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Adaptive Control of Steady States and Slowly Varying States in the Duffing-Holmes Type System with Unstable High-Pass Filter

Elena Tamaševičiūtė[1,2], Arūnas Tamaševičius[1], Gytis Mykolaitis[1], and Skaidra Bumelienė[1]

[1]Plasma Phenomena and Chaos Laboratory, Semiconductor Physics Institute
LT-01108 Vilnius, Lithuania
[2]Department of Radiophysics, Faculty of Physics, Vilnius University
LT-10222 Vilnius, Lithuania
Email: tamasev@pfi.lt, elena.tamaseviciute@ff.vu.lt, gytis@pfi.lt, skaidra@pfi.lt

Abstract—We describe a very simple feedback controller for stabilizing unknown unstable steady states of dynamical systems. The feedback loop contains an unstable first order high-pass filter. The controller is reference-free. It does not require knowledge of the location of the states in the phase space and is suitable not only for fixed steady states but for slowly varying states as well. As a specific example we consider the damped Duffing-Holmes type oscillator and investigate it analytically, numerically and experimentally. Experiments have been performed using a simplified version of the electronic Young-Silva circuit imitating dynamical behavior of the Duffing-Holmes system.

1. Introduction

Many techniques for stabilization of unstable periodic orbits (UPOs) of dynamical systems are overviewed in [1, 2]. The problem of stabilizing unstable steady states (USS) is also of great importance, especially in engineering applications. Simple control methods of stabilizing nonoscillatory states, e.g proportional feedback technique require as a reference point the coordinates of the USS. In many practical cases the location of the USS is either unknown or it slowly varies with time because of changes in the ambient conditions. Therefore adaptive, reference-free methods automatically locating the USS are preferable.

The simplest adaptive technique for stabilizing USS is based on derivative controller. A perturbation in the form of a derivative dx/dt derived from an observable $x(t)$ does not change the original system, since it vanishes when the variable approaches the steady state $x(t) = const.$ [3, 4, 5].

Another adaptive method for stabilizing USS employs low-pass filter in the feedback loop [6, 7, 8, 9]. Provided the cut-off frequency of the filter is low enough, the filtered image $v(t)$ of the observable $x(t)$ asymptotically approaches the USS and therefore can be used as a reference point in the proportional feedback. This method has been successfully applied to several experimental systems, including electronic circuits [6, 7] and lasers [8], also to control unknown unstable spirals in the Lorenz system [9]. However, if an USS is neither a node nor a spiral but a saddle (characterized by an odd number of real positive eigenvalues) the conventional filter technique does not work. More sophisticated controllers involving *unstable* low-pass filters should be used. The idea of using auxiliary unstable degree of freedom in the feedback loop has been introduced in [10] in a slightly different context and has been experimentally verified for stabilizing torsion-free UPOs (characterized by an odd number of real positive Floquet exponents) of autonomous [11] and nonautonomous [12] dynamical systems. Unstable low-pass filter has been also demonstrated to be useful for stabilization of the saddle type USS as well [13, 14]. Simulations and experiments have been carried out for an electrochemical oscillator [13], studied analytically and numerically for the case of the saddles in the mechanical pendulum and the Lorenz system [14].

In this paper, we demonstrate that an *unstable high-pass* filter can be applied to stabilize saddles in the Duffing type system. Using high-pass filter instead of low-pass filter makes the controller technically simpler. We note that a high-pass filter (a stable version) has been used already to stabilize a laser [15]. However, since it is a stable filter, it is unable to control saddles. In addition to the fixed USS we investigate the case of slowly varying USS.

2. Electronic Circuit and Equations

Full circuit diagram of the experimental setup is shown in Fig. 1. An electronic circuit imitating dynamics of the Duffing-Holmes oscillator is composed of the elements OA1, R1...R3, R, L, C, D1 and D2. Actually it is a simplified version of the Young-Silva oscillator used to demonstrate stabilization of the UPO [12] and characterized in more details in [16]. The rest of the circuit is a controller. The OA2 stage is a buffer. The OA3 stage is a negative impedance converter. It

