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A Brief Overview of Some Recent Advances in Pinning Control of Complex Networks

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Abstract—Complex networks are everywhere. Over the last ten years, various approaches have been proposed for controlling complex networks. Pinning control, as an effective method for controlling complex networks, has received increasing attention in recent years. This paper will briefly review some main advances in pinning control of complex networks, with emphasis on the potential applications in power electronic grid.

1. Introduction

Complex networks are everywhere today [1, 2]. Typical examples include the Internet, World Wide Web, power grid networks, communication networks, scientific citation networks, social networks, cellular neural networks, genetic regulatory networks, and so on [3, 4]. As we know now, the representative characteristics of complex networks are a large number of interconnected nodes and complex topological structure [5, 6, 7].

It is well known that the immune, vascular, endocrine, and nerve systems of our bodies regulate chemical reactions to keep equilibria in the face of ongoing attacks from disease and diet [4]. The above process is called homeostasis in biology. Here, the similar regulation processes of complex networks are called control [8, 9, 10].

Over the last ten years, numerous approaches have been proposed to control or intervene the dynamical behaviors of various real-world complex networks [11, 12, 13]. Since the real-world complex networks often have a large number of nodes, it is very difficult or even impossible to control all nodes to realize a given control goal [14, 15]. Therefore, we hope to control a portion of nodes to achieve the same control goal. In fact, the above idea of control of a portion of nodes is very effective in many real-world complex networks. Thus the above control technique of a portion of nodes is called pinning control. In 1997, Grigoriev, Cross, and Schuster introduced the pinning control of spatiotemporal chaos [5]. In 2004, Li, Wang, and Chen presented the pinning control of a complex dynamical network to its equilibrium [6]. In 2008, Zhou, Lu, and Lü studied the pinning adaptive synchronization of a general complex dynamical network [13]. In 2009, Wu, Zhou, and Chen further investigated the cluster synchronization of linearly coupled complex networks under pinning control [8]. Moreover, there are numerous results reported over the last few years. The intended purpose of this paper is to briefly review some recent advances in pinning control of complex networks. We hope to reflect the current state of the pinning control of complex networks.

This paper is then organized as follows. Section 2 introduces the basic idea of pinning control and its challenging questions. The adaptive pinning synchronization of complex dynamical networks is presented in Section 3. In Section 4, the pinning synchronization of undirected and directed complex dynamical networks is then discussed. Moreover, the global pinning controllability of complex networks is further investigated in Section 5. Finally, some potential applications are explored in Section 6.

2. Preliminary

This section will briefly review the basic idea of pinning control and its challenging questions.

As we know now, complex networks often have a large number of network nodes. The pinning control is proposed based on the following two main motivations: i) It is usually impossible to achieve a given control goal by controlling every node; ii) It is likely possible to reduce the number of controllers under the condition of the same control goal. Therefore, the basic idea of pinning control is to realize the same or even better control goal by employing a portion of network nodes. In general, there are two interesting basic questions in pinning control of complex networks: (i) How many nodes should a network with a given topological structure and coupling strength be pinned to realize the desired control goal? (ii) How much coupling strength should a network with a given topological structure and pinning nodes be applied to achieve the desired control goal? In 2008, Zhou, Lu, and Lü gave a positive answer to the above two fundamental questions for a special case [13].

In pinning control, the selection of network nodes is also an interesting question. It is well known that there are

