2011 International Symposium on Nonlinear Theory and its Applications
NOLTA2011, Kobe, Japan, September 4-7, 2011

Adaptive Synchronization of Nonlinear Systems Using Sliding Mode Observer

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Abstract–This paper presents a methodology for designing an adaptive sliding mode observer to achieve synchronization of a class of nonlinear systems with unknown parameters. The observer's states are driven by a sliding-mode error signal and the estimation of unknown parameters is updated according to a dynamic minimization algorithm. The convergence of the synchronization errors and parameter estimation errors has been justified by the conditional Lyapunov exponents and the effectiveness of the design has also been confirmed by simulations.

1. Introduction

Adaptive synchronization of nonlinear systems has received considerable attentions recently. The main objective of adaptive synchronization is to design a dynamical system (also called adaptive observer) synchronizing with a targeted system, even though some of system parameters are unknown [1]. Usually, the achievement of synchronization also implies the revealing of the unknown parameters, leading to practical and interesting applications, such as secure communications [2], cryptanalysis [3], system modeling [5,6], to name a few.

The common design strategy of adaptive observer is based on the concept of adaptive feedback control, while the global convergence for state and parameter estimation errors is justified by Lyapunov stability theorem [1,7]. However, in order to assure the stability, restrictions on the targeted system are imposed. Specifically, the unknown parameters can only reside in the dynamical equation of the observable states or the system has to be transformed into some particular forms, such as state-affine form [1].

On the other hand, sliding mode observer (SMO) has been widely applied for achieving finite time synchronization [8]. SMO is also well-known for its robustness with respect to uncertainties [9], and its stability has been mathematically proved for some cases, such as those given in [8]. Although SMO is useful for achieving synchronization, its uses in adaptive synchronization need further exploration, as the stability condition may not be easily obtainable. Thus, the objective of this paper is to propose an effective design methodology to solve this problem. It is to incorporate the SMO design together with the dynamic minimization algorithm, so that a sliding mode control signal with parametric updating rules can be designed. The resultant adaptive SMO (ASMO) can then achieve synchronization and estimate unknown parameter simultaneously.

The organization of this paper is as follows. In Sec. 2, a design of ASMO is proposed with the analysis of its stability. To further verify its performance, simulation results for achieving adaptive synchronization of Lorenz system are presented in Sec. 3. Finally, conclusions are drawn in Sec. 4.

2. A Design of Adaptive Sliding-Mode Observer

Consider a class of nonlinear systems expressed as below:

$$\begin{cases} \dot{x} = A(p)x + F(x) \\ y = Cx \end{cases} \tag{1}$$

where $x = [x_1 \quad x_2 \quad \cdots \quad x_n]^T \in \Re^n$ is the state vector; $A(p)$ is a square matrix of size n depending on p with $p = [p_{11} \cdots p_{1m_1} \quad \dots \quad p_{n1} \cdots p_{nm_n}]^T \in \Re^M$ ($M = \sum_{i=1}^{n} m_i$); $F(x) = [f_1(x) \quad f_2(x) \quad \cdots \quad f_n(x)]^T$ is the vector of nonlinear functions, satisfying the Lipshitzian condition, i.e. there exists a positive constants $\gamma < \infty$, such that

$$\|F(x) - F(\hat{x})\| \le \gamma \|x - \hat{x}\| \; ; \tag{2}$$

y is the observable output; C is the output matrix; (A, C) is observable and X^T denotes the transpose of X. In this paper, we focus on the case that only one system state is measurable, i. e. $C = [0_{1\times(l-1)} \quad 1 \quad 0_{1\times(n-l)}]$ or $y = x_l$.

