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A Combinatorial Optimization Method which Combines Ant Colony Optimization and Chaotic Dynamical

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Abstract– There are various combinatorial optimization problems in various fields, such as science, engineering, economy and so on. For heuristic solution search in combinatorial optimization problems, the chaotic dynamics has been shown effective by many studies. Such chaotic search algorithm is based on an improvement of the solutions by some local search heuristics. In order to improve the performance of algorithm, the tabu search and gradient search have been combined with global search algorithms, such as the genetic algorithms, ant colony optimization and so on. In such hybrid approach, the ant colony optimization has the better performance than the genetic hybrid. In this paper, we propose a novel hybrid method that combines the chaotic neural tabu search, which is besed on neighboring search and the ant colony optimization. Our results show that the proposed hybrid algorithm has better performance than the original chaotic search and its hybrid with the genetic algorithm.

1. Introduction

There are various combinatorial optimization problems in various fields. In those problems, exact solution of the NP-hard problems cannot be solved in reasonable time. For such problems, it is important to develop effective heuristic methods for solving better solutions in reasonable time.

As one of the heuristic approaches, effectiveness of the chaotic dynamics has been shown by various studies [5,6,7,8]. There are three major methods in chaotic optimization approach. The first method is to add chaotic noise in the mutually connected neural network minimizing energy function [5]. It has been shown more effective than the stochastic noise, such as the white Gaussian noise [6]. The second method is to apply high dimensional chaotic neural dynamics to the mutually connected neural networks [7]. However, these two methods based on the mutually connected neural networks are applicable only to very small toy problems. To enable applications of such chaotic dynamics to much larger problems, the third method combines effective searching ability of the optimal solution by the chaotic dynamics with the heuristic methods that can be easily applied to large scale combinatorial optimization problems. This third chaotic method has been shown more effective than existing tabu searches, or stochastic searches, even for the large scale problems [8,9].

Such chaotic search algorithms neighboring solution search are based on chaotic updating of local effective. To improve the performance of local search based methods, such algorithms have been combined with global searches, such as the genetic algorithms (GA), ant colony optimization (ACO), and so on. The chaotic dynamical method has also been combined with a global search algorithm, the GA [13]. The global searching methods much improves the performance of the tabu search and the local search, and solved the best known solutions in many benchmark problems in the QAPLIB [18]. In Ref. [14,15], the ACO, the GH and other algorithms have been compared it has been shown that the ACO has the best solving performance.

Therefore, in this paper, we propose a novel hybrid method that combines chaotic neural tabu search proposed in Ref.[9], which has the better performance than the tabu search or the stochastic search, and the ACO proposed in Ref.[15] , which may be the best for the hybrid algorithms. The proposed method is applied to the QAPs and its performance is compared with the original chaotic neural tabu search, hybrid method that combines the GA and the chaotic neural tabu search.

2. Local search and global search algorithms

2.1. Local search algorithms

The local search algorithms are the methods to search neighboring solutions in a searching space. In this paper, such neighboring solution search is based on exchanges of the elements. The methods based on such neighboring solutions are effective for searching in detail in narrow space. However, it is difficult to jump to the states far from the current solution. Therefore, in this paper such local search algorithms are utilized for searching deeply in a limited area.

There are a lot of existing methods based on neighboring solution search, such as steepest descent method, tabu search [1], chaotic neural tabu search [8] and so on. Among such methods, the chaotic dynamics has been shown effectiveness in such approach [5-12]. As one of the more effective chaotic methods, chaotic neural tabu search has been shown more effective than existing tabu searches and stochastic searches [9].

In this paper, the chaotic neural tabu search is realized by extending the Taillard's tabu search [16] using the chaotic neural network [9]. It can be realized by the following chaotic neural network updating a local search heuristics,

$$\xi_{ij}(t+1) = \beta\Delta(t), \tag{1}$$

$$\eta_{ij}(t+1) = -W \sum_{k=1}^{n} \sum_{l=1, (k \neq i \vee l \neq j)}^{n} x_{kl}(x) + W, \tag{2}$$

$$\begin{aligned}\gamma_{ij}(t+1) = k_r \varsigma_{p(j)q(i)}(t) \\ - \alpha\{x_{p(j)q(i)}(t) + z_{p(j)q(i)}(t)\} + R,\end{aligned} \tag{3}$$

$$\varsigma_{ij}(t+1) = k_r \varsigma_{ij}(t) - \alpha\{x_{ij}(t) + z_{ij}(t)\} + R, \tag{4}$$

$$\begin{aligned}x_{ij}(t+1) = f\{\xi_{ij}(t+1) + \eta_{ij}(t+1) \\ + \gamma_{ij}(t+1) + \varsigma_{ij}(t+1)\},\end{aligned} \tag{5}$$

where, the output function is the sigmoidal function $f(y) = \dfrac{1}{1+\exp(-\frac{y}{\varepsilon})}$, α and β are scaling parameter of the refractory effects and gain inputs, k_r is decay parameter of the tabu effects, W is connection weights, n is the size of the problem and R is the positive bias, respectively.