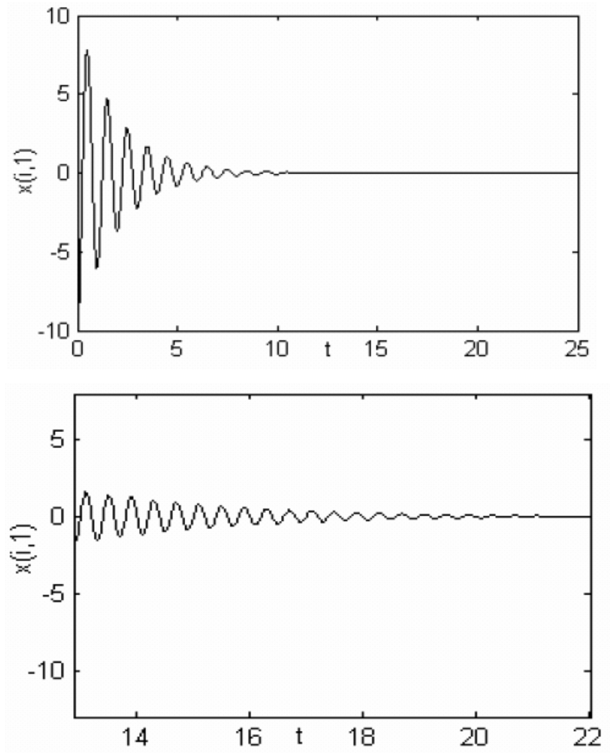

Figure 1: [14] (a) Plot of sub-state of the largest-degree node for the specifically pinning control scheme; (b) plot of sub-state of the node with largest degree for the randomly pinning control scheme.

two basic selective schemes: random scheme and specific scheme. The random scheme is to pin a portion of randomly selected network nodes. And the specific scheme is to pin a portion of network nodes by following a given rule, such as the degrees of network nodes and the betweenness of network nodes. A natural question is: "which kind of pinning schemes is much better?".

Fig. 1 shows the control effectiveness of the random scheme and specific scheme for a scale-free network [14]. Here, the scale-free network has 60 network nodes with the coupling strength $c = 8.246$ and $l = 15$. In specific scheme, one selects the top 15 largest-degree nodes and control gain is 29.7603. In random scheme, one randomly select 15 nodes and the control gain is 513.3709. It means that the specific scheme is much more effective than the random scheme for a scale-free network. However, it is not always true for all cases. Sometimes, the random scheme is much more effective than the specific scheme. It depends on the detailed network structure and node dynamics.

Moreover, the pinning control technique can be combined with some traditional or modern control methods, such as switching control, adaptive control, and robust control.

3. Adaptive Pinning Synchronization of Complex Dynamical Networks

To answer the above two fundamental questions in Section 2, this Section provides a simply approximate formula for estimating the detailed number of pinning nodes and the magnitude of the coupling strength for a given general complex dynamical network [13]. In this Section, all notations are described in [13].

Consider a general complex dynamical network consisting of N identical nodes with linearly diffusive couplings [13], which is given by

$$\dot{\mathbf{x}}_i = \mathbf{g}(\mathbf{x}_i, t) + \sum_{j=1}^{N} c_{ij}\,\mathbf{A}\,\mathbf{x}_j + \mathbf{v}_i(\mathbf{x}_1, \cdots, \mathbf{x}_N), \quad (1)$$

where $1 \le i \le N$, $\mathbf{x}_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T \in \mathbf{R}^n$ is the state vector of the ith node, $\mathbf{g} : \Omega \times \mathbf{R}^+ \rightarrow \mathbf{R}^n$ is a nonlinear smooth vector field, $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, t)$ is the node dynamics, $\mathbf{v}_i \in \mathbf{R}^n$ are the control inputs satisfying $\mathbf{v}_i(\mathbf{x}, \cdots, \mathbf{x}) = \mathbf{0}$. And $\mathbf{A} \in \mathbf{R}^{n\times n}$ is the inner-coupling matrix and $\mathbf{C} = (c_{ij})_{N\times N} \in \mathbf{R}^{N\times N}$ is the coupling configuration matrix. If there exists a link from node i to node j ($j \neq i$), then $c_{ij} > 0$ and c_{ij} is the coupling strength; otherwise, $c_{ij} = 0$. Suppose that $\mathbf{C}$ is an irreducibly diffusive matrix satisfying

$$\sum_{j=1}^{N} c_{ij} = 0\,.$$

Let $\mathbf{x} = \mathbf{s}(t\,;\, t_0, \mathbf{x}_0) \in \mathbf{R}^n$ with $\mathbf{x}_0 \in \mathbf{R}^n$, denoted as $\mathbf{s}(t)$, be a solution of the node system $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, t)$. Thus $\mathbf{S}(t) = (\mathbf{s}^T(t), \mathbf{s}^T(t), \cdots, \mathbf{s}^T(t))^T \in \mathbf{R}^{n\times N}$ is a synchronous solution of the general complex dynamical network (1). Here, $\mathbf{s}(t)$ can be an equilibrium point, a periodic orbit, an aperiodic orbit, even a chaotic orbit in the phase space [3, 13].

