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A Complex Network Perspective to Volatility in Stock Markets

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Abstract—In this paper we examine the interaction of stock markets of different countries by constructing networks that connect 32 selected stock market indices from different countries. In the network being constructed, the nodes are the stock market indices and the edges are connections between the indices. Each edge has an edge weight equal to the cross-correlation between the pair of connecting indices over a window of w days. We consider the period from 7 March 2005 to 23 April 2009, i.e., 1078 days with $w < 1078$. In this period, networks are constructed for all w-day windows at 1-day intervals. By examining the variation of the network parameters as time elapses, we show that the dynamics of network connectivity is related to the fluctuation of the stock markets. Specifically, a form of network synchronization is found to be correlated with the volatility of the stock markets. Our study thus reveals that the stock markets in different countries generally behave in a synchronous manner when the markets experience fluctuation.

Keywords—Complex network, stock market, network dynamics, market volatility.

1. Introduction

The total domestic market capitalization of world equity markets has exceeded US$ 60 trillion in 2007 but dropped 46.5% in 2008 [1]. The phenomenon seemed to be global as the 2008 financial crisis swept almost every country [2]. It is thus clear that stock markets in different countries do not operate independently, and their interactions have a significant role to play in shaping the overall world stock market performance. The co-movement of world exchange indices has been studied since the 1970s [3]. Prior work uses variations of ARCH (autoregressive conditional heteroskedasticity) models [4, 5, 6] to study the correlations between stock market indices. Directed acyclic graphs are used to represent the structure of interdependence in international stock markets [7]. It has been found that a relationship exists between the structure of international stock markets and the market volatility [4, 8]. However, in much of the previous research, only a small number of stock markets from developed countries have been studied, resulting in somewhat biased conclusions on relationship between individual markets' volatility and their correlations to the peer markets. Also, the previous approaches have over-simplified the structure of international stock markets.

In this paper, we construct a network of stock markets of 32 member countries of the World Federation of Exchanges.[1] The network nodes are the representative indices of the 32 stock markets.[2] Our study considers the daily closing value of each index during the 1078 working day period from 7 March 2005 to 23 April 2009. In case a stock market is closed on a working day, the day's closing value inherits from the last available working day. In this 1078-day period, the stock markets network is constructed for all w-day windows at 1-day intervals. In the network, each pair of nodes are connected by an edge, with weight equal to the Pearson's correlation coefficient between the two adjacent indices over a window of w days. It is obvious that the weight of edges evolve chronologically as the window slides in forward time. In this paper we will examine the network dynamics based on the variation of edge weights as time elapses. For each w-day window, we also calculate the properties of each stock market index, including its return, mean value and volatility in the window period. Our study focuses on investigating the relationships between network dynamics and financial properties of the indices under different choices of window size.

We begin with the network construction procedure in Section 2. Then, we examine the network properties and introduce the definition of network dynamics in Section 3. We will examine the dynamics of the stock market indices in Section 4 and show the relationship between network dynamics and stock markets' financial dynamics in Section 5. Finally the discussion of our results will be presented in Section 6.

2. Network construction

In the 1078 working days from 7 March 2005 to 23 April 2009, the network is constructed for each of the w-day windows at 1-day intervals. Hence, the entire period is divided into M windows: $W_1, W_2, \cdots, W_M$, where $M = 1079 - w$. Let $P_i(m)$ be the series of closing values of stock index i in the mth window. We consider the node of stock index i in the mth window. In the network construction procedure, each pair of network nodes are connected by an edge, with the edge weight equals to the Pearson's correlation between the pair of adjacent indices [9]. Specifically, the edge weight $\rho_{i,j}(m)$ between node $P_i(m)$ and $P_j(m)$ in the

[1] They include Brazil, Mexico, Argentina, the USA and Canada from the Americas; the Netherlands, Austria, Belgium, France, Germany, the UK, Ireland, Spain, Denmark, Sweden, Portugal, Italy, Switzerland and Norway from Europe; Australia, India (both National Stock Exchange of India and Bombay Stock Exchange), Hong Kong, Indonesia, Malaysia, New Zealand, Japan, South Korea, China, Singapore from Asia/Pacific Region; Egypt and Israel from Africa and Middle East.

