2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

An improved generalized Hamiltonian systems approach via LMI criterion for chaotic synchronization

Huiling Xi†‡ and Simin Yu†

†College of Automation, Guangdong University of Technology
Guangzhou, Guangdong, 510006 P. R. China
‡Department of Mathematics, North University of China
Taiyuan, Shanxi, 030051 P. R. China
Email: cxhhhl@126.com, gdutsiminyu@yahoo.com.cn

Abstract—In this paper, a generalized Hamiltonian systems approach is improved via the Linear Matrix Inequality (LMI) criterion for chaotic synchronization. By using the Lyapunov stability theory and some matrix techniques, a new sufficient criterion, formulated in the LMI form, is established. The new sufficient criterion can guarantee that chaotic synchronization is achieved at an exponential convergence rate. Theoretical analysis and numerical simulations are presented to verify the effectiveness of this approach.

1. Introduction

The discovery of the Lorenz chaotic system has led to a new era in the study of nonlinear dynamical systems [1]. During the last two decades, some of the chaos study has devoted to the chaos control and synchronization problems. The well-known Chua's circuit is the first chaotic system realized by real electrical circuits [2, 3]. The circuit can generate a double-scroll chaotic attractor [4]. The study of double-scroll Chua's circuit has been extended to the study of the n-scroll modified Chua's circuit lately. In 2002, özoguz et al. proposed a modified Chua's circuit model, in which the piece-wise linear function was replaced by smooth hyperbolic tangent function. The modified Chua's circuit, which can generate chaotic attractors with arbitrary many scrolls, has promoted the study and application of chaos [5].

Recently, many methods have been proposed for controlling and synchronizing of chaos such as OGY method [6], backstepping design [7], sliding mode control [8], nonlinear control [9], adaptive control [10], neural network control [11], fuzzy logic control [12], and LMI technique [13, 14].

In [15], the synchronization problem of a modified Chua's circuits generator of 5-scroll chaotic attractors is numerically studied by using Hamiltonian systems and state observer approach. The Sylvester's Criterion is applied to provide a test for negative definite of a matrix. We need the complex calculations to determine the constant matrix required by this criterion. However, if the generalized Hamiltonian approach is improved based on LMI criterion, the constant matrix required can be easily verified and resolved by using the LMI Toolbox in MATLAB software. In view of this, the main goals of this paper are: (i) to improve the generalized Hamiltonian systems approach proposed in [15] via LMI criterion. And, (ii) to obtain synchronization of two modified Chua's circuit with hyperbolic tangent functions in master-slave configuration. This objective is achieved by appealing to improved generalized Hamiltonian system approach via LMI criterion.

The organization of this paper is as follows: in section 2, we first give a brief review on chaos synchronization by the generalized Hamiltonian system approach, then this approach is improved by LMI criterion. In section 3, the effectiveness of the proposed approach is demonstrated by the modified Chua's circuit with hyperbolic tangent functions. Finally, some conclusions are drawn in section 4.

2. Improved generalized Hamiltonian approach via LMI criterion

Consider the following dynamical system:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{x}(t) \in \Re^n$ is the state vector, $f(\boldsymbol{x}) : \Re^n \to \Re^n$ is a nonlinear vector function.

The Generalized Hamiltonian canonical form is as follows [16]:

$$\dot{\boldsymbol{x}} = \mathscr{J}(\boldsymbol{x})\partial H/\partial \boldsymbol{x} + \mathscr{S}(\boldsymbol{x})\partial H/\partial \boldsymbol{x} + \mathscr{F}(\boldsymbol{x}), \boldsymbol{x} \in \Re^n. \tag{2}$$

In the context of observer design, a special class of Generalized Hamiltonian forms with linear output map $\boldsymbol{y}(t)$ is given by [15]

$$\dot{\boldsymbol{x}} = \mathscr{J}(\boldsymbol{y})\partial H/\partial \boldsymbol{x} + (\mathscr{I} + \mathscr{S})\partial H/\partial \boldsymbol{x} + \mathscr{F}(\boldsymbol{y}), \boldsymbol{x} \in \Re^n,$$

$$\boldsymbol{y} = \mathscr{C} \cdot \partial H/\partial \boldsymbol{x}, \boldsymbol{y} \in \Re^m, \tag{3}$$