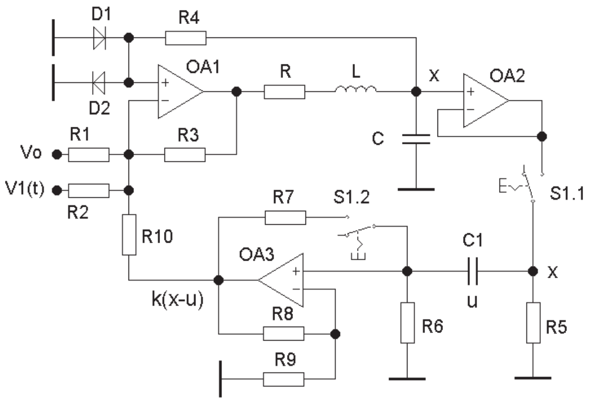

Figure 1: Duffing-Holmes circuit with an unstable controller in the feedback. $R_1 = 1$ MΩ, $R_2 = 100$ kΩ, $R_3 = R_4 = R_9 = R_{10} = 10$ kΩ, $R_5 ... R_8 = 20$ kΩ (R_7, R_8 adjustable, used to set the gain $k = R_8/R_9 + 1$), $L = 19$ mH, $C = 470$ nF. OA1...OA3 – LM741; D1, D2 – 1N4148. S1.1–S1.2 is an electronically controlled double switch. R and C_1 are specified in the captions to Figs. 5,6.

introduces negative resistance R^- (if $R_7 = R_8$ then $R^- = -R_9$) and makes the high-pass $R_f C_f$ filter unstable. Here $R_f = -R_9||R_6 = -R_6R_9/(R_6 - R_9)$, $C_f = C_1$. Location of the USS can be varied by means of the DC voltage V_0 and the AC voltage $V_1(t)$.

The closed-loop circuit is described by

$$\dot{x} = y, \quad (1)$$

$$\dot{y} = -by + F(x) + \xi(t) + k(u - x), \quad (2)$$

$$\dot{u} = \omega_f (u - x). \quad (3)$$

Here $\xi(t) = \xi_0 + \xi_1 sin(\omega_1 t)$, $F(x)$ is approximated by three-segment piecewise linear function:

$$F(x) = \begin{cases} -(x+1), & x < -0.5 \\ x, & -0.5 \le x \le 0.5 \\ -(x-1), & x > 0.5 \end{cases} \quad (4)$$

Here b is the damping coefficient of the Duffing-Holmes oscillator, ξ_0 and ξ_1 are unknown DC and AC biasing parameters, respectively. The $\omega_1 << 1$ is the normalized frequency of the slow AC perturbation, ω_f is the normalized cut-off frequency of the high-pass filter. The variables and parameters are given by

$$x = \frac{V_C}{1V},\ y = \frac{\rho I_L}{1V},\ u = \frac{V_{C1}}{1V},\ \rho = \sqrt{\frac{L}{C}}, \quad (5)$$

$$b = \frac{R}{\rho},\ \omega_1 = \Omega_1 \sqrt{LC},\ \omega_f = \frac{\sqrt{LC}}{|R_f|C_f}. \quad (6)$$

3. Linear Analysis

In this Section we assume for simplicity that $\xi_1 = 0$. When the feedback is OFF ($k = 0$) and the DC biasing force ξ_0 is not too large ($|\xi_0| < 0.5$) Eqs. (1,2) have

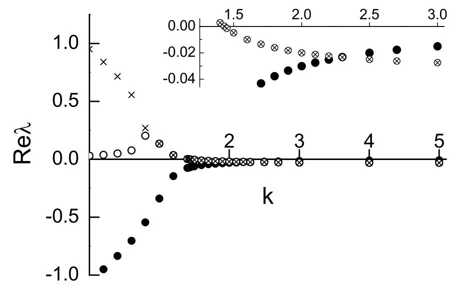

Figure 2: Dependence of the real part of the eigenvalues Reλ on the control gain k from Eq. (10). $b = 0.1$, $\omega_f = 0.03$.

three real steady state solutions $(x_0, y_0) = (x_{01,02,03}, 0)$ with $x_{01} = \xi_0 - 1$, $x_{02} = -\xi_0$, $x_{03} = \xi_0 + 1$. Two of them, with x_{01} and x_{03} are stable, while the middle one with x_{02} is a saddle type unknown USS.