[2] They are Bovespa (Brazil), IPC (Mexico), MerVal (Argentina), S&P 500 (USA), S&P TSX Composite (Canada), AEX (Netherlands), ATX (Austria), BEL-20 (Belgium), CAC 40 (France), DAX (Germany), FTSE 100 (United Kingdom), ISEQ20 (Ireland), Madrid General (Spain), OMX Copenhagen 20 (Denmark), OMX Stockholm 30 (Sweden), PSI 20 (Portugal), S&P Mib (Italy), Swiss Market (Switzerland), Total Share (Norway), All Ordinaries (Australia), BSE 30 (India), Hang Seng (Hong Kong), Jakarta Composite (Indonesia), KLSE Composite (Malaysia), NZSE 50 (New Zealand), Nikkei 225 (Japan), S&P CNX NIFTY (India), Seoul Composite (South Korea), Shanghai Composite (China), Strait Times (Singapore), CASE 30 (Egypt) and TA-100 (Israel). All data are retrieved from Yahoo! Finance.

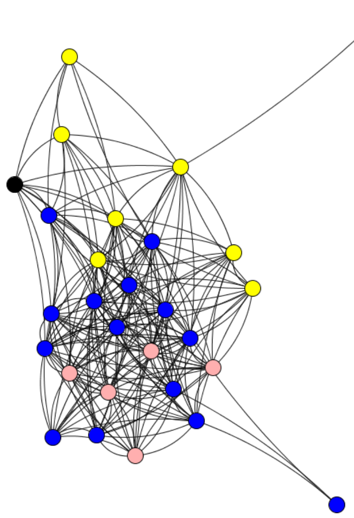


Figure 1: A network for world stock market indices constructed from a 20-day window from 12 July 2008 to 2 August 2008. Edges with weights below 0.95 are excluded (unconnected). Yellow nodes represent stock market indices from the Asia Pacific region; red for the Americas; blue for Europe; and black for the Africa/Middle East region.

mth window is given by

$$\rho_{i,j}(m) = \frac{\text{cov}(P_i(m), P_j(m))}{\sigma_{P_i(m)}\sigma_{P_j(m)}} \quad (1)$$
$$= \frac{\langle P_i(m)P_j(m)\rangle - \langle P_i(m)\rangle\langle P_j(m)\rangle}{\sqrt{\langle P_i(m)^2\rangle - \langle P_i(m)\rangle^2}\sqrt{\langle P_j(m)^2\rangle - \langle P_j(m)\rangle^2}}$$

where $\sigma_{P_i(m)}$ and $\sigma_{P_j(m)}$ are the standard deviations of the constituent closing values in $P_i(m)$ and $P_j(m)$, respectively; cov means covariance; and $\langle\cdots\rangle$ denotes the expected value. During the whole 1078-day period, the network construction procedure is repeated M times. Each edge will have its weight recalculated for every w-day window and hence has an edge weight series of length M. In the next section we will define network dynamics based on the weight series of edges.

The edge weight $\rho_{i,j}$ ranges from -1 to 1. Furthermore, for better clarity of presentation of the network, we may optionally omit edges whose weights are less than a threshold value θ. Fig. 1 shows a sample network.

3. Network properties and dynamics

With all edges included, the number of nodes and edges in each of the constructed network are 32 and 496, respectively. We may now describe the connectivity and structure of the networks in terms of the distributions of the edge weights. Fig. 2 shows the edge weight distribution of the network for a particular window. The following two properties are particularly useful in characterizing the dynamics of the networks.

Definition 1: The *node strength* $s_i(m)$ of node i in the mth window is the average of the weights of all the edges connected to node i, i.e.,

$$s_i(m) = \frac{1}{31}\sum_{j=1,i\neq j}^{32} \rho_{ij}(m). \quad (2)$$

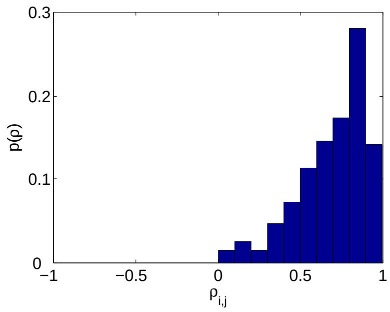


Figure 2: Edge weight distribution of the network constructed for the 20-day window of 17 Feb 2009 to 16 Mar 2009. $\rho_{i,j}$ is the weight of edge between each pair of network nodes and $p(\rho)$ is the probability of an edge weight falling in a 0.1 interval. This edge weight distribution resembles a normal distribution with mean 0.69 and standard deviation 0.21.

