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A New memristor Based Chaotic System

Yuxia Li[*,‡], Lanying Zhao, Wenqing Chi, Shuli Lu, Xia Huang

Key Laboratory for Robot and Intelligent Technology of Shandong Province,
College of Information & Electrical Engineering,Shandong
University of Science and Technology, Qingdao 266510, P. R. China
[*] Corresponding author
[‡] Email: yuxiali2004@yahoo.com.cn

Abstract: In this work, we present a new memristor based chaotic circuit, which is obtained by replacing the nonlinear resistor in the canonical Chua's circuit with a charge-controlled memristor. This chaotic circuit uses only the four basic circuit elements, and has only one negative element in addition to the nonlinearity. The existence of the chaos is not only demonstrated by computer simulations, but also verified with Lyapunov exponents, bifurcation, poincaré mapping and power spectrum analysis.

Keywords: memristor; chaotic circuit; Chua's diode; charge-controlled.

1. Introduction

Resistor R, capacitor C, and an inductor L are the three well-known basic two-terminal circuit elements. From the circuit theoretic point of view, the memristor (short for memory resistor), with memristance M, was postulated as a fourth fundamental element based on symmetry principle by Leon O. Chua in 1971 [1]. Almost 40 years later, Strutkov et al., a team led by R. Stanley Williams from the Hewlett–Packard Company claimed their invention of a physical memristor device together with a useful physical model of the memristor [2], and thereby cemented its place as the fourth circuit element. This nanometer-size solid-state two-terminal passive device has generated worldwide interest because of its potential applications [3], much research is already focused on them [4-8]. Since a memristor is a fundamental circuit element, circuit applications of memristors are also active topics of research [9-13].

By definition, a memristor is said to be charge-controlled if the nonlinear relation between the charge q and the flux φ can be expressed as a single-valued function $\varphi = \varphi(q)$ of the charge q.

In this paper, we assume that the charge-controlled memristor is characterized by a smooth continuous cubic monotone-increasing nonlinearity as follows:

$$\varphi(q) = aq + bq^3 \tag{1}$$

Consequently, the memristance $M(q)$ are defined by

$$M(q) = \frac{d\varphi(q)}{dq} = a + 3bq^2 \tag{2}$$

2. The new chaotic circuit with charge-controlled memristor and system equations

Figure 1 shows a chaotic circuit with charge-controlled memristor, which is a dual circuit of the canonical Chua's oscillator [9, 13] by replacing the Chua's diode with a charge-controlled memristor. The charge-controlled memristor shown in Fig. 1 is a passive two-terminal electronic device described by Eq. (1).

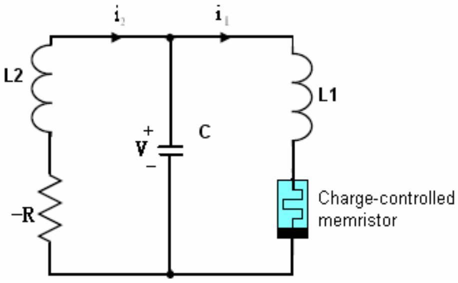


Fig. 1. Chaotic circuit with a charge-controlled memristor

Applying *Kirchhoff's voltage laws* and *current laws* to the circuit in Fig.1, we obtain a set of four first-order differential equations, which define the relation among the four circuit variables i_1, i_2, v, q :

$$\begin{cases} L_1 \dfrac{di_1}{dt} = v - M(q)i_1 \\ C\dfrac{dv}{dt} = i_2 - i_1 \\ L_2 \dfrac{di_2}{dt} = Ri_2 - v \\ \dfrac{dq}{dt} = i_1 \end{cases} \tag{3}$$

where the φ–q characteristic curve of the charge-controlled memristor is given by Eq. (1) and $M(w) = \dfrac{d\varphi(w)}{dw}$.

By let in $x = i_1$, $y = v$, $z = i_2$, $C = 1$, $w = q$, $m = 1/L_1$, $n = 1/L_2$, $k = R/L_2$, and defining the nonlinear functions $\varphi(w)$ and $M(w)$ as

$$\varphi(w) = aw + bw^3 \tag{4}$$

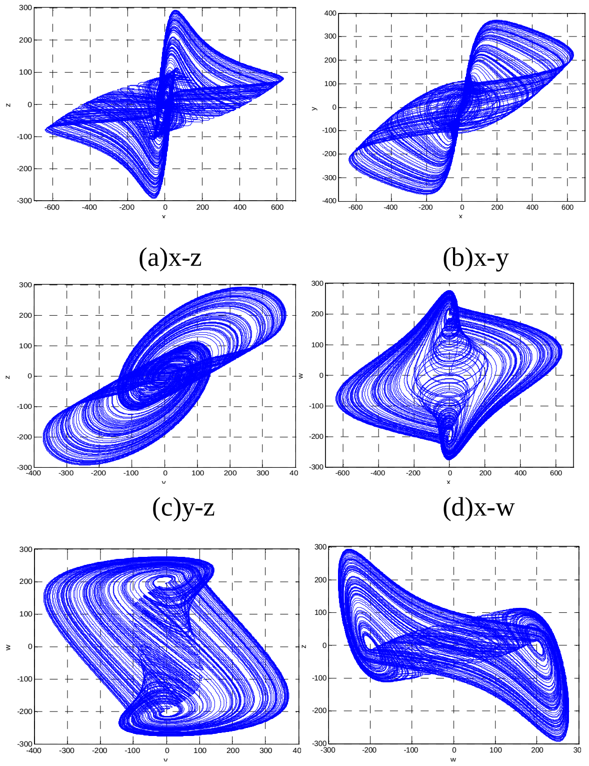
$$M(w) = \frac{d\varphi(w)}{dw} = w + 3bw^2 \tag{5}$$