When the output of the (i, j) th neuron, $x_{ij}(t+1)$, becomes larger than 0.5, the (i, j) neuron fires, and the element i is assigned to the jth index as shown in Fig.1. At the same time the element in the jth index, $p(j)$, is assigned to the $q(i)$th index. Therefore, when the (i, j) th neuron fires, not only the assignment of (i, j) but also the $(p(j), q(i))$ must be tabu. $z_{ij}(t)$ and $z_{p(j)q(i)}(t)$ are prepared to memories such assignments as $(p(j), q(i))$, which is updated by the following $z_{ij}(t+1)$ is reset to 0, when the (i, j) th neuron is updated; $x_{p(j)q(i)}(t+1)$ is added to $z_{ij}(t+1)$, when updating other neurons.

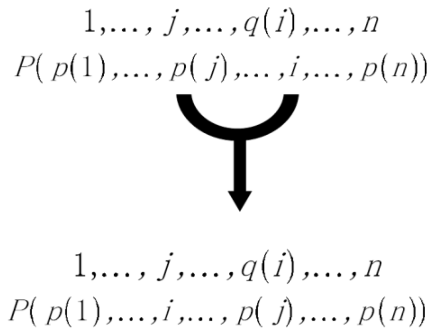

Figure1: Update of the solution by firing of the (i, j) th neuron.

The chaotic neural tabu search described above is a searching method that has not only features of the tabu search but also features of the chaotic dynamics that is known effective for combinatorial optimization. That is the reason why the chaotic neural tabu search is superior to the tabu search [9].

However, since the local search algorithms are based on the neighborhood search, they are not good at searching globally in a search space. To overcome such an issue, the local search algorithms have been combined with global search heuristic methods, such as the GA, ACO, and so on. Such Hybrid methods have been shown much better performance [14,17]. The chaotic method has also combined with the GA and improvement of the performance has been shown [13].

2.2. Hybrid algorithms

In the hybrid approaches, it has been shown that, the hybrid method combines the ant system and a local search has better performance than the GA and a local search [14,15]. Therefore in this paper, we propose the chaotic neural tabu search, which has good searching ability in local searches, combined with the ACO. The origin of the ant systems is to simulate the behavior of ants' searching food. The ants find the sources of food in the following way: First, they explore the area surrounding their nest in a random manner. While they are moving, the ants left a pheromone (chemical trace) on the floor, in such a way that they can find their way back to the nest. When they find a source of food, the ants bring food back to the nest following the pheromone traces, leaving additional pheromone during the return trip. After a while, the paths between the nest and sources of food will be indicated by an amount of pheromone in relation with the length of the path. Indeed, short paths will be travelled at a higher rate than long ones and the amount of pheromone will grow faster on the short ways. Therefore, the ants are able to optimize their paths by this process. In this paper, we apply the ACO effective for global search to the QAPs.

3. Hybrid Method Combining Chaotic Search and Ant Colony Optimization

Our proposed algorithm is a combination of the chaotic neural tabu search as a local search proposed in Ref.[9] and the ACO as a global search proposed in Ref.[15]. The procedure of the proposed algorithm is shown in Fig.2.

We prepare the pheromone matrix ***T***, whose size is $n \times n$ to memorize previous better solutions. The (i, j) th element of the matrix ***T***, τ_{ij} , is corresponding to the probability of the assignment of the element *i* to the index *j*. $p(j) = i$. The pheromone matrix ***T*** is used for generating the initial solutions for local search.

First, in the step 1 in Fig.2, all of the elements τ_{ij} in the pheromone matrix ***T*** are set 1. In the step 2, the ACO generate initial solution for the chaotic neural tabu search according to the pheromone matrix ***T***, by the following procedure.

1) $I = \phi, J = \phi$

2)While $|I| < n$ repeat:

2a)Choose *j*, randomly, uniformly, $1 \le j \le n, \quad j \notin J$.