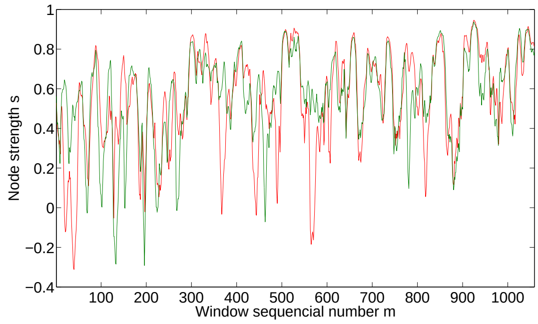


Figure 3: Node strengths of Hong Kong stock market and the US stock market with window size equal to 20 days. Hang Seng Index is shown in red and S&P500 Index in green.

Fig. 3 shows the node strengths of Hang Seng Index and Standard & Poor 500 (S&P500) Index versus time with window size equal to 20 days.

Definition 2: *Network synchronization* $s_{\text{NET}}(m)$ in the m-th window is the average of weights of all the edges in the network, i.e.,

$$s_{\text{NET}}(m) = \frac{1}{496}\sum_{i=1}^{32}\sum_{j=1,i\neq j}^{32} \rho_{ij}(m) \quad (3)$$

Fig. 4 shows the network synchronization versus time with window size of 20 days.

The node strength describes the connectivity of a node to its peer nodes. It reveals to what extent the financial system of a country is fused into the world financial system. The network synchronization, in a larger scale, describes how closely the financial systems of different countries are collaborating. Given the window size w, we can study the network dynamics in terms of node strengths $S_i(w)$ and network synchronization $S_{\text{NET}}(w)$ in all the windows, with the lengths of the two series $M = 1079 - w$.

4. Stock market properties and dynamics

By constructing the network of stock market indices for all the w-day windows, we can study how the individual stock markets interact. Here, we are interested to know whether the network

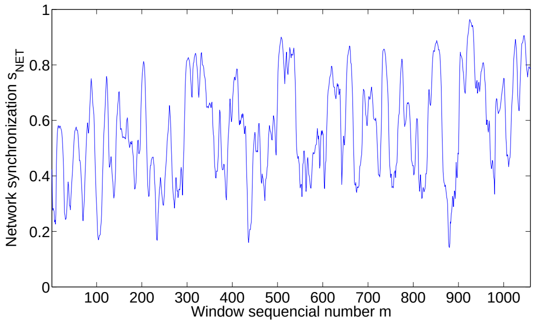


Figure 4: Network synchronization of stock markets with window size equal to 20 days.

properties are related to any financial phenomena in the same windows. In particular, we calculate the *return*, *average value* and *volatility* of individual indices.

Let $p_i(t)$ be the closing value of index i on day t. The window return $r_i(m)$ of index i in the mth w-day window W_M, starting from t_m to t_{m+w-1}, is given by

$$r_i(m) = \frac{p_i(t_{m+w-1}) - p_i(t_m)}{p_i(t_m)} \times 100\%. \quad (4)$$

Likewise, the average closing value $\mu_i(m)$ of index i in the mth w-day window W_M is given by

$$\mu_i(m) = \frac{\sum_{t=t_m}^{t_{m+w-1}} p_i(t)}{w}. \quad (5)$$

In stock markets, the stock prices and market indices change whenever stocks are traded. Moreover, there are many factors which could influence stock prices and hence make the fluctuation irregular. Such fluctuation, usually known as *volatility*, can be measured by calculating the standard deviation of the stock price or market index over a period of time. Specifically, the volatility $\sigma_i(m)$ of index i in the mth w-day window W_M is given by

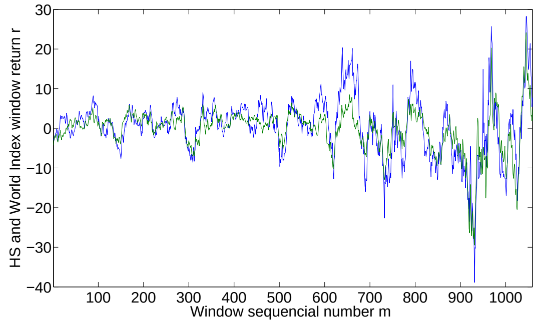
$$\sigma_i(m) = \sqrt{\frac{\sum_{t=t_m}^{t_{m+w-1}} (p_i(t) - \mu_i(m))^2}{w-1}} \quad (6)$$

Here, we take the MSCI AC World Index (World Index) as the stock market index of the world financial system. Dynamics of the window return, average value and volatility of this World Index is an indication of how the world financial system behaves. Fig. 5 shows the return, average value and volatility of Hang Seng Index and World Index versus time, based on 20-day windows.