In order to achieve adaptive synchronization of (1), the following ASMO is proposed:

$$\dot{\hat{x}} = A(q)\hat{x} + F(\hat{x}) + U(y, \hat{y}) \tag{3a}$$

$$\hat{y} = C\hat{x} \tag{3b}$$

$$\dot{q}_{ij} = \delta_{ij}\mu_{ij}(\hat{x})h_{ij}(\hat{x}, e_y) \tag{3c}$$

where $e_y = y - \hat{y} = Ce = e_l$ denotes the output error with $e = x - \hat{x}$; q_{ij} is the estimate of p_{ij} and $q = [q_{11} \cdots q_{1m_1} \quad \dots \quad q_{n1} \cdots q_{nm_n}]^T \in \Re^M$. Consider $S = e_y$ as the sliding surface, the sliding mode control signal $U(y, \hat{y})$ is then designed as:

$$U = Ke_y + \alpha\, d(t)\Phi^{-1}C^T \,\mathrm{sgn}(e_y) \quad (4)$$

where α is a positive constant; $K \in \Re^n$ is the gain matrix such that $A_0 = A(p) - KC$ is a strict Hurwitz matrix, and the following Lyapunov equation has a positive solution Φ for a positive definite matrix $\Omega = \Omega^T > 0$, so that

$$A_0^T \Phi + \Phi A_0 = -\Omega; \quad (5)$$

$d(t)$ is an adaptive gain governed by:

$$\dot{d}(t) = \left|e_y\right| - \beta d(t) \quad (6)$$

with β >0 and $d(0) > 0$.

The design of ASMO (3) is based on the following three key points:

(1) Similar to our previous design [3,4] for linear-feedback adaptive observer, the feedback gain K is set to stabilize the linear part of the system such that if all the parameters are known and the Thau condition holds, we have the syncrhonization errors converge to zeros asymptotically based on Lyapunov stability theorem. Since $A(p)$ is unknown in real situation, K is chosen so that $\tilde{A}_0 = (A(q^{(0)}) - KC)$ is stabilized, where $q^{(0)}$ is the initial auxiliary value of p.

(2) h_{ij} is designed based on dynamic minimization algorithm [10]. According to the dependence of unknown parameters on the measurable system output and following the idea presented in [3,4,10], the parametric updating laws are summarized as follows:

Case 1: If $i = l$, we have

$$h_{ij} = \mathrm{sgn}\left[\frac{\partial F_i(\hat{x}, q)}{\partial q_{ij}}\right] e_y. \quad (7)$$

Case 2: If q_{ij} appears in $F_i(\hat{x}, q)$, $i \neq l$ while the time evolution of $\hat{x}_l$ explicitly depends on $\hat{x}_i$,

$$h_{ij} = \mathrm{sgn}\left[\frac{\partial F_l(\hat{x}, q)}{\partial \hat{x}_i}\frac{\partial F_i(\hat{x}, q)}{\partial q_{ij}}\right] e_y. \quad (8)$$

Case 3: If q_{ij} does not belong to the above two cases,

$$h_{ij} = \mathrm{sgn}\left\{\sum_k \left[\frac{\partial F_l(\hat{x}, q)}{\partial \hat{x}_k}\frac{\partial F_k(\hat{x}, q)}{\partial \hat{x}_i}\right]\frac{\partial F_i(\hat{x}, q)}{\partial q_{ij}}\right\} e_y. \quad (9)$$

(3) μ_{ij} is designed such that the synchronization errors converge with a similar rate. It is approximated by the case of only single unknown and details can be referred to [3,4]. Let $r_{ij} = p_{ij} - q_{ij}$ denoting the parameter errors. As μ_{ij} is independent on the output error e_y, both $d(t)$ and $r_{ij}(t)$ depend on the observable state error e_y with the same order, which further ensures that the synchronization errors converge with a similar rate.

3. Simulation Results

The Lorenz system is used for evaluating the performance of ASMO, as it is well-known to be a difficult synchronization problem if the unknown parameters reside in all the three dynamical equations and only the first state is observable.