Proposition 1 *[13] (**P1**) Assume that $\|\boldsymbol{Dg}(\boldsymbol{s})\|_2$ is bounded, where $\boldsymbol{Dg}(\boldsymbol{s})$ is the Jacobian of $\boldsymbol{g}$ evaluated at $\boldsymbol{x} = \boldsymbol{s}$. That is, there exists a nonnegative constant α satisfying $\|\boldsymbol{Dg}(\boldsymbol{s})\|_2 \le \alpha$.*

Theorem 1 *[13] Assume that **P1** holds. If there exists a natural number $1 \le l < N$ satisfying $\lambda_{l+1} < -\frac{\alpha}{\gamma}$, then the synchronous solution $\boldsymbol{S}(t)$ of the general complex network (1) is locally asymptotically stable under the pinning adaptive controllers*

$$\begin{cases} \boldsymbol{v}_i = -p_i\boldsymbol{e}_i,\ \dot{p}_i = q_i\|\boldsymbol{e}_i\|_2^2, & 1 \le i \le l \\ \boldsymbol{v}_i = 0, & (l+1) \le i \le N\,, \end{cases} \quad (2)$$

where q_i are positive constants for $1 \le i \le l$.

Rewrite the general complex network (1) as follows:

$$\dot{\mathbf{x}}_i = \mathbf{G}\mathbf{x}_i + \mathbf{h}(\mathbf{x}_i, t) + \sum_{j=1}^{N} c_{ij}\,\mathbf{A}\,\mathbf{x}_j + \mathbf{v}_i(\mathbf{x}_1, \cdots, \mathbf{x}_N), \quad (3)$$

where $1 \le i \le N$.

Proposition 2 *[13] (***P2***) Assume that $\boldsymbol{h}(\boldsymbol{x}, t)$ is Lipschitz continuous. That is, there exists a Lipschitz constant μ satisfying $\|\boldsymbol{h}(\boldsymbol{x}_i, t) - \boldsymbol{h}(\boldsymbol{s}, t)\|_2 \le \mu \|\boldsymbol{e}_i\|_2$ for $1 \le i \le N$.*

Theorem 2 *[13] Assume that* **P2** *holds. If there exists a natural number $1 \le l < N$ satisfying $\lambda_{l+1} < -\frac{\beta+\mu}{\gamma}$, then the synchronous solution $\boldsymbol{S}(t)$ of the general complex network (3) is globally asymptotically stable under the pinning adaptive controllers*

$$\begin{cases} \boldsymbol{v}_i = -p_i\boldsymbol{e}_i,\ \dot{p}_i = q_i\|\boldsymbol{e}_i\|_2^2, & 1 \le i \le l \\ \boldsymbol{v}_i = 0, & (l+1) \le i \le N, \end{cases} \tag{4}$$

where q_i are positive constants for $1 \le i \le l$.

4. Pinning Synchronization of Complex Dynamical Networks

In this Section, the pinning synchronization of undirected and directed complex dynamical networks will be further investigated, where all notations are given in [7].

A general pinning controlled network is described by [7]

$$\dot{x}_i(t) = f(x_i(t), t) + c\sum_{j=1}^{N} G_{ij}\Gamma x_j(t) + u_i, i = 1, 2, \ldots, l,$$

$$\dot{x}_i(t) = f(x_i(t), t) + c\sum_{j=1}^{N} G_{ij}\Gamma x_j(t), i = l+1, 2, \ldots, N. \tag{5}$$

where

$$u_i = -cd_i\Gamma(x_i - s(t)) \in R^n, i = 1, 2, \ldots, l, \tag{6}$$

are n-dimensional linear feedback controllers with all the control gains $d_{i_k} > 0$.

Proposition 3 *[7]* **P3** *There exists a constant matrix K satisfying*

$$(x-y)^T(f(x,t) - f(y,t)) \le (x-y)^T K\Gamma(x-y), \tag{7}$$

where $\forall x, y \in R^n$.

Theorem 3 *[7] Assume that* **P3** *holds. The controlled undirected network (5) is globally synchronized if the following condition is satisfied:*

$$I_N \otimes (K\Gamma) + c(G-D)\otimes\Gamma < 0, \tag{8}$$

where $\otimes$ is the Kronecker product,

$$D = \operatorname{diag}(\underbrace{d_1, \ldots, d_l}_{l}, \underbrace{0, \ldots, 0}_{N-l}),$$

and I_N is the N-dimensional identity matrix.