where $H(\boldsymbol{x})$ denotes a smooth energy function which is globally positive definite in $\Re^n$. The gradient vector of H, denoted by $\partial H/\partial \boldsymbol{x}$, is assumed to exist everywhere. We use quadratic energy function $H(\boldsymbol{x}) = \boldsymbol{x}^T \mathscr{U} \boldsymbol{x}/2$ with $\mathscr{U}$ being a constant, symmetric positive definite matrix. In such case, $\partial H/\partial \boldsymbol{x} = \mathscr{U}\boldsymbol{x}$. The matrix $\mathscr{J}(\boldsymbol{y})$ satisfies the property $\mathscr{J}(\boldsymbol{y}) + \mathscr{J}^T(\boldsymbol{y}) = 0$. The vector field $\mathscr{J}(\boldsymbol{y})\partial H/\partial \boldsymbol{x}$ exhibits the conservative part of the system and it is also referred to as the workless forces of the system. On the other hand, the matrix $\mathscr{S}$ is a constant symmetric matrix, not necessarily of definite sign. The matrix $\mathscr{I}$ is a constant skew symmetric matrix. The vector field $(\mathscr{I}+\mathscr{S})\partial H/\partial \boldsymbol{x}$ is considered as the dissipative part of system, that is, it is the work part. Finally, $\mathscr{F}(\boldsymbol{x})$ represents a locally destabilizing vector field, and $\mathscr{C}$ is a constant matrix.

We denote the estimate of the state $\boldsymbol{x}(t)$ by $\tilde{\boldsymbol{x}}(t)$, and consider the Hamiltonian energy function $H(\tilde{\boldsymbol{x}})$ to be the particularization of H in terms of $\tilde{\boldsymbol{x}}(t)$. Similarly, we denote by $\tilde{\boldsymbol{y}}(t)$ the estimated output, computed in terms of the estimated state $\tilde{\boldsymbol{x}}(t)$. The gradient vector $\partial H(\tilde{\boldsymbol{x}})/\partial \tilde{\boldsymbol{x}}$ is, naturally, of the form $\mathscr{U}\tilde{\boldsymbol{x}}$ with $\mathscr{U}$ being a constant, symmetric positive definite matrix.

In [15], the nonlinear state observer for the Generalized Hamiltonian form (3) is modified by

$$\dot{\tilde{\boldsymbol{x}}} = \mathscr{J}(\boldsymbol{y})\partial H/\partial\tilde{\boldsymbol{x}} + (\mathscr{I}+\mathscr{S})\partial H/\partial\tilde{\boldsymbol{x}} + \mathscr{F}(\tilde{\boldsymbol{y}}) + \mathscr{B}(\boldsymbol{y}-\tilde{\boldsymbol{y}}),$$
$$\tilde{\boldsymbol{x}} \in \Re^n, \tilde{\boldsymbol{y}} = \mathscr{C}\cdot\partial H/\partial\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}} \in \Re^m, \tag{4}$$

where $\mathscr{C}$ is the observer gain, and the matrix $\mathscr{B}$ is chosen to make $(\mathscr{I}+\mathscr{S},\mathscr{B})$ controllable.

Remark 1. One can choose a special matrix $\mathscr{B}$ with simple configuration, which just ensures that $(\mathscr{I}+\mathscr{S},\mathscr{B})$ is controllable, to make sure the error dynamical system to be asymptotically stable.

Our aim is to find a suitable $\mathscr{B}$ and $\mathscr{C}$ to achieve chaos synchronization.

The state estimation error is defined as $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \tilde{\boldsymbol{x}}(t)$ and the output estimation error is defined as $\boldsymbol{e}_y(t) = \boldsymbol{y}(t) - \tilde{\boldsymbol{y}}(t)$. We set, when needed, $\mathscr{I}+\mathscr{S} = \mathscr{A}$. Then from (3) and (4) we can get the following error dynamical system

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \mathscr{J}(\boldsymbol{y})\partial H/\partial\boldsymbol{e} + \mathscr{A}\cdot\partial H/\partial\boldsymbol{e} + (\mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}})) - \mathscr{B}\cdot\boldsymbol{e}_y,$$
$$\boldsymbol{e} \in \Re^n, \boldsymbol{e}_y = \mathscr{C}\cdot\partial H/\partial\boldsymbol{e}, \boldsymbol{e}_y \in \Re^m, \tag{5}$$

where the vector $\partial H/\partial\boldsymbol{e}$ actually stands, with some abuse of notation, for the gradient vector of the modified energy function, $\partial H/\partial\boldsymbol{e} = \partial H/\partial\boldsymbol{x} - \partial H/\partial\tilde{\boldsymbol{x}} = \mathscr{U}(\boldsymbol{x}-\tilde{\boldsymbol{x}}) = \mathscr{U}\boldsymbol{e}$.