Consider the dynamics of a Lorenz system described by:

$$\dot{x} = A(p)x + F(x) \quad (10)$$

where $A(p) = \begin{bmatrix} -p_1 & p_1 & 0 \\ p_2 & -1 & 0 \\ 0 & 0 & -p_3 \end{bmatrix}$ with $p = [p_1 \quad p_2 \quad p_3]^T$ being the unknown parameters. The vector of nonlinear functions is given by $F(x) = [0 \quad -x_1x_3 \quad x_1x_2]^T$. Let $y = x_1$ be the available output (i.e. $C = [1 \quad 0 \quad 0]$), and the sliding mode surface $S = Ce = e_1$.

Based on Sec. 2, an ASMO can be designed as follows:

$$\dot{\hat{x}} = A(q)\hat{x} + F(\hat{x}) + U(y, \hat{y}) \quad (11)$$

with the dynamics of the unknown parameters q_i governed by:

$$\dot{q}_i = \delta_i \mu_i(\hat{x}) h_i(\hat{x}, e_1) \quad (12)$$

where $A(q) = \begin{bmatrix} -q_1 & q_1 & 0 \\ q_2 & -1 & 0 \\ 0 & 0 & -q_3 \end{bmatrix}$ with $q = [q_1 \quad q_2 \quad q_3]^T$.

The detailed design procedures are described as follows:

Step 1: Determine the sliding mode control signal U

In the simulation, it is letting the true values of the unknown parameters $p_1 = 10$, $p_2 = 28$, $p_3 = 2.667$, while their initial estimates are set to be $q^{(0)} = [16 \quad 25 \quad 2.4]^T$. By setting $K = [80 \quad 30 \quad 1]^T$, we obtain the eigenvalues of $\tilde{A}_0 = (A(q^{(0)}) - KC)$ as $\lambda_{1,2,3} = -1.8497, -2.400, -95.1503$, which are all negative. Then, by setting $\Omega = 10I_{3\times3}$ ($I_{3\times3}$ is the 3×3 identity matrix), the solution of LE equation (5) according to Kronecker tensor product yields:

$$\Phi = \begin{bmatrix} 0.0593 & -0.1354 & -0.0171 \\ -0.1354 & 2.8332 & -0.0804 \\ -0.0171 & -0.0804 & 2.0833 \end{bmatrix}.$$

Then, we have the Thau condition implying that $\gamma < \dfrac{\lambda_{\min}(\Omega)}{2\lambda_{\max}(\Phi)} = 1.7556.$

Step 2: Design parametric updating laws

The functions h_i are derived by using the dynamic minimization algorithm [3,4,6,7]. According to different cases given in (7)-(9), it can be obtained that:

$h_1 = \mathrm{sgn}(\hat{x}_2 - \hat{x}_1)e_1$, $h_2 = \mathrm{sgn}(\hat{x}_1)e_1$, and $h_3 = \mathrm{sgn}(\hat{x}_1\hat{x}_3)e_1$.

Moreover, to make all the state errors e_i converge in a similar way as e_1, the auxiliary functions μ_i are introduced as given in (12). Following the same method as adopted in

[3, 4], it can be derived that $\mu_1 = \mu_3 = 1$ and $\mu_2 = |\hat{x}_2|$ (due to the page limitation, the procedures are omitted here).

Substituting h_i and μ_i into (12), the final parametric updating laws are expressed as:

$$\begin{cases} \dot{q}_1 = \delta_1 \operatorname{sgn}(\hat{x}_2 - \hat{x}_1) e_1 \\ \dot{q}_2 = \delta_2 |\hat{x}_2| \operatorname{sgn}(\hat{x}_1) e_1 \\ \dot{q}_3 = \delta_3 \operatorname{sgn}(\hat{x}_1 \hat{x}_3) e_1 \end{cases} \quad (13)$$

and that completes our ASMO design.

To verify the feasibility of our approach, the largest conditional Lyapunov exponents (CLE) are computed. The use of CLE for the justification of synchronization was firstly proposed by Pecora and Carroll in [11,12], stating that synchronization can only be achieved if all the CLEs are negative. In our previous works [3,4], the impact of the feedback gains K on the stability of the linear-feedback based observers have been investigated, showing that the stability of the proposed design can be ensured if a negative largest CLE is obtained.