Let us consider the steady states and their stability properties of the system under control ($k > 0$). The x- and y-projections of the steady states remain unchanged in the case of the closed loop (the u-projections coincide with the x-projections). However, the originally unstable state (x_0, y_0) with x_{02} under certain conditions can become a stable steady state $(x_0, y_0, u_0) = (x_{02}, 0, x_{02})$. To check the stability of the system we linearize Eqs. (1-3) around $(x_{02}, 0, x_{02})$:

$$\dot{x} = y, \quad (7)$$

$$\dot{y} = -by - (k-1)x + ku, \quad (8)$$

$$\dot{u} = \omega_f (u - x). \quad (9)$$

and analyse its characteristic equation

$$\lambda^3 + (b - \omega_f)\lambda^2 + (k - 1 - b\omega_f)\lambda + \omega_f = 0. \quad (10)$$

The system is stable if all the real parts of three eigenvalues of the Eq. (10) are negative. This can be checked using the Routh-Hurwitz criteria. The eigenvalues Re$\lambda_{1,2,3}$ are all negative if the diagonal minors of the Routh-Hurwitz matrix are all positive:

$$\Delta_1 = b - \omega_f > 0, \quad (11)$$

$$\Delta_2 = (b - \omega_f)(k - 1 - b\omega_f) - \omega_f > 0, \quad (12)$$

$$\Delta_3 = \omega_f \Delta_2 > 0. \quad (13)$$

These inequalities yield the following stability criteria:

$$0 < \omega_f < b,\ k > k_{th} = \frac{b}{b - \omega_f} + b\omega_f. \quad (14)$$

At small b the threshold gain $k_{th} \approx b/(b - \omega_f)$. For example, at $b = 0.1$ and $\omega_f = 0.03$ the gain $k_{th} \approx 1.43$. The |Reλ| has a maximum at $k_{opt} \approx 2.3$ (Fig. 2). Note, that k_{th} in Fig. 2 well coincides with the value obtained from the Routh-Hurwitz criteria.

4. Numerical Results

Results obtained by means of numerical integration of Eqs. (1-3) are shown in Figs. 3,4. The main difference between the results in Fig. 3 and the investigations in [13, 14] is in the following. The electrochemical oscillator [13], the pendulum, and the Lorenz system [14] all exhibit oscillating behavior, either periodic or chaotic before the control is turned on. While the control of the system in Fig. 3 is started from the stable steady states (either stable spirals or stable nodes). Although it is stated nowhere in the text of [13, 14], the presented illustrations of originally oscillating systems give an inadequate impression that a saddle point can be stabilized if it is approached by the trajectories of the limit cycles or trajectories of chaotic attractors. In our case the original steady states and the USS are fixed and rather remote objects in the phase space. In addition, the simulations of the over-damped system (bottom plot in Fig. 3) show that in order to switch from a stable state to a saddle it is not necessary even for the transient trajectories to twist around the USS. In contrast, the target can be reached point-blank.

In addition, we demonstrate in Fig. 4, that slowly varying states can also be controlled by means of the unstable high-pass filter.

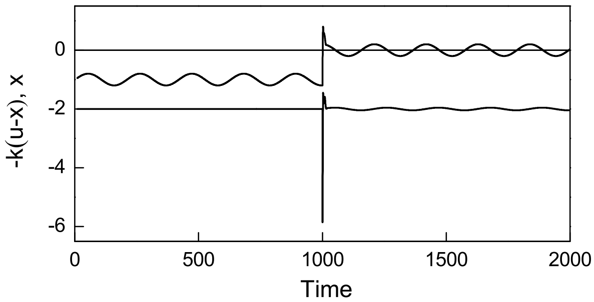

Figure 4: Stabilizing the slowly varying state of the Duffing-Holmes oscillator from Eq. (1-3) with $k = 3$, $\xi_0 = 0$, $\xi_1 = 0.2$, $\Omega_1 = 0.03$. $b = 2$, $\omega_f = 0.4$. Upper trace is the main observable x, lower trace (shifted down by 2 for clarity) is the control term $k(u - x)$. Control is turned on at $t = 1000$.

5. Experimental Results

Experimental photos taken from the screen of a multichannel oscilloscope are presented in Figs. 5,6. Experimental results are in a good agreement with the numerical simulations.