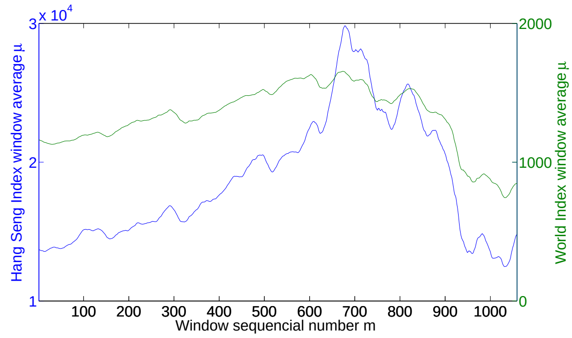
5. Application of networks: Connecting networks and stock markets

The key challenge for applying the study of complex networks in real-life applications is whether the properties found in networks have any corresponding physical meanings that would shed light on how the actual system behaves [10]. In this section we will explore the connection between the network dynamics (complex networks) and stock market dynamics (real systems).

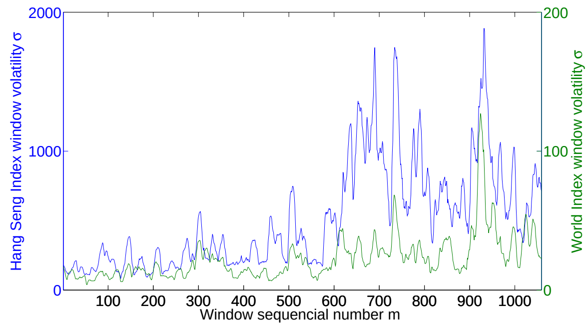
First, we compare the network synchronization with dynamics of the World Index, which is used to characterize the world financial system. Specifically, given a w-day window, the correlations



(a)



(b)



(c)

Figure 5: (a) Window return (in percentage) of Hang Seng Index (blue) and World Index (green). (b) Average value of of Hang Seng Index (blue) and World Index (green). (c) Volatility of Hang Seng Index (blue) and World Index (green). All market properties are calculated based on 20-day windows.

among series of network synchronization, index window return, average and volatility of length $M = 1079 - w$ are calculated. Again we adopt the Pearson's correlation. For example, the correlation coefficient $\rho_{s,r}$ between network synchronization s and World Index return r is given by

$$\rho_{s,r} = \frac{\langle sr\rangle - \langle s\rangle\langle r\rangle}{\sqrt{\langle s^2\rangle - \langle s\rangle^2}\sqrt{\langle r^2\rangle - \langle r\rangle^2}} \quad (7)$$

where $\langle\cdots\rangle$ denotes the expected value. We use different window sizes, i.e., w= 10, 20, 40, 60 and 120 days, to examine the correlation between the network and stock market dynamics in different time scales. The results of the calculation are shown in Table 1. We see from Table 1 that regardless of the choice of window size, the network synchronization and World Index volatility is strongly correlated with correlation coefficients around 0.6,

Table 1: Pearson's correlation coefficients between each pair of dynamics of network synchronization s, World Index window return r, average μ and volatility σ.

w	$\rho_{s,r}$	$\rho_{s,\mu}$	$\rho_{s,\sigma}$	$\rho_{r,\mu}$	$\rho_{r,\sigma}$	$\rho_{\mu,\sigma}$
10	-0.18	-0.10	0.65	0.07	-0.49	-0.21
20	-0.28	-0.13	0.65	0.12	-0.63	-0.21
40	-0.33	-0.10	0.65	0.24	-0.75	-0.20
60	-0.34	-0.05	0.61	0.31	-0.83	-0.19
120	-0.29	-0.38	0.55	0.38	-0.88	-0.33