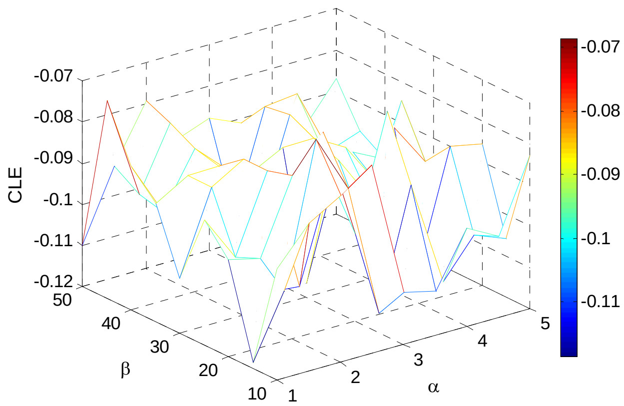
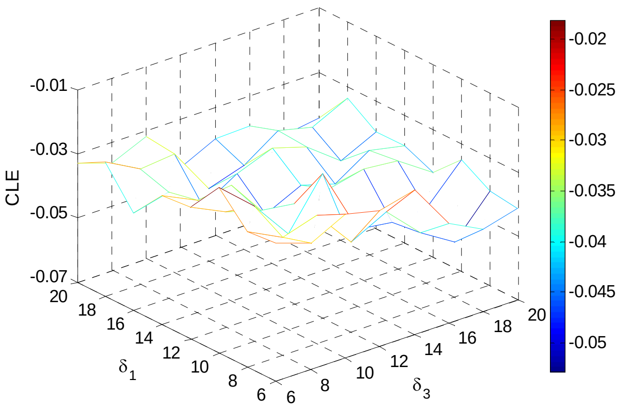


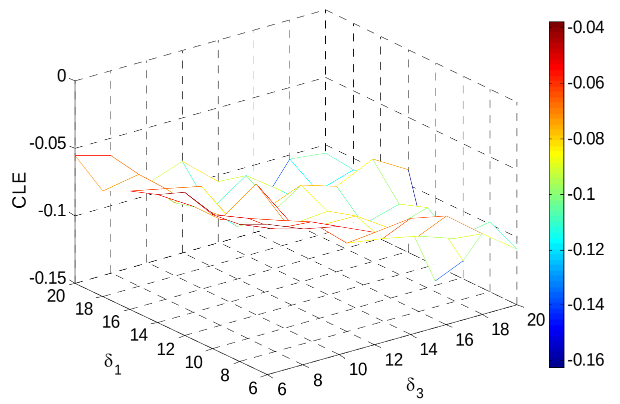
Fig.1 The largest CLEs for different α and β with the feedback gains $K = [80 \quad 30 \quad 1]^T$, and the stiffness constants $\delta = [12 \quad 4 \quad 12]^T$.

Obviously, the main difference between the ASMO and the linear-feedback based adaptive observer is the introduction of the sliding mode control signal $\alpha\, d(t) \Phi^{-1} C^T \operatorname{sgn}(e_y)$ in (6) which is affected by the parameters α and β. Therefore, the effects of α and β to the performance of the estimation are studied and the results are shown in Fig. 1. As shown in Fig.1, it is noticed that a negative CLE is obtained for a large regime of both α and β.

The impact of the stiffness constants on the largest CLE is also obtained as presented in Fig. 2. It is noticed that in a relative large region of stiffness constants δ, all the CLEs remain negative, and hence the stability of the proposed design is confirmed.