2b)Choose *i*, randomly, $1 \le i \le n, \quad i \notin I,$ with probability $\tau_{ij} / \sum_{1 \le k \le n, k \notin I} \tau_{kj}$ and set $p(j) = i$.

2c) $I = I \cup \{i\}, \quad J = J \cup \{j\}$

Then, in the step3, the chaotic neural tabu search described in the previous section is applied to the generated initial solution in the step 2. During the solution search by the chaotic algorithm, the best solution found in the search in this step is memorized as ***P****. If the ***P**** is better than the best solution found so far in whole procedure, it is memorized as ***P*****. After the chaotic neural tabu search, in step the 4, the pheromone matrix is updated, by the following procedure.

1) For *i* = 1 to *n* do:

1a) $\tau_{ip^*(i)} = \tau_{ip^*(i)} + r^*$

1b) $\tau_{ip^{**}(i)} = \tau_{ip^{**}(i)} + r^{**},$

Where, *r** and *r*** are the parameters corresponding to reinforcement of the probability of the assignments in the solution obtained by the chaotic neural tabu search, and those in the best solution found so far, respectively. By repeating from the steps 2 to 4, better solutions become easy to be found by applying chaotic searches to more appropriate initial solutions generated by the pheromone matrix ***T***.

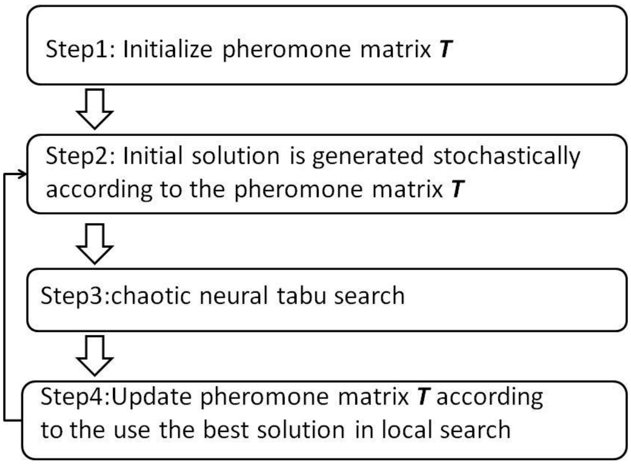

Figure2: Flow chart of proposal method.

4. Numerical Experiments

We apply the proposed hybrid algorithm to asymmetric QAPs, whose sizes are 20 to 100 from QAPLIB [18]. We compare the performance of the proposed method that combines the chaotic neural tabu search and the ACO with the original chaotic neural tabu search and the genetic hybrid combining the GA, and the chaotic neural tabu search.

Table1: Results of the proposed hybrid algorithm combing chaotic neural tabu search and ACO, the original chaotic neural tabu search the chaotic neural tabu search combined with the GA, on QAPs. The results are shown by percentages of average gaps from the best known solutions.

	combining chaotic neural tabu search and ant colony optimization	chaotic neural tabu search	combining the GA and the chaotic neural tabu search
Tai20b	**0**	**0**	**0**
Tai25b	**0**	**0**	0.0651
Tai30b	0.0588	0.147	**0.0181**
Tai35b	**0.0102**	0.430	0.349
Tai40b	0.0217	0.210	**0.0153**
Tai50b	**0.0452**	0.216	0.155
Tai60b	**0.00887**	0.172	0.123
Tai80b	**0.0310**	0.146	0.558
Tai100b	**0.107**	0.653	0.198
average	**0.0314**	0.216	0.165

Table 1 shows the results of those three methods on Tai20b whose size is 20 to Tai100b whose size is 100. From the Table 1, that the proposed hybrid algorithm combining chaotic neural tabu search and ACO has better results than the original chaotic neural tabu search. This result confirms that the proposed hybrid algorithm improves the performance of the original chaotic search method. By comparing the proposed hybrid algorithm combining chaotic neural tabu search and ACO with the hybrid combining the chaotic neural tabu search and the GA, the proposed algorithm using ACO has better performance especially in the large scale problems. For hybrid approach using the chaotic neural tabu search, our results show that the ACO is the best and much improves the performance.

5. Conclusions

We propose a novel hybrid method that combines the chaotic neural tabu search and the ACO. By combining the chaotic search which improves the solution by efficiently moving it, with an efficient global search, the ACO, the performance can be much improved. Our simulation results show that the proposed hybrid algorithm has the better performance than the original chaotic search and its hybrid with GA. Our algorithm is effective especially for the large scale problems.

As future works, we would like to improve hybrid method by introducing other ACO hybrid methods. We also would like to apply this proposed approach to the large scale TSPs.

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