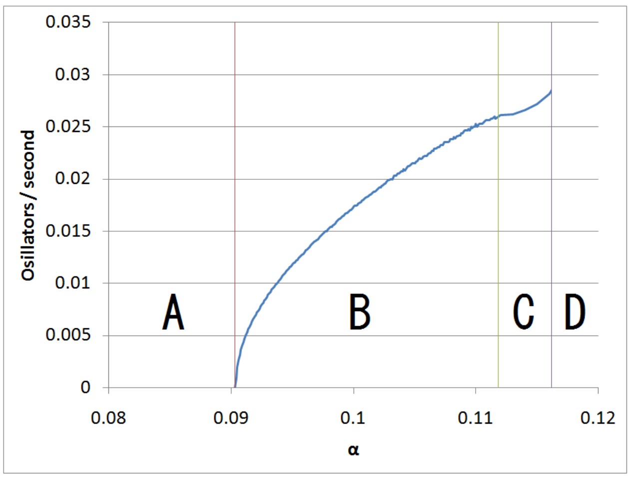


Fig. 4 Propagating speed in terms of α region A: standing pulse wave, region B: non-chaotic propagating pulse wave, region C: chaotic propagating pulse wave. The fixed parameters are $\varepsilon = 0.36$ and $\beta = 3.18$.

3. Statistical characteristics

Figures 5 (a), (b) and (c) demonstrate the probability density of the one-section time length for three values of α. This probability density is calculated by using the kernel density estimation method with band width equals to 1 and with Gaussian kernel [4]. Here, one-section time length denotes the time in which a chaotic pulse propagates to one direction. The probability of occurrence of small time length and that of large time length are both large, while, that of medium time length is small. Moreover, the maximum time length for small α (the time corresponding to peak a), is larger than that for large α (the time corresponding to peak a' and a''). This means that the probability of propagating direction change is small for smaller values of α compared to larger values of α. One of the characteristic features of the probability density in Fig.5 is its tooth-like structure. The time Δt in Fig.3 (a) is equal to the time between two peaks in Fig.5. This is the time for a pulse to move one oscillator unit. Namely, a pulse stays in one oscillator for a long time and quickly moves to the next one.

Figures 6 (a), (b) and (c) denote the probability density of one-section length. Same as Fig.5 the probability density is large in both sides and it is small in the middle. In particular, comparing three peaks b, b' and b'', it is recognized that for smaller α, the distance of one-section is longer.

4. Conclusions

We investigate properties of chaotic propagating pulse wave in a ring of six coupled oscillator system. Namely, we calculate propagating distance in term of time, propagating speed in terms of coupling strength, and probability density of the pulse direction change phenomenon. In the future, we investigate the same characteristics for larger number of oscillator cases.

References

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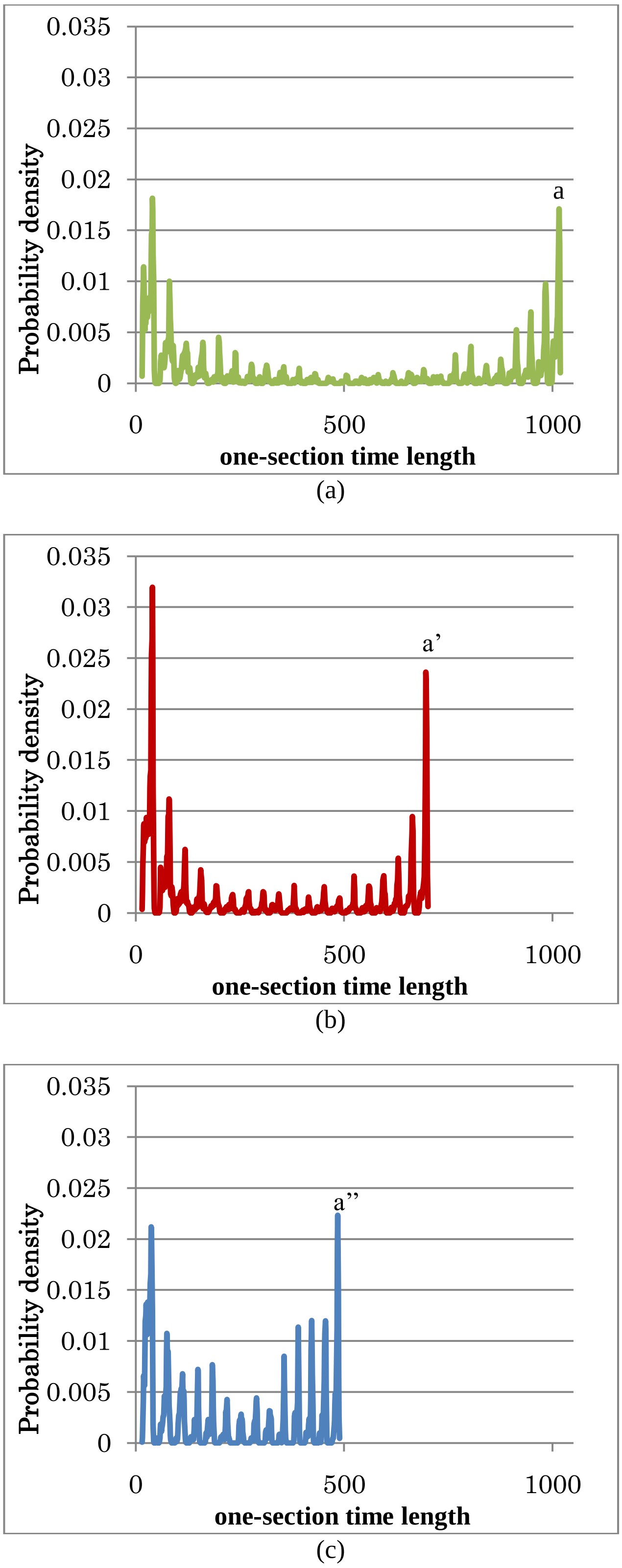

