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Practical Preprocessing in Realizing Errorless and Compressive Description Method of Digital Sounds in Cellular Automata

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Abstract—In this paper, we investigate practical preprocessing in realizing errorless and compressive description method of digital sounds by means of rule sequences of one dimensional cellular automaton with two states and three neighbors, hereafter referred as 1-2-3 CA. We have succeeded to describe digital sounds compressively without any reproducing error except for several sounds. In order to accomplish the errorless and compressive description, we investigate the relationship between the time development of bit-patterns and the length of the rule sequences describing the development. It is shown that the following two preprocessing is necessary; (i) applying the subtraction between successive bit-patterns and (ii) relocating the most significant bit to the lowest order bit. From computer simulations, we succeed to realize the errorless and compressive description for all the digital sounds by applying the above two preprocessing. Therefore, the two preprocessing are practical in describing the digital sound compressively without any reproducing error by means of 1-2-3 CA.

1. Introduction

We have proposed a novel description method of digital signals by making use of rule sequences of 1-2-3 CA [1]-[5]. The method enables us to describe the digital signals without any reproducing error. In addition, except for several cases, the amount of resultant coding is less than the original data. Thus the method is compressive for almost cases. The coding shows attractor dynamics analogous with conventional nonlinear dynamical systems. The attractor dynamics means that even if starting from any initial condition of CA, original data can be recovered by applying the coding of rule sequences of CA.

We proposed the method based on the idea that there exist certain deterministic simple rules behind complex dynamics even if the system has a large degree of freedom. In our method, thus, we focus on real digital sounds as example of complex dynamics and we employ rules of 1-2-3 CA as deterministic simple rules. In other words, the resultant rule sequences of the description could reflect the dynamical features of the target sound. For instance, the data amount of the resultant coding of the description could represent the complexity of the the target sound from the view point of Kolgomorov complexity [6]. The larger data amount of the description is necessary for more complex signals. In our method, longer length of rule sequences means the larger data amount of the description.

In order to evaluate the complexity of the data in our method from the view point of Kolgomorov complexity, it is important to achieve a compressive and errorless description. From our results, the description with the two-rules set of (#90, #180) in 1-2-3 CA for the data applied to the XOR preprocessing gives best results. However, for several data, the data amount becomes lager than the original one. In this paper, therefore, we aim to realize the errorless and compressive description of digital sounds. In realizing the errorless and compressive description, one of the strategies is to find out a practical preprocessing to the data. The purposes of this paper are (i) to investigate the relationship between the time development of bit-patterns of the data and the length of the rule sequences describing the development, and (ii) based on the result (i), to propose a useful preprocessing in realizing the errorless and compressive description of 1-2-3 CA.

2. Basic Idea for Describing Digital Sounds

2.1. 1-2-3 CA

Let us explain 1-2-3 CA, briefly. The variables a_i^t takes 0 or 1 and represent the state of the ith cell in the one-dimensional chain at time step t. The updating rule of the state is defined as follows:

$$a_i^{t+1} = f(a_{i-1}^t, a_i^t, a_{i+1}^t), \quad (1)$$

where a function $f(\cdot)$ is called a transition function and specified as follows:

$$\begin{aligned} f(0,0,0) &= f_0\ ,\ f(1,0,0) = f_1, \\ f(0,1,0) &= f_2\ ,\ f(1,1,0) = f_3, \\ f(0,0,1) &= f_4\ ,\ f(1,0,1) = f_5, \\ f(0,1,1) &= f_6\ ,\ f(1,1,1) = f_7, \end{aligned} \quad (2)$$

where $f_i = 0$ or 1 ($i = 0, 1, \cdots, 7$). By choosing each of f_i to be 0 or 1, we can determine a certain specified rule. Thus total number of rules in 1-2-3 CA is $2^8 = 256$.

In order to identify the one specific rule of 256, one introduces a rule number,

$$\text{rule}\# = f_0 + 2f_1 + 2^2 f_2 + 2^3 f_3 + 2^4 f_4 + 2^5 f_5 + 2^6 f_6 + 2^7 f_7, \quad (3)$$

which takes from 0 to 255. If we assign the two states of each cell to 0 or 1 and specify the rule number, the time development of a certain initial state in 1-2-3 CA gives a sequence of one-dimensional bit-patterns consisting of 0 or 1.