Remark 2. Because the chaotic system to discuss is usually dissipative, the vector field $\mathscr{J}\partial H/\partial\boldsymbol{e}$ needs not be considered.

Theorem 2.1 *If the vector field $\mathscr{F}(\boldsymbol{y})$ satisfies Lipschitz condition, namely, $\|\mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}})\| \le \rho\|\boldsymbol{y}-\tilde{\boldsymbol{y}}\|$, $\rho \in \Re^+$, a suitable matrix $\mathscr{B}$ is chosen such that $(\mathscr{A},\mathscr{B})$ is controllable, and a suitable observer gain $\mathscr{C}$ is selected such that*

$$\begin{aligned}&\mathscr{U}^T\mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A}\mathscr{U} - \mathscr{U}^T\mathscr{C}^T\mathscr{B}^T\mathscr{P} - \mathscr{P}\mathscr{B}\mathscr{C}\mathscr{U}\\ &+\rho^2\mathscr{P}\mathscr{P} + (\mathscr{C}\mathscr{U})^T(\mathscr{C}\mathscr{U}) + 2\delta\mathscr{P} < 0,\end{aligned} \tag{6}$$

where $\mathscr{P}$ is a symmetric positive definite matrix, I is the identity matrix, and δ is a positive constant, then the error dynamical system (5) *is globally exponentially stable, implying that the coupled system* (3) *and* (4) *are globally exponentially synchronization.*

proof: Define $V = \boldsymbol{e}^T\mathscr{P}\boldsymbol{e}$, where $\mathscr{P}$ is a symmetric positive definite constant matrix. Differentiating V along the error dynamical trajectory (5) and using (2) yield

$$\begin{aligned}\dot{V} &= \dot{\boldsymbol{e}}^T\mathscr{P}\boldsymbol{e} + \boldsymbol{e}^T\mathscr{P}\dot{\boldsymbol{e}}\\ &= [(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}\boldsymbol{e} + \mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}})^T]\mathscr{P}\boldsymbol{e} + \boldsymbol{e}^T\mathscr{P}[(\mathscr{A}-\mathscr{B}\mathscr{C})\\ &\quad \mathscr{U}\boldsymbol{e} + \mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}})]\\ &= \boldsymbol{e}^T\{[(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}]^T\mathscr{P} + \mathscr{P}(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}\}\boldsymbol{e} + 2[\mathscr{F}(\boldsymbol{y})\\ &\quad -\mathscr{F}(\tilde{\boldsymbol{y}})]^T\mathscr{P}\boldsymbol{e}\\ &\le \boldsymbol{e}^T\{[(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}]^T\mathscr{P} + \mathscr{P}(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}\}\boldsymbol{e} + 2\rho\|\mathscr{C}\mathscr{U}\boldsymbol{e}\|\\ &\quad \cdot\|\mathscr{P}\boldsymbol{e}\|.\end{aligned}$$

Since $2\rho\|\mathscr{C}\mathscr{U}\boldsymbol{e}\|\cdot\|\mathscr{P}\boldsymbol{e}\| \le \rho^2\|\mathscr{P}\boldsymbol{e}\|^2 + \|\mathscr{C}\mathscr{U}\boldsymbol{e}\|^2$, using (6) we further have

$$\begin{aligned}\dot{V} &\le \boldsymbol{e}^T\{[(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}]^T\mathscr{P} + \mathscr{P}(\mathscr{A}-\mathscr{B}\mathscr{C})\mathscr{U}\}\boldsymbol{e} + \rho^2\|\mathscr{P}\boldsymbol{e}\|^2\\ &\quad +\|\mathscr{C}\mathscr{U}\boldsymbol{e}\|^2\\ &= \boldsymbol{e}^T[\mathscr{U}^T\mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A}\mathscr{U} - \mathscr{U}^T\mathscr{C}^T\mathscr{B}^T\mathscr{P} - \mathscr{P}\mathscr{B}\mathscr{C}\mathscr{U}\\ &\quad +\rho^2\mathscr{P}\mathscr{P} + (\mathscr{C}\mathscr{U})^T(\mathscr{C}\mathscr{U})]\boldsymbol{e}\\ &\le -2\delta\boldsymbol{e}^T\mathscr{P}\boldsymbol{e} = -2\delta V < 0.\end{aligned}$$

Based on the Lyapunov stability theory, the error dynamical system (5) is globally exponentially stable, and hence, the coupled system (3) and (4) are globally exponentially synchronized.