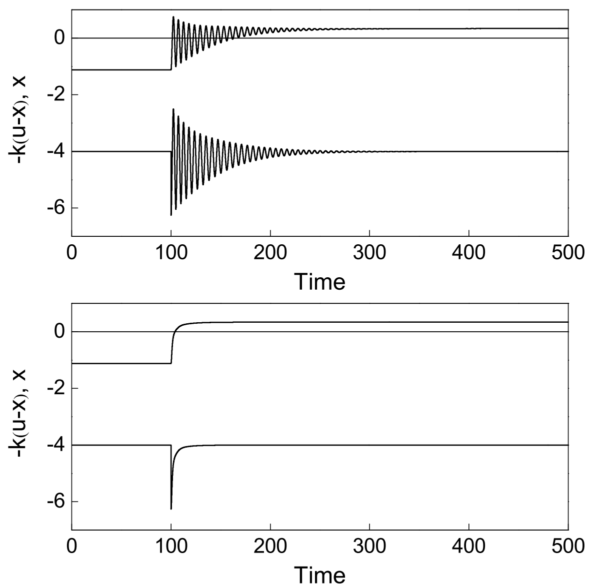

Figure 3: Stabilizing the unstable state of the Duffing-Holmes oscillator from Eq. (1) with $k = 2$, $\xi_0 = -0.3$, $\xi_1 = 0$. Top: $b = 0.1$, $\omega_f = 0.03$. Bottom: $b = 2$, $\omega_f = 0.1$. Upper traces in the top and bottom plots is the main observable x, lower traces (shifted down by 4 for clarity) is the control term $k(u - x)$. Control is turned on at $t = 100$.

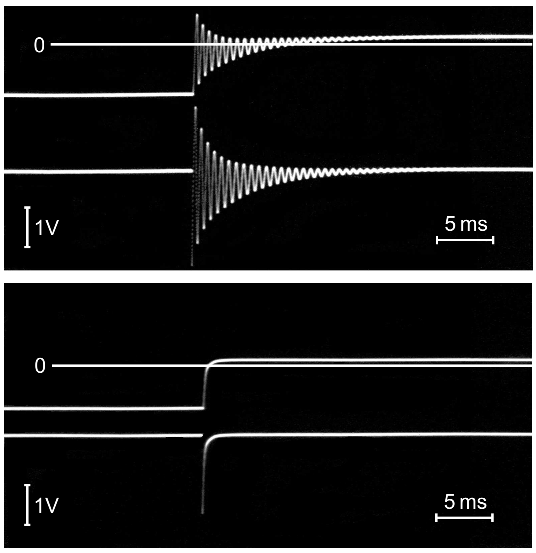

Figure 5: Experimental control of the steady states of the Duffing-Holmes electrical circuit shown in Fig. 1 with $R_7 = R_8 = 10$ kΩ ($k = 2$), $R_f = -20$ kΩ, $V_0 = 30$ V, $V_1 = 0$. (Top) $R = 20$ Ω ($b = 0.1$), $C_1 = 330$ nF ($\omega_f = 0.03$); (bottom) $R = 400$ Ω ($b = 2$), $C_1 = 100$ nF ($\omega_f = 0.1$). Upper traces in the top and bottom photos is the main signal $V_C \propto x$, lower traces is the control signal $V_{contr} \propto -k(u - x)$ taken from the OA3.

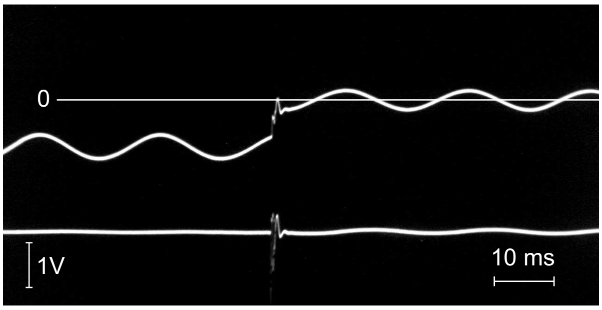

Figure 6: Experimental control of slowly varying state of the Duffing-Holmes electrical circuit shown in Fig. 1 with $R_7 = R_8 = 20$ kΩ ($k = 3$), $V_0 = 0$, $V_1 = 2$ V, $f_1 = 50$ Hz ($\Omega_1 = 0.03$). $R = 400$ Ω ($b = 2$), $R_f = -20$ kΩ, $C_1 = 13$ nF ($\omega_f = 0.4$); Upper trace is the main signal $V_C \propto x$, lower trace is the control signal $V_{contr} \propto -k(u - x)$ taken from the OA3.

6. Conclusion

We have demonstrated the efficiency of an adaptive control technique using unstable high-pass filter to stabilize saddle type steady state in a dynamical system. The controller automatically locates unknown and/or slowly varying unstable state and uses it as a reference point in the proportional feedback.

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