Theorem 4 *[7] Assume that the condition (7) holds and Γ is a positive definite matrix. Then, the adaptively controlled undirected network (9) is globally synchronized for a small constant $\alpha > 0$.*

5. Global Pinning Controllability of Complex Networks

In this Section, the global pinning controllability of complex dynamical networks will be further explored, where all notations are given in [11].

Proposition 4 *[11] (***P4***) If the feedback gain matrix K, the inner linking matrix B, and the coupling strength σ are chosen such that for every $t \ge t_0$, and for every $y_1, \ldots, y_N \in \boldsymbol{R}^n$*

$$\lambda_i(y, t) < -\mu, \quad i = 1, \ldots, nN,$$

where $y = [y_1^T, \ldots, y_N^T]^T$, $\mu > 0$, $\{\lambda_i(y, t)\}_{i=1}^{nN}$ are the eigenvalues of the matrix $H(y, t)$ defined by

$$H(y, t) = \mathcal{D}(y, t) - 2(\sigma L \otimes sym\, QB + P \otimes sym\, QK)$$

with Q positive definite symmetric matrix in $\mathbb{R}^{n\times n}$, and

$$\mathcal{D}(y, t) = 2Diag[symQF_{s(t),s(t)-y_1}, \ldots, symQF_{s(t),s(t)-y_N}].$$

Then, the dynamical system

$$\dot{e}(t) = \mathcal{F}(e(t), t)e(t) - (\sigma L \otimes B + P \otimes K)e(t),$$

where

$$\mathcal{F}(e(t), t) = Diag[F_{s(t),s(t)-e_1(t)}, \ldots, F_{s(t),s(t)-e_N(t)}],$$

is globally exponentially stable about the origin, implying that the network

$$\dot{x}_i(t) = f(x_i(t)) - \sigma B\sum_{j=1}^{N} l_{ij}x_j(t) + u_i(t)$$

is globally pinning-controllable.

Corollary 1 *[11] If, for some Q positive definite symmetric matrix in $\mathbb{R}^{n\times n}$, condition $sym\, QK = \kappa\, sym\, QB$ is satisfied, $symQB$ is a positive definite matrix and*

$$\lambda_{min}(\sigma L + \kappa P)\lambda_{min}(symQB) > \alpha\|Q\|,$$

where the positive constant α satisfies $\|F_{\xi,\bar{\xi}}\| \le \alpha$, then

$$\dot{x}_i(t) = f(x_i(t)) - \sigma B\sum_{j=1}^{N} l_{ij}x_j(t) + u_i(t)$$

is globally pinning- controllable.

Corollary 2 *[11] If for some Q positive definite symmetric matrix in $\mathbb{R}^{n\times n}$ condition $sym\, QK = \kappa\, sym\, QB$ is satisfied and $symQB$ is a positive definite matrix, and the feedback gain κ satisfies*

$$\frac{\sigma_\kappa(\frac{\lambda_2(L)}{N})}{\sigma(\frac{\lambda_2(L)}{r}) + \kappa} > \alpha\|Q\|\frac{1}{\lambda_{min}(symQB)}$$

then

$$\dot{x}_i(t) = f(x_i(t)) - \sigma B\sum_{j=1}^{N} l_{ij}x_j(t) + u_i(t)$$

is globally pinning-controllable.

$$
\begin{aligned}
\dot{x}_i(t) &= f(x_i(t),t) + c(t)\sum_{j=1}^{N} G_{ij}\Gamma x_j(t) - c(t)d_i\Gamma(x_i(t)-s(t)), i=1,2,\ldots,l,\\
\dot{x}_i(t) &= f(x_i(t),t) + c(t)\sum_{j=1}^{N} G_{ij}\Gamma x_j(t), i=l+1,2,\ldots,N,\\
\dot{c}(t) &= \alpha\sum_{j=1}^{N}(x_j(t)-s(t))^T\Gamma(x_j(t)-s(t)).
\end{aligned}
\tag{9}
$$

6. Concluding remarks

This paper has briefly reviewed some recent advances in the pinning control of complex dynamical networks. It is certain that the pinning control will have a good prospect of application. For example, the operator of an electric power grid hopes to find an effective network model with pinning control design that will help form predictions of supply and demand to keep the stability of the whole power network [4], which requires the combined expertise of statisticians, economists, and power engineers. Moreover, the cost and effects of pinning control for network performance should be further investigated in the near future.

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