Lemma 2.1 (Schur Complements [17]) *For a given symmetric matrix $S = \begin{pmatrix} S_{11} & S_{12}\\ S_{21} & S_{22}\end{pmatrix}$, where $S_{11} = S_{11}^T, S_{12} = S_{21}^T, S_{22} = S_{22}^T$, the condition $S < 0$ is equivalent to $S_{22} < 0$, and $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.*

Using Lemma 2.1, the condition (6) can be easily transformed to be

$$\begin{pmatrix} \mathscr{U}^T\mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A}\mathscr{U} - \mathscr{U}^T\mathscr{C}^T\mathscr{B}^T\mathscr{P} & \\ -\mathscr{P}\mathscr{B}\mathscr{C}\mathscr{U} + (\mathscr{C}\mathscr{U})^T(\mathscr{C}\mathscr{U}) + 2\delta\mathscr{P} & \rho\mathscr{P}\\ \rho\mathscr{P} & -I\end{pmatrix} < 0, \tag{7}$$

If (7) is multiplied by $\begin{pmatrix}\mathscr{P}^{-1} & 0\\ 0 & I\end{pmatrix}$ from the left-hand and right-hand side, respectively, and letting $X = \mathscr{P}^{-1}\mathscr{U}^T$ and $W = (\mathscr{C}\mathscr{U}\mathscr{P}^{-1})^T = X\mathscr{C}^T$, then (7) can easily be further transformed into the following LMI form (8).

Theorem 2.2 *If suitable matrices X and W are selected such that the following LMI*

$$\begin{pmatrix} X\mathscr{A}^T + \mathscr{A}X - W\mathscr{B}^T - \mathscr{B}W^T & \\ +WW^T + 2\delta X\mathscr{U}^{-1} & \rho I\\ \rho I & -1\end{pmatrix} < 0, \tag{8}$$

is satisfied, the error dynamical system (5) *with the observer gain $\mathscr{C} = W^TX^{-1}$, is globally exponentially stable, implying that the coupled systems* (3) *and* (4) *are globally exponentially synchronized.*

Remark 3. The feasible sets of X and W satisfying (8) can be easily found by using the LMI Toolbox in MATLAB software.

3. Applications to synchronization of chaotic systems

Consider the following modified Chua's circuit proposed by özoguz et al. [5]:

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= x_3,\\ \dot{x}_3 &= -a(x_2 + x_3 + f(x_1)),\end{aligned} \tag{9}$$

where

$$f(x_1) = \sum_{j=-N}^{M} (-1)^{j-1} tanhk(x_1 - \sigma_j), \tag{10}$$

and $\sigma_j = 2j$. Here, M and N are odd integers, determining the number of scrolls in the chaotic attractors as $n = (M + N + 2)/2$. The values of other coefficients are chosen as: $a = 0.25$ and $k = 2$.

The state equations describing the modified Chua's circuit with hyperbolic tangent functions in Hamiltonian form (3) with destabilizing vector field (as master circuit), are given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a & -a \end{pmatrix} \partial H/\partial \boldsymbol{x} + \begin{pmatrix} 0 \\ 0 \\ -af(x_1) \end{pmatrix}, \tag{11}$$

with $f(x_1)$ given by (10) for $M = N = 3$, and Hamiltonian energy function given by

$$H(\boldsymbol{x}) = (x_1^2 + x_2^2 + x_3^2)/2. \tag{12}$$

The destabilizing vector field calls for x_1 signal to be used as the output of the master circuit (11). We have $\boldsymbol{y} = (x_1, 0, 0)^T$. The matrices $\mathscr{A}$ and the vector field $\mathscr{F}(\boldsymbol{y})$ are given by

$$\mathscr{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a & -a \end{pmatrix}, \mathscr{F}(\boldsymbol{y}) = \begin{pmatrix} 0 \\ 0 \\ -af(x_1) \end{pmatrix}.$$

Choose $\mathscr{B} = (b_1, 0, 0)^T$, where $b_1 \neq 0$. Obviously, $(\mathscr{A}, \mathscr{B})$ is controllable. According to (4), the slave circuit can be designed as

