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Binary generalized synchronization and its application to communication systems

A.A. Koronovskii[†§], O.I. Moskalenko[†§], V.I. Ponomarenko[‡§], M.D. Prokhorov[‡] and A.E. Hramov[§†]

†Saratov State University — Astrakhanskaya, 83, Saratov, 410012, Russia
§Saratov State Technical University — Politehnicheskaja 77, Saratov 410056, Russia
‡Saratov Branch of the Institute of RadioEngineering and Electronics of Russian Academy of Sciences — Zelyonaya, 38, Saratov 410019, Russia
Email: alexey.koronovskii@gmail.com, o.i.moskalenko@gmail.com, ponomarenkovi@gmail.com, mdprokhorov@yandex.ru, hramovae@gmail.com

Abstract—In this report the binary generalized synchronization is considered, when for the certain values of the coupling strength two unidirectionally coupled dynamical systems generating the aperiodic binary sequences are in the generalized synchronization regime. The presence of the binary generalized synchronization may be revealed with the help of both the auxiliary system approach and the largest conditional Lyapunov exponent calculation. The binary generalized synchronization regime seems to be useful in the binary systems of data transmission.

1. Introduction

Among the different types of the synchronous behavior of chaotic systems the generalized synchronization (GS) [1, 2, 3] stands out due to its interesting features. This kind of synchronous behavior means the state vectors of the interacting chaotic systems being in the generalized synchronization regime are related with each other by the functional [4]. As well as the other chaotic synchronization types, GS may be used for the chaotic communication [5, 6], with the certain advantages taking place in comparison with the complete or phase synchronization regimes [7, 8].

In this work we report on the generalized synchronization between two unidirectionally coupled binary systems (binary generalized synchronization — BGS). We believe that the binary generalized synchronization phenomenon may be useful in the broad class of the digital schemes for the data transmission.

2. Binary Generalized synchronization

We have observed *the binary generalized synchronization* in two unidirectionally coupled systems whose equations read as

$$\begin{aligned} x_{n+1} &= H(\eta_{n+1}), \quad \eta_{n+1} = 1 - \lambda_d \eta_n^2, \\ y_{n+1} &= H(\zeta_{n+1}), \quad \zeta_{n+1} = 1 - \lambda_r \zeta_n^2 + \varepsilon \zeta_n^2 x_n, \end{aligned} \qquad (1)$$

where x_n, y_n are the binary sequences under study, η_n and ζ_n are supposed to be the interior (hidden) variables, $\lambda_d = 1.6$ and $\lambda_r = 1.54$ are the control parameters of the drive and response systems, respecively, ε is the coupling strength and $H(\xi)$ is the Heaviside function.

To detect the generalized synchronization regime in the unidirectionally coupled systems the different techniques have been proposed, e.g., the nearest neighbor method [1] or the conditional Lyapunov exponent calculation [9].

Among these techniques the auxiliary system approach proposed for the unidirectionally coupled chaotic oscillators may be generally considered as the most easy, clear and powerful tool to study the generalized synchronization regime in chaotic systems. Starting from the seminal paper of *Abarbanel et al.* [10], the auxiliary system approach has become de-facto the standard of generalized synchronization studies. Although the auxiliary system approach is not applicable for the mutual type of coupling [11] it is the very effective tool to detect GS regime in the unidirectionally coupled chaotic systems. The auxiliary system approach has been used in the plenty of theoretical and experimental works (see, e.g., [2, 12, 13, 14]).

In the present work to reveal the BGS regime in (1) we have used the auxiliary system with the same values of the control parameters

$$z_{n+1} = H(\varsigma_{n+1}), \quad \varsigma_{n+1} = 1 - \lambda_r \varsigma_n^2 + \varepsilon \varsigma_n^2 x_n, \qquad (2)$$

which starts from the different initial condition in comparison with the response system, i.e., $\varsigma_0 \neq \zeta_0$.