2.2. Errorless Description Method

In the standard data format of a musical CD, digital signals are sampled at the frequency of 44.1 kHz and the amplitudes are quantized with 16 bits. Therefore, digital Sound data are also represented as one-dimensional bit-pattern sequences of 16 bits.

Our idea about describing digital sound data by 1-2-3 CA are stated as follows:

- $\mathbf{a}^t = \{a_i^t | i = 1, \ldots, 16\}$ at time step t of 1-2-3 CA can be regarded as "an amplitude of quantized sound signals in binary coding" taken with appropriately sampling frequency.
- Time development of digital sound data ($\mathbf{a}^t \Rightarrow \mathbf{a}^{t+1}$) can be generated by applying rule to $\mathbf{a}^t$, that is, $\mathcal{R} \circ \mathbf{a}^t = \mathbf{a}^{t+1}$, where $\mathcal{R}$ is a certain rule sequence.

Developing our idea, it was discovered that only two rules, which are appropriately chosen from all the $2^8 = 256$ rules in 1-2-3 CA, are sufficient to generate all the patterns consisting of 16 bits starting from an arbitrary initial pattern of them [2].

In the errorless description, we employ the two-rules set (#90, #180) and fixed boundary condition where the boundary cell of the left cell takes 1 and one of the right cell takes 0. In our method, it is important how to find out rule sequences which can reproduce the time development of the bit-patterns of the target sound. The strategy as follows:

Method: Let us denote the two-rules set as $\mathcal{R}_1 = \{\#90, \#180\}$, and the set with the length k of rule sequences denotes as $\mathcal{R}_k = \{c_1 c_2 \cdots c_i \cdots c_k \mid c_i \in \mathcal{R}_1\}$. The former is a set of specified two rules which For instance, $\mathcal{R}_2 = \{\#90\#90, \#90\#180, \#180\#90, \#180\#180\}$. An initial state in the 1-2-3 CA is taken from the original sound data. At each time step, we search for the rule sequence with the shortest length by applying the elements of rule sets $\mathcal{R}_k$ $(k = 1, 2, \cdots)$. It means that, starting from $k = 1$, we increase k until getting the target pattern. The rule sequence obtained by this procedure rigorously gives the cell pattern at the next time step. If the correct pattern is not obtained within the maximum sequence length $N_{\max} = 15$, then we give up description with use of the rule at that time step and employ the original pattern. This procedure is repeated until the end of the data.

Table 1: Averaged length of rule sequences and compression rate. We perform the description after we applied XOR preprocessing between successive bit-patterns of the data.

	sports	news	pocket
man 1	11.36 (0.921)	12.60 (0.996)	9.59 (0.826)
man 2	11.34 (0.920)	11.59 (0.944)	10.00 (0.852)
woman 1	11.52 (0.933)	12.42 (0.990)	9.18 (0.803)
woman 2	11.24 (0.912)	12.57 (0.994)	9.02 (0.796)

	sample1	sample2	sample3	sample4
J-pop	12.82 (1.009)	12.93 (1.013)	12.73 (1.003)	12.90 (1.013)

First, sound data in t step is given. Next, preprocessed data （XOR and subtraction）in 0 step and 1 step is given. After that, rule sequences and length of rule sequences are given. The above-mentioned coding becomes it. therefore, The data amount becomes small if rule sequences is long. and the data amount grows if rule sequences is short.

In table 1, we present description results of spoken words supplied by ATR (Advanced Telecommunication Research Institute International) and JPOP (Japanese Popular) musics taken from CD with one second time interval. The data is represented by the binary coding with the sign applied to the XOR preprocessing. which results in the highest compression rate among all the possible codings, the binary coding, the binary coding with the sign, the gray coding, the gray coding with the sign, the binary coding applied to the XOR preprocessing, and so on. Small value of compression rate means high compressive performance. For JPOP, unfortunately, our method miss to realize compressive description without any reproducing error.