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \mathscr{A}\partial H/\partial \boldsymbol{z} + \mathscr{F}(\tilde{\boldsymbol{y}}) + \mathscr{B}\mathscr{C}(\boldsymbol{y} - \tilde{\boldsymbol{y}}),$$

where $\tilde{\boldsymbol{y}}$ is taken as $(z_1, 0, 0)$, $\mathscr{F}(\tilde{\boldsymbol{y}}) = (0, 0, -af(z_1))$, $\mathscr{C}$ is the observer gain. Then we have

$$\begin{aligned}\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a & -a \end{pmatrix} \partial H/\partial \boldsymbol{z} + \begin{pmatrix} 0 \\ 0 \\ -af(z_1) \end{pmatrix}\\ &+ \begin{pmatrix} b_1 \\ 0 \\ 0 \end{pmatrix} \mathscr{C}\mathscr{U} \begin{pmatrix} x_1 - z_1 \\ 0 \\ 0 \end{pmatrix}.\end{aligned} \tag{13}$$

Then the synchronization error dynamics is governed by

$$\begin{aligned}\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a & -a \end{pmatrix} \partial H/\partial \boldsymbol{e} + \begin{pmatrix} 0 \\ 0 \\ -a(f(x_1) - f(z_1)) \end{pmatrix}\\ &- \begin{pmatrix} b_1 \\ 0 \\ 0 \end{pmatrix} \mathscr{C}\mathscr{U} \begin{pmatrix} e_1 \\ 0 \\ 0 \end{pmatrix},\end{aligned} \tag{14}$$

where $e_i = x_i - z_i (i = 1, 2, 3)$. Furthermore, (14) can be written as

$$\dot{\boldsymbol{e}} = \mathscr{A}\mathscr{U}\boldsymbol{e} + \mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}}) - \mathscr{B}\mathscr{C}\mathscr{U}\boldsymbol{e}, \tag{15}$$

where $\boldsymbol{e} = (e_1, e_2, e_3)^T$.

Consider

$$\begin{aligned}\mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}}) &= \begin{pmatrix} 0 \\ 0 \\ -a(f(x_1) - f(z_1)) \end{pmatrix}\\ &= \begin{pmatrix} 0 \\ 0 \\ -af'(\xi)(x_1 - z_1) \end{pmatrix}\\ &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -af'(\xi) & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix}\\ &= M_{x,z} \cdot (\boldsymbol{y} - \tilde{\boldsymbol{y}}),\end{aligned}$$

where $\xi \in (x_1, z_1)$ or $\xi \in (z_1, x_1)$. Therefore,

$$\begin{aligned}\|\mathscr{F}(\boldsymbol{y}) - \mathscr{F}(\tilde{\boldsymbol{y}})\| &= \|M_{x,z} \cdot (\boldsymbol{y} - \tilde{\boldsymbol{y}})\| \leq ak(M + N + 1)\|\boldsymbol{y} - \tilde{\boldsymbol{y}}\|\\ &= \rho\|\boldsymbol{y} - \tilde{\boldsymbol{y}}\|,\end{aligned} \tag{16}$$

where $\rho = ak(M + N + 1)$. That is, the vector field $\mathscr{F}(\boldsymbol{y})$ satisfies Lipschitz condition.

Remark 4. It is noted that, the choice of the matrix $\mathscr{B}$ is not unique, and can be selected as any other values provided that $(\mathscr{A}, \mathscr{B})$ is controllable.

In the following, chaos synchronization of the modified Chua's circuit with hyperbolic tangent functions is demonstrated. Take $b_1 = 0.8$, and $\delta = 0.5$. We have $\rho = 3.5$ from (16). Then we can get $\mathscr{C} = (1.9444, 5.6424, 1.3283)$ from (8) by using MATLAB LMI Toolbox. In numerical simulations, we have used a fourth-order Runge-Kutta integration algorithm with time step of 0.001. The initial values of the master system (11) and the slave system (13) are taken as $x_1(0) = 180, x_2(0) = -180, x_3(0) = 180$ and $z_1(0) = 185, z_2(0) = -185, z_3(0) = 185$, respectively. *Fig.*1 shows the time responses of the synchronization errors $e_i(t) = x_i(t) - z_i(t) (i = 1, 2, 3)$. Numerical simulations show that chaotic synchronization of the modified Chua's circuit can be achieved.

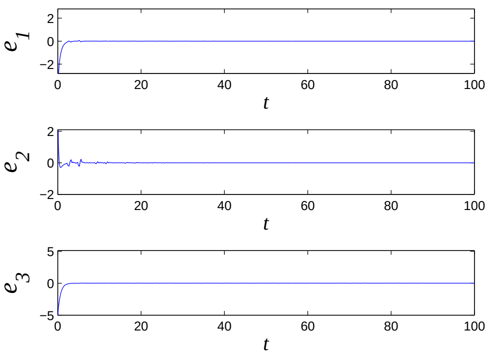

Fig.1 Synchronization errors.