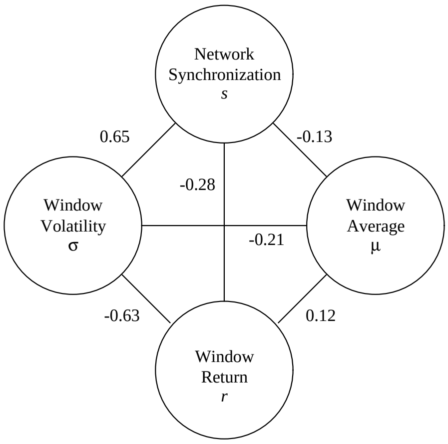


Figure 6: Graphical representation of relationships among network synchronization s, World Index window return r, average μ and volatility σ, with window size $w = 20$ days.

comparing to those between other pairs of dynamical properties. A graphic representation of the relationships among the network and stock market dynamics is shown in Fig. 6.

The network synchronization is actually an assembly of all the individual node strengths as shown in equations (2) and (3). Thus, it is of interest to know whether the volatilities of the individual stock markets are also related to their node strengths in the stock markets network. A similar calculation discussed earlier in this section is adopted. This time the Pearson's correlation coefficient of individual node strengths to each of the index's window return, average and volatility are calculated. Table 2 summarizes the results of our calculation for all the 32 indices taking window size $w = 20$ days. It is found that most stock markets in developed countries have their node strengths correlated to indices' volatility while for stock markets in developing countries, no definite conclusion can be drawn.

6. Conclusion

Networks have been constructed for 32 important stock markets based on connecting each pair of stock markets according to the correlation between their representative indices. We have studied the dynamics of the networks during the period from 7 March 2005 to 23 April 2009. In order to make the study useful for application, we have established the relationship between network dynamics and stock markets dynamics. By comparing the network synchronization and node strength to the indices' window return, average and volatility, we discover that the network synchronization and most of the node strength dynamics are strongly correlated to the volatility of stock markets, with the exception of stock markets from developing countries. Thus, we may conclude that individual markets generally react in a synchronous fashion when the markets experience fluctuation.

Table 2: Pearson's correlation coefficients between each pair of node strength s, index window return r, average μ and volatility σ. The table is sorted by Pearson's correlation coefficient $\rho_{s,\sigma}$ between node strength s and index window volatility σ.

Index	Country	$\rho_{s,r}$	$\rho_{s,\mu}$	$\rho_{s,\sigma}$
ATX	Austria	-0.23	-0.10	0.64
All Ordinaries	Australia	-0.16	0.09	0.59
FTSE 100	United Kingdom	-0.25	-0.14	0.57
CAC 40	France	-0.26	-0.16	0.57
DAX	Germany	-0.24	0.00	0.56
S&P Mib	Italy	-0.14	-0.25	0.55
TA-100	Israel	-0.03	0.17	0.55
AEX	Netherlands	-0.24	-0.18	0.54
OMX Stockholm 30	Sweden	-0.22	-0.11	0.53
Swiss Market	Switzerland	-0.11	-0.08	0.53
Hang Seng	Hong Kong	-0.13	0.10	0.52
OMX Copenhagen 20	Denmark	-0.26	0.00	0.52
S&P 500	USA	-0.16	-0.08	0.52
BEL-20	Belgium	-0.18	-0.16	0.51
Nikkei 225	Japan	-0.11	-0.26	0.49
Total Share	Norway	-0.25	-0.06	0.49
NZSE 50	New Zealand	-0.11	-0.04	0.48
Bovespa	Brazil	-0.01	0.17	0.48
Strait Times	Singapore	-0.08	-0.02	0.46
Madrid General	Spain	-0.13	-0.24	0.45
IPC	Mexico	-0.02	-0.08	0.45
MerVal	Argentina	-0.04	-0.06	0.44
KLSE Composite	Malaysia	-0.05	0.16	0.44
BSE 30	India (BSE)	-0.22	-0.14	0.43
S&P TSX Composite	Canada	-0.11	-0.16	0.42
Seoul Composite	South Korea	-0.05	0.03	0.41
S&P CNX NIFTY	India (NSE)	-0.20	-0.14	0.40
PSI 20	Portugal	-0.15	-0.07	0.40
Jakarta Composite	Indonesia	0.00	0.04	0.34
ISEQ20	Ireland	-0.06	-0.22	0.28
EGX30.CA.	Egypt	0.06	-0.06	0.13
Shanghai Composite	China	0.02	0.03	0.04

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