3. Preprocessing in Realizing Errorless and Compressive Description

We investigate the relationship between the time development of bit-patterns and the length of the rule sequences describing the development. In details, please see discussions. From the investigations, we conclude that the following two preprocessing are necessary.

1. Application of the subtraction between successive bit-patterns. Thus, resultant bit-pattern changes to 17 bits from 16 bits of the original data.
2. Relocation of the most significant bit to the lowest order bit.

After two preprocessing, the description method given by sub-section 2.2 is applied using the two-rules set (#90, #180) . In table 2, we give compression rate of our descrip-

Table 2: Averaged length of rule sequences and compression rate after applying the tow preprocessing.

	sport	news	pocket
man 1	10.36 (0.889)	11.55 (0.961)	9.18 (0.821)
man 2	10.38 (0.892)	11.12 (0.932)	9.58 (0.846)
woman 1	10.83 (0.913)	12.09 (0.983)	8.83 (0.795)
woman 2	10.41 (0.882)	11.47 (0.957)	8.72 (0.789)

	sample1	sample2	sample3	sample4
JPOP	11.92 (0.989)	11.89 (0.988)	11.77 (0.980)	12.03 (0.996)

Table 3: The number of the transitions from 0 to 1 or from 1 to 1 at the most significant bit, and the number of such the transitions with the length of 16 or more the in rule sequences for JPOP.

	from 0 to 1 from 1 to 1	the length of 16 or more	ratio (%)
sample 1	2702	2242	83.0
sample 2	2493	2260	90.7
sample 3	2594	2364	91.1
sample 4	2638	2393	90.7

tion method with two preprocessing. For JPOP adding to the other data, the compression rate takes less than 1. We evaluate the compression rate for various sound data of spoken words, classic music and JPOP music. At least, for all the data, we succeeded to realize compressive and errorless description. Please see the discussions why the two preprocessing is practical in achieving compressive and errorless description.

4. Discussions

It described it by using all two-rules set. As a result, rule sequences that two-rules set of (#90, #180) is the shortest was given. In the following sub-sections, Description Difficulty are described for two-rules set of (#90, #180).

4.1. Description Difficulty Originating in the Most Significant Bit

Let us discuss the effect of the preprocessing (ii), relocation of the most significant bit to the lowest order bit. At first, we investigate characteristic features of the bit-pattern sequences, where the length of rule sequences reproducing them is lager than 16. Such the bit-pattern sequences almost change from 0 to 1, or 1 to 1 in the most significant bit. Results is given in table 3, where we investigate the number of the transitions from 0 to 1 or from 1 to 1 at the most significant bit, and the number of such the transitions with the length of 16 or more the in rule sequences.

At last, we investigate the relationship between the two-rules set of (#90, #180) and such the transitions of bit-patterns. The two rules of #90 and #180 are given as folows:

rule #90

$$\begin{aligned} f(0,0,0) = f_0 = 0\ ,\ f(1,0,0) = f_1 = 1 \\ f(0,1,0) = f_2 = 0\ ,\ f(1,1,0) = f_3 = 1 \\ f(0,0,1) = f_4 = 1\ ,\ f(1,0,1) = f_5 = 0 \\ f(0,1,1) = f_6 = 1\ ,\ f(1,1,1) = f_7 = 0 \end{aligned} \tag{4}$$

rule #180

$$\begin{aligned} f(0,0,0) = f_0 = 0\ ,\ f(1,0,0) = f_1 = 0 \\ f(0,1,0) = f_2 = 1\ ,\ f(1,1,0) = f_3 = 0 \\ f(0,0,1) = f_4 = 1\ ,\ f(1,0,1) = f_5 = 1 \\ f(0,1,1) = f_6 = 0\ ,\ f(1,1,1) = f_7 = 1 \end{aligned} \tag{5}$$

In our description method, we employ the binary coding with the sign applied to the XOR preprocessing, where the most significant bit represents the sign and the bit is at the cell site of the right hand side as shown in figure 1. In addition, the right boundary condition in our method takes 0, and the cell at the left hand side of the most significant bit takes 0 almostly.