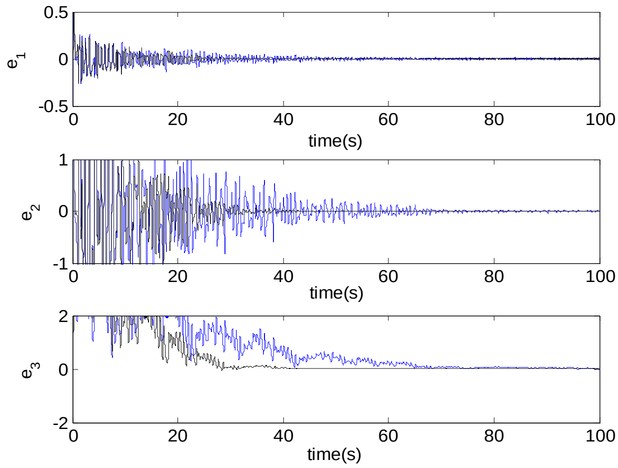
$\delta_2 = 2$



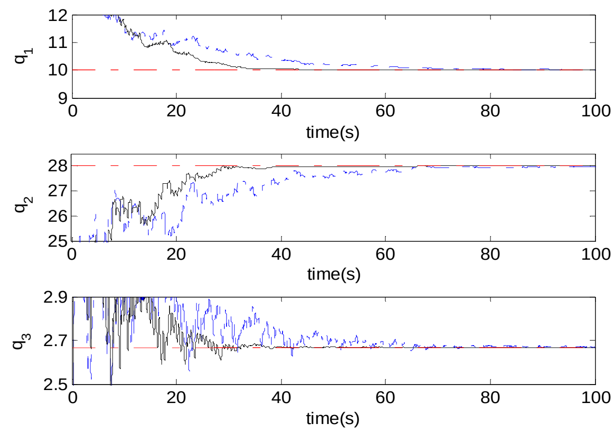
$\delta_2 = 4$

Fig. 2 The largest CLE with different stiffness constants, where $\delta_{1,3} \in [6 \quad 20]$, and $\delta_2 = 2,4$ respectively.

Based on the above study, we choose $\alpha = 2.5$, $\beta = 40$, $K = [80 \quad 30 \quad 1]^T$, and $\delta = [12 \quad 4 \quad 12]^T$ for the simulation, and the initial value of d is set to be $d(0) = 100$. Figure 3 depicts the convergences of state errors e_i and system parameters q_i. The performance of the linear feedback adaptive observer proposed in [3], i.e. using $U = Ke_y$, is also presented for comparison. To have a fair comparison, the same parameter settings have been used. It is clearly observed that the convergence of ASMO outperforms that of the linear feedback adaptive observer. The synchronization errors converge to very small values after initial transient time of about 30s using ASMO while 70s is needed for the linear feedback adaptive observer. The unknown parameters are also accurately estimated by ASMO. For referencing, after $t = 100$s, the parameter estimation errors are: $r_1 = 1.24 \times 10^{-4}$, $r_2 = -2.94 \times 10^{-4}$, $r_3 = 1.52 \times 10^{-4}$ for ASMO, while $r_1 = 2.78 \times 10^{-3}$, $r_2 = -5.06 \times 10^{-3}$, $r_3 = 3.26 \times 10^{-4}$ for a linear feedback adaptive observer. It should be emphasized that the proposed ASMO also works fine for a large class of nonlinear systems specified by (1), including the Genesio system, the Rössler system and so on. Due to page limitation, the results are not included here.



e_i vs time



q_i vs time

Fig.3. Convergences of e_i and q_i for different observers, where the blue lines denote the linear-feedback adaptive observer and the black lines denote the ASMO.

4. Conclusions

In this paper, a systematic adaptive sliding mode observer is proposed for synchronizing nonlinear systems based on a scalar time series. The convergence of the proposed observer is justified by the conditional Lyapunov exponents and simulation results clearly illustrate that it outperforms the linear-feedback adaptive observer both in terms of accuracy and speed of convergence.

Acknowledgments

This work is fully supported by a grant from the Research Grants Council of Hong Kong S.A.R., China, (CityU120708), Chinese Universities Scientific Fund (2010QNA5031), and Scientific Research Fund of Zhejiang Provincial Education (Y201017301).

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