Fig. 5 Probability density of one-section time length of a chaotic propagating pulse in different values of α: (a) α = 0.113, (b) α = 0.114, (c) α = 0.116. The total time for (a), (b) and (c) is 800000 seconds. The fixed parameters are ε = 0.36 and β = 3.18.

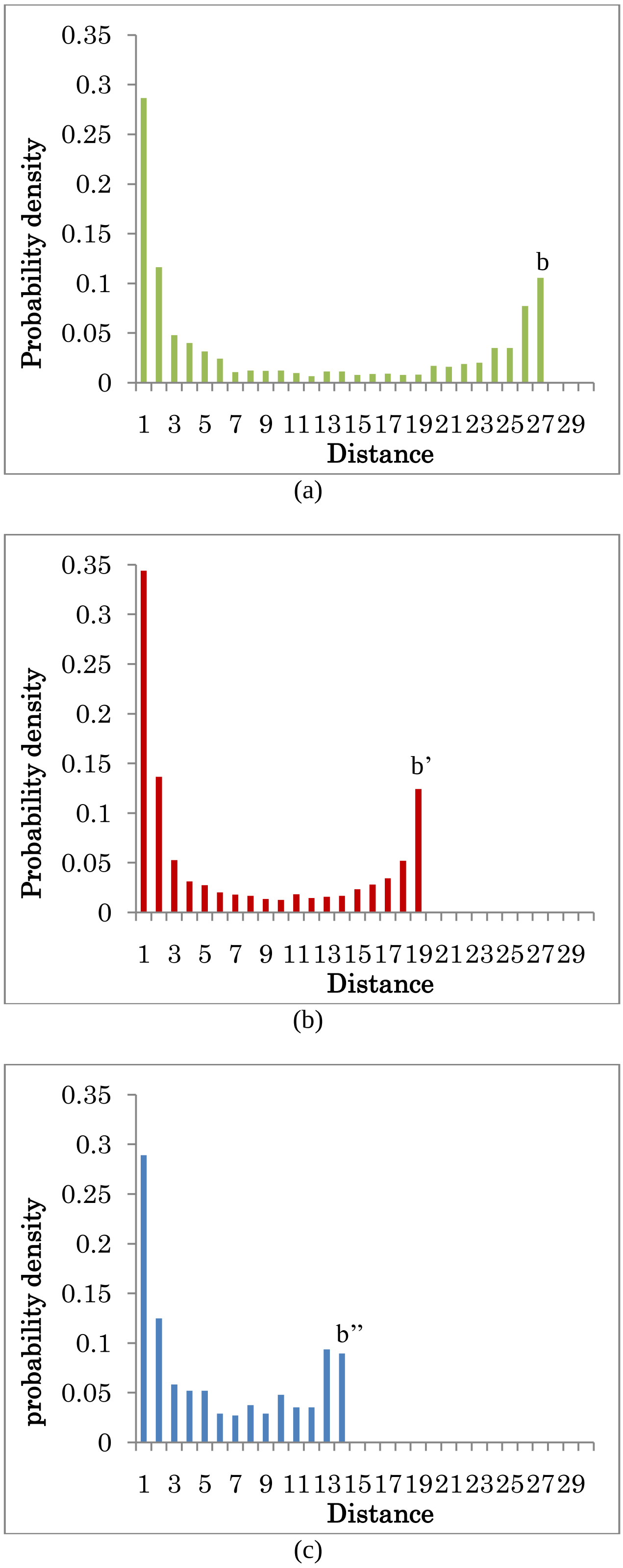

Fig. 6 Probability density of the distance of one section of a chaotic propagating pulse in different values of α:. (a) α = 0.113, (b) α = 0.114, (c) α = 0.116. The total time for (a), (b) and (c) is 800000 seconds. The fixed parameters are ε = 0.36 and β = 3.18.