State equations of (3) can be written in dimensionless form with a time scale factor k as follows:

$$\begin{cases} \dfrac{dx}{dt} = m(y - M(w)x) \\ \dfrac{dy}{dt} = z - x \\ \dfrac{dz}{dt} = kz - ny \\ \dfrac{dw}{dt} = x \end{cases} \tag{6}$$

where m, k, n > 0.

Let $m = 4, n = 1, a = -0.59\times10^{-3}, b = 0.02\times10^{-3}$, and k=0.83. For initial conditions $(0, 10^{-10}, 0, 0)$, the system(6) is chaotic with Lyapunov exponents $\lambda_1 = 0.1506, \lambda_2 = 0, \lambda_3 = -0.0003, \lambda_4 = -8.7096$, and the Lyapunov dimension is dL = 3.0179, the corresponding attractor is depicted in Fig.5.



(a)x-z (b)x-y (c)y-z (d)x-w (e)y-w (f)z-w

Fig. 2 Phase portraits of the chaotic system (6)

3. Bifurcation analysis of the new system (6)

The spectrum of Lyapunov exponents on the (k-λ) plane is obtained as shown in Fig.3, while the corresponding bifurcation diagram of state x with respect to k is given in Fig.4.

It can be observed that the bifurcation diagram well coincides with the spectrum of the Lyapunov exponents. As k increases, system(4) undergoes the following route:

1) 0.5＜k≤0.6, λ1=0, λ2, λ3, λ4＜0; system(6) is periodic (Fig.5(a)).

2) 0.6＜k≤0.715, λ1=λ2=0, λ3, λ4＜0; system(6) is quasi-periodic(Fig.5(b)) .

3) 0.715＜k≤0.76, λ1>0, λ2=0, λ3, λ4＜0; system(6) is chaotic (Fig.5(c)).

4) 0.76＜k≤0.775, λ1=λ2=0, λ3, λ4＜0; system(6) is quasi-periodic.

5) 0.775＜k≤0.84, λ1>0, λ2=0, λ3, λ4＜0; system(6) is chaotic.

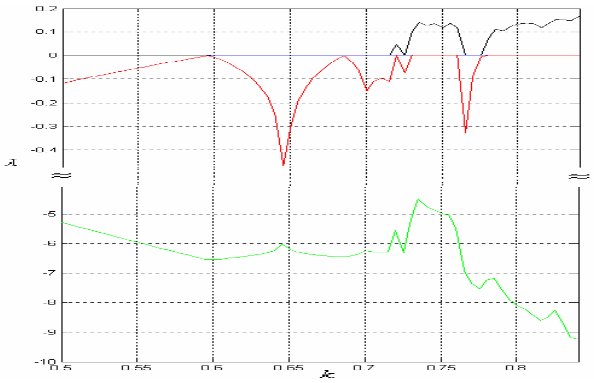


Fig.3 The Lyapunov exponents versus k in the (k-$\lambda_{1,2,3,4}$) plane

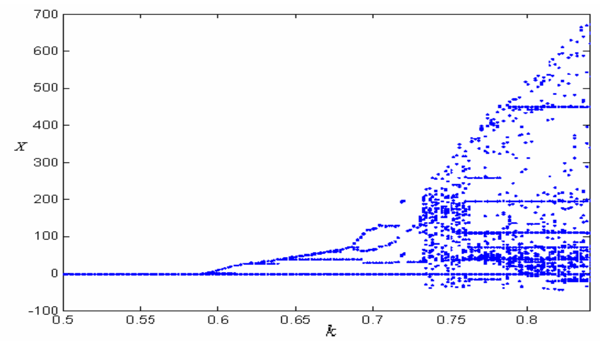
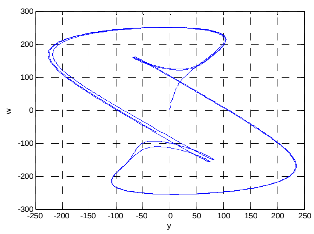
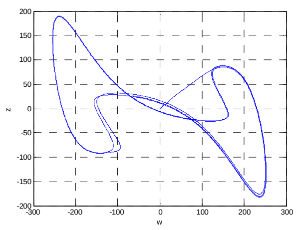


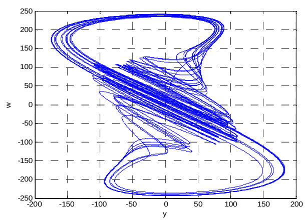
Fig. 4 Bifurcation diagram for increasing k