4. Conclusions

Based on the Lyapunov stability theory and LMI technique, the generalized Hamiltonian systems approach in [15] is improved to establish a new LMI criterion for chaotic synchronization. Then this improved approach is used to synchronize n-scroll chaotic attractors in a modified Chua's circuit with hyperbolic tangent functions. Numerical simulations show that the proposed method works effectively. It is to be noted that the proposed sufficient criterion can also be applied to other chaotic systems, such as Chen system, Rössler system, and chaotic Murali-Lakshmanan-Chua system etc.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grants 60572073 and 60871025, the Natural Science Foundation of Guangdong Province under Grants 8151009001000060 and 8351009001000002, and the Science and Technology Program of Guangdong Province under Grant 2009B010800037.

References

[1] E. N. Lorenz, "Deterministic non-periodic flow," *J. Atoms. Sci.*, vol.20, pp.130–141, 1963.

[2] L. O. Chua, M. Komuro, T. Matsumoto, "The double scroll family, Part I: Rigorous proof of chaos," *IEEE Trans. Circuits Syst.*, vol.33, pp.1072–1096, 1986.

[3] L. O. Chua, G. N. Lin, "Canonical realization of Chuas circuit family," *IEEE Trans. Circuits Syst.*, vol.37, pp.885–902, 1990.

[4] T. Matsumoto, L. O. Chua, M. Komuro, "The double scroll circuits and systems," *IEEE Trans. Circuits Syst.*, vol.32, pp.797–818, 1985.

[5] S. özoguz, A. S. Elwakil, K. N. Salama, "N-scroll chaos generator using nonlinear transconductor," *Electron Lett.*, vol.38, pp.685–686, 2002.

[6] E. F. Ott, C. Grebogi, J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol.64, pp.1196–1199, 1990.

[7] B. Wang, G. J. Wen, "On the synchronization of a class of chaotic systems based on backstepping method," *Phys. Lett. A.*, vol.370, pp.35–39, 2007.

[8] M. Haeri, M. S. Tavazoei, M. R. Naseh, "Synchronization of uncertain chaotic systems using active sliding mode control," *Chaos, Solitons and Fractals*, vol.33, pp.1230–1239, 2007.

[9] Q. J. Zhang, J. A. Lu, "Chaos synchronization of a new chaotic system via nonlinear control," *Chaos, Solitons and Fractals*, vol.37, pp.175–179, 2008.

[10] S. H. Chen, J. H. Lü, "Synchronization of an uncertain unified chaotic system via adaptive control," *Chaos, Solitons and Fractals*, vol.14, pp.643–647, 2002.

[11] S. Kuntanapreeda, "An observer-based neural network controller for chaotic Lorenz system," *Lect. Notes Comput. Sci.*, vol.5370, pp.608–617, 2008.

[12] L. L. Zhang, L. H. Huang, Z. Z. Zhang, Z. Y. Wang, "Fuzzy adaptive synchronization of uncertain chaotic systems via delayed feedback control," *Phys. Lett. A.*, vol.372, pp.6082–6086, 2008.

[13] J. H. Park, S. M. Lee, H. Y. Jung, "LMI optimization approach to synchronization of stochastic delayed discrete-time complex networks," *J. Optim. Theory Appl.*, vol.143, pp.357–367, 2009.

[14] J. G. Lu, D. J. Hill, "Global asymptotical synchronization of chaotic Lur'e systems using sampled data: A linear matrix inequality approach," *IEEE Trans. Circ. Syst. II: Exp. Briefs*, vol.55, pp.586–590, 2008.

[15] L.Gámez-Guzmán, C.Cruz-Hernández, R. M. López-Gutiérrez, E. E. García-Guerrero, "Synchronization of chuas circuits with multi-scroll attractors: Application to communication," *Commun. Nonlinear Sci. Numer. Simulat.*, vol.14, pp.2765–2775, 2009.

[16] H. Sira-Ramłrez, C. Cruz-Hern¢ndez, "Synchronization of chaotic systems: a generalized Hamiltonian systems approach," *Int. J. Bifurc. Chaos*, vol.11, pp.1381–1395, 2001.

[17] S. Boyd, L. E. Ghaoui, E. Feron, V. Balakrishnan, "Linear matrix inequalities in system and control theory," *Philadelphia: SIAM*, 1994.