With the increase of the coupling strength ε, the interacting systems undergo into the generalized synchronization regime. Whereas the behavior of the drive system remains unchanged (due to the unidirectional type of coupling), after the short transient (approx. 100 units of the discrete time) the dynamics of the response and auxiliary systems becomes identical that is the evidence of the generalized synchronization of the binary drive and response systems (1). Since there is the broad class of the schemes for the data transmission, namely, the *digital* schemes, where the *binary* signals consisting of bits "0" or "1" are used, the binary generalized synchronization phenomenon may be useful in these systems.

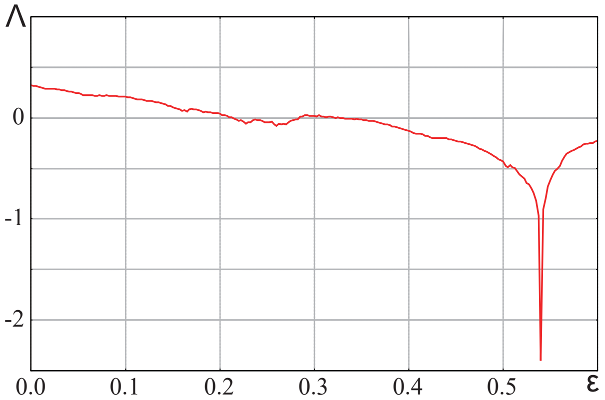

Figure 1: (Color online) The dependence of the conditional Lyapunov exponent Λ of the response system on the coupling strength ε

To validate the presence of the generalized synchronization regime, in parallel with the auxiliary system approach, we have also used the calculation of the conditional Lyapunov exponent (CLE) of the response system [9], see Fig. 1. Although for the binary signals there is no possibility to get the Lyapunov exponent value (because the output variable takes only two possible values "0" and "1" and one can not find the small variable variation which is necessary for the LE calculation), we can calculate CLE of system (1) as

$$\Lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln |2(\varepsilon x_i - \lambda_r)\zeta_i|, \qquad (3)$$

since for the the model system we can suppose that the hidden variable ζ is known.

The consideration of the conditional Lyapunov exponent allows also to understand the relationship between the synchronous dynamics (from the point of view of the generalized synchronization regime) of the hidden and binary variables. It is obviously, that the generalized synchronization of the hidden variables (i.e., η_n and ζ_n) implies definitely the generalized synchronization in terms of the binary variables, x_n and y_n. At the same time, the reverse relationship is not so obvious, but, fortunately, CLE allows one to solve this problem. Indeed, the increase/decrease of the difference between the hidden variables of the response and auxiliary systems, $\delta = \zeta - \varsigma$, is determined completely by the sign of CLE. When CLE is positive (the generalized synchronization regime is not observed), the difference δ between the values of the hidden variables of the response and auxiliary system increases, and, since the values of the hidden variables are bounded ($-\zeta_m < \zeta < \zeta_m$, $-\zeta_m < \varsigma < \zeta_m$, $0 < \zeta_m \leq 1$), δ-variable exhibits the chaotic behavior, with its value located within the range $(-2\zeta_m, 2\zeta_m)$. Obviously, when $-2\zeta_m < \delta < -\zeta_m$ and $\zeta_m < \delta < 2\zeta_m$ the hidden variables ζ and ς are characterized by the different signs, and, as a consequence, the variables y and z corresponding to the response and auxiliary systems, respectively, are also different. So, the binary variables are synchronized (in terms of the generalized synchronization regime) if and only if the hidden variables are in the generalized synchronization regime.

3. Conclusion

In conclusion, in this work we have reported on the binary generalized synchronization, when for the certain values of the coupling strength two unidirectionally coupled dynamical systems generating the aperiodic binary sequences are in the generalized synchronization regime. Along with the previously revealed the possibility of the binary systems to demonstrate the complete synchronization regime [15], the results of our paper concerning generalized synchronization, suggest the possibility of the development of a theory of chaotic synchronization of binary systems. We believe that the finding discussed in this paper gives a strong potential for new applications under many relevant circumstances including the chaotic communication field (first of all, in the digital schemes for the data transmission).

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