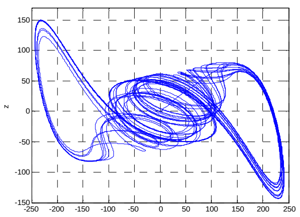
a(i)y-w



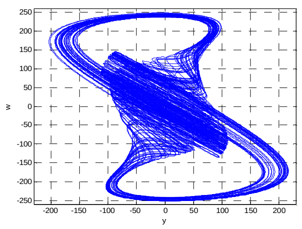
a(ii)z-w



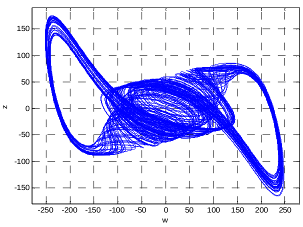
b(i)y-w



b(ii)z-w



c(i)y-w



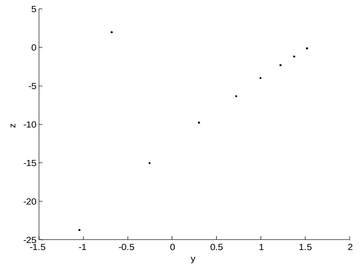
c(i)z-w

Fig.5 Phase portraits of system (6) in: (i) y-w plane and (ii) z-w plane with different k: (a) period orbit, k=0.55; (b) quasi-period orbit, k=0.65; (c) chaotic attraction, k=0.745; (d) chaotic attraction, k=0.83.

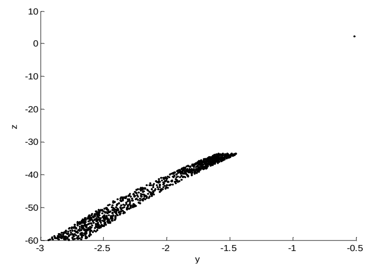
4. Poincaré mapping and power spectrum analysis.

On the poincaré section, when it has only one point or few dispersed points, the motion is periodic; when it has a closed curve, the motion is quasi-periodic; when it has a lot of concentrated points, the motion is chaotic.

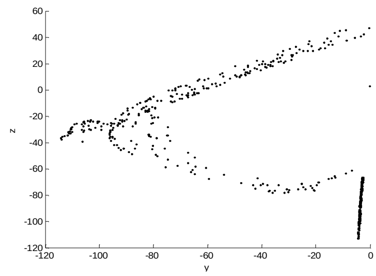
The poincaré mapping of system (6) with different k is obtained as shown in Fig.6.



（a）period orbit



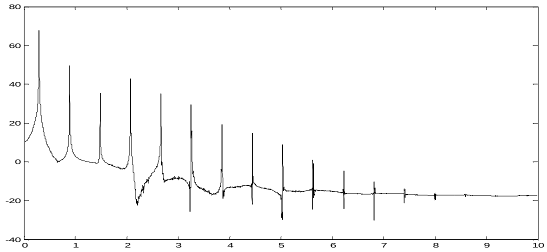
(b) quasi-period orbit



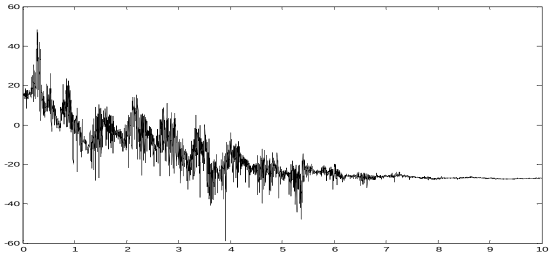
(c) chaotic attraction

Fig. 6 Poincaré mapping of system (6) in: y-z plane with different k: (a) period orbit, k=0.55; (b) quasi-period orbit, k=0.65; (c) chaotic attraction, k=0.745.

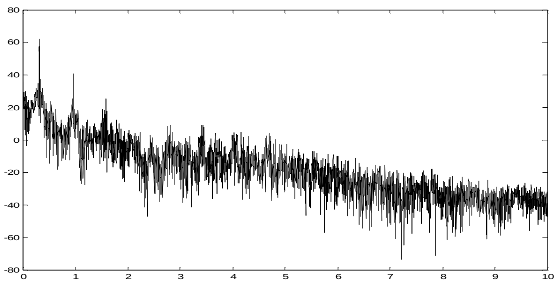
The power spectrum of system (6) with different k is obtained as shown in Fig.7.



(a) period orbit k=0.55



(b) quasi-period orbit k=0.71



(c) chaotic attraction k=0.795

Fig. 7 Signal spectrum of system (6)

7. Conclusions

In this paper, a memristor oscillator, which is extended from a Chua's oscillator by replacing the Chua's diode with a charge-controlled memristor, is presented and studied. This chaotic circuit uses only the four basic circuit elements, and has only one negative element in addition to the nonlinearity. The resulting chaotic system is not only demonstrated by computer simulations but also verified with Lyapunov exponents, bifurcation, poincaré mapping and power spectrum analysis.

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