In this situation, if we reproduce the transition from 0 to 1 of the most significant bit by the two-rules set of (#90, #180), the transition function of $f(0,\ 0,\ 0) = 1$ is necessary. Unfortunately, the two-rules set of (#90, #180) do not include such the transition function. The difficulty in reproducing the transition from 0 to 1 originates in the result. Similarly, if we reproduce the transition from 1 to 1, we have to apply the transition function of $f(0,\ 1,\ 0) = 1$ in the rule #180. However, in reproducing the transition at the region of the lower bit, the rule #90 would be applied, resulting in the disappearance of the transition from 1 to 1. Thus, it is difficult to reproduce the transition from 1 to 1 by the two-rules set of (#90, #180).

In order to overcome the difficulty, one notices that the most significant bit should be relocated to the lowest order bit. In moving the the most significant bit to the lowest order bit, the transition of the bit at the right hand side takes

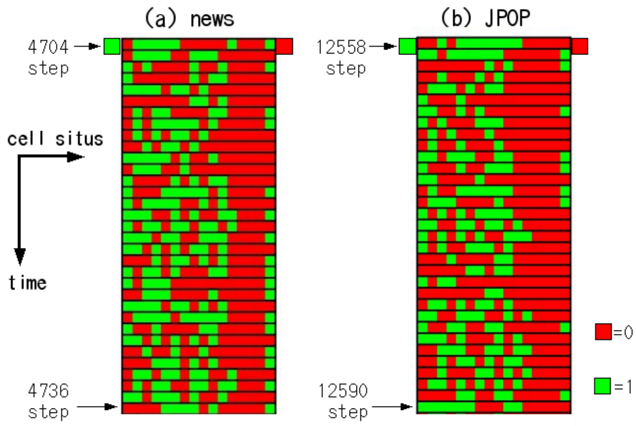

Figure 1: Time development of bit-patterns of the word “news” spoken by man 1 and sample 3 in JPOP.

Figure 2: Typical bit pattern sequences of JPOP, where the length of rule sequences reproducing them is lager and shoter than 16. (a) The case of the longer lenth of rule sequences. (b) The case of the shorter lenth of rule sequences.

0 to 0 almostly. If we reproduce the transition from 0 to 0, we have to apply the transition function of $f(0,\ 0,\ 0) = 0$, which is included in the rule #90 and the rule #180. Therefore, the reproducing procedure of time development of bit-patterns by the two-rules set of (#90, #180) would become easy and the length of rule sequences could become shorter.

4.2. Description Difficulty of the Other Contributing Factors of Bit-Patterns Sequences

Let us discuss the effect of the preprocessing (i), application of the subtraction between successive bit-patterns. We investigate the other contribution factors of description difficulty in bit-patterns sequences. Thus, we studies characteristic features of the bit-pattern sequences with the transion from 0 to 0 of the most significant bit, where the length of rule sequences reproducing them is lager than 16. Typical bit-pattern sequences are given in figure 2.

In our coding, the cells on the band-like region at the left hand side of the most significant bit take 0 almostly as shown in figure 2. In the case of longer length of rule sequences, the band-like region with 0 becomes narrower. In the case of shorter length, on the other hand, the band-like region is broader. The narrower band-like region means that the amplitude of the target sound is larger. Thus, by applying the subtraction between successive bit-patterns, the amplitude becomes smaller, and then the length of rule sequences becomes shorter. The fact shows the effectiveness of the preprocessing (i), application of the subtraction between successive bit-patterns.

It should be noted that in order to introduce the subtraction effect, we have applied the XOR preprocessing between successive bit-patterns. However, in our description method, the XOR preprocessing is not practical but the subtraction preprocessing.

5. Conclusions

In this paper, we investigate practical preprocessing method in order to realize errorless and compressive description method of digital sounds by means of rule sequences of 1-2-3 CA. The practical two preprocessing are as follows:

1. Application of the subtraction between successive bit-patterns. Thus, resultant bit-pattern changes to 17 bits from 16 bits of the original data.
2. Relocation of the most significant bit to the lowest order bit.

By applying the two preprocessing to digital sounds, we succeeded the errorless and compressive description for all the data with our method.

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