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Augmented Automatic Choosing Control of Nonlinear Observer Type for Nonlinear Systems with Linear Measurement and Its Application

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Abstract—This paper deals with a feedback control using automatic choosing functions and observer-control design procedure, for nonlinear systems with linear measurement. A constant term which arises from linearization of a nonlinear equation is treated as a coefficient of a stable zero dynamics. A given nonlinear system is linearized piecewise so as to be able to design the linear optimal controllers with observer. By the automatic choosing functions, these controllers are smoothly united into a single nonlinear feedback controller, which is called an augmented automatic choosing control of nonlinear observer type. This controller is applied to improve the transient stability of power systems, whose simulation results show that the new controller enables to expand the stable region well.

1. Introduction

The problem of nonlinear control design has been studied for many years[1-8]. Most controllers are synthesized by linearizing a given nonlinear system so that the linear estimation and control theory is applicable when some of the state variables of the system are not measurable. One of them is based on a truncation at the first order of the Taylor expansion[1,2]. This control law is easy to implement to many practical nonlinear systems, but is only useful in small region or in almost linear ones. Controllers based on a change of coordinates in differential geometry [3,4] are effective in wider region, but not easy to implement to practical systems. Controllers based on Fuzzy reasoning[5] are more practical, but usually need a lot of divisions.

This paper is concerned with a nonlinear feedback controller by using the automatic choosing functions and the linear control theory for nonlinear systems with linear measurement. This controller well works even in nonlinear systems with high nonlinearity and wider region. Considering the nonlinearity, we define some separative variables whose inverse domain associated with the region of system is divided into some subdomains. On each subdomain, the system equation is linearized by the Taylor expansion so as to be applied the linear observer and LQ control theory [2,8]. Constant terms by this linearization are treated as coefficients of a stable zero dynamics[7]. The resulting linear controls are smoothly united by the automatic choosing function to make a single nonlinear feedback control, whose estimator is a nonlinear observer. This controller is called an augmented automatic choosing control of nonlinear observer type (AACCNO).

A power system in transient stability problem is one of the typical nonlinear systems with high nonlinearity, so the proposed method is successfully applied to it. Experimental results indicate that the transient stability by AACCNO is much improved than by the ordinary linear optimal controller (LOC).

2. Statement of Problem

The plant is assumed to be described by a nonlinear dynamic equation and a linear measurement equation

$$\dot{x} = f(x) + Bu, \quad x \in \mathbf{D} \subset R^n \tag{1}$$

$$y = Hx \tag{2}$$

where $\cdot = d/dt$, $x = [x_{[1]}, \cdots, x_{[n]}]^T$ is an n-dimensional state vector, $u = [u_{[1]}, \cdots, u_{[r]}]^T$ is an r-dimensional control vector, $y = [y_{[1]}, \cdots, y_{[m]}]^T$ is an m-dimensional measurement vector, f is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, B is an $n \times r$ constant driving matrix, H is an $m \times n$ constant measurement matrix, and T denotes transpose.

Considering the nonlinearity of the system (1), introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x_{[2]}$ is the element which has the highest nonlinearity of (1), then

$$C(x) = x_{[2]} \in D \subset R \quad (L = 1).$$

The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \leq i \leq M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L}[a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

We here introduce an automatic choosing function of sigmoid type:

$$I_i(x) = \prod_{j=1}^{L} \left\{ 1 - \frac{1}{1+\exp\left(2N\left(C_j(x)-a_{ij}\right)\right)} - \frac{1}{1+\exp\left(-2N\left(C_j(x)-b_{ij}\right)\right)} \right\} \quad (3)$$

where N is positive real value, $-\infty \le a_{ij} < b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Figure 1).

The aim of the paper is to design a nonlinear feedback control AACCNO by smoothly uniting the sectionwise observer-controls which make use of (3).

3. Design of AACCNO

The nonlinear function f of (1) is linearized by the Taylor expansion truncated at the first order about a point $\hat{\chi}_i \in C^{-1}(D_i)$ and $\hat{\chi}_0 = 0$ on each subdomain D_i (see Figure 2):

$$f(x) \simeq f(\hat{\chi}_i) + A_i(x - \hat{\chi}_i) = A_i x + w_i$$

where

$$A_i = \partial f(x)/\partial x^T|_{x=\hat{\chi}_i}, \quad w_i = f(\hat{\chi}_i) - A_i\hat{\chi}_i.$$

Introduce a stable zero dynamics :

$$\dot{\hat{x}}_{[n+1]} = -\sigma \hat{x}_{[n+1]} \quad (4)$$

$$(\hat{x}_{[n+1]}(0) \simeq 1, \quad 0 < \sigma < 1),$$

where the value of σ shall be selected so that $\sigma = -\dot{\hat{x}}_{[n+1]}/\hat{x}_{[n+1]} \le -\dot{x}_{[k]}/x_{[k]}$ holds for all k ($k = 1, \cdots, n$). This tries to keep $\hat{x}_{[n+1]} \simeq 1$ for a good while when the system (1) is not on $C^{-1}(D_0)$. We approximate f as

$$f(x) \simeq A_i x + w_i \simeq A_i x + w_i \hat{x}_{[n+1]}. \quad (5)$$

Assume that the control is designed by using (3) as

$$u = \sum_{i=0}^{M} u_i I_i(\hat{x}) \quad (6)$$

where $\hat{x}$ is an estimate of x.

Substituting (5) and (6) into (1), the dynamic equation becomes

$$\begin{aligned} \dot{x} &= f(x) + Bu \\ &= \sum_{i=0}^{M} f(x) I_i(\hat{x}) + B\sum_{i=0}^{M} u_i I_i(\hat{x}) \\ &= \sum_{i=0}^{M} (A_i x + w_i \hat{x}_{[n+1]} + Bu_i + \varepsilon_i(x)) I_i(\hat{x}) \quad (7) \end{aligned}$$

where ε_i is approximation error. This suggests the following sectionwise linear observer:

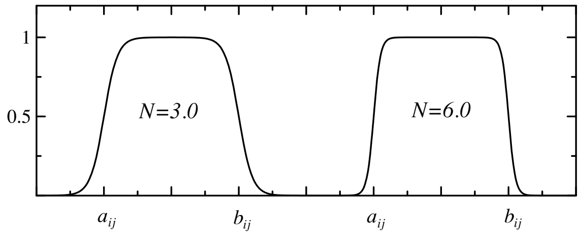

Figure 1: Automatic Choosing Function (N = 3.0, 6.0)

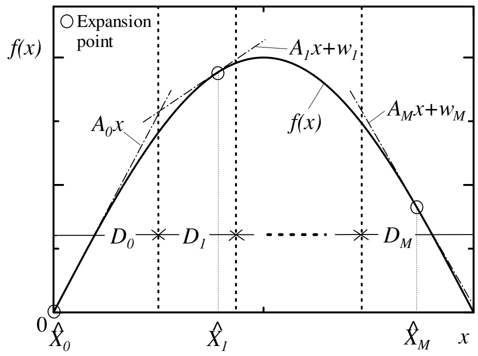

Figure 2: Sectionwize linearization

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}) + Bu + K(y - H\hat{x}) \\ &= \sum_{i=0}^{M} (A_i\hat{x} + w_i\hat{x}_{[n+1]} + Bu_i + \varepsilon_i(\hat{x}) \\ &\quad + K_i(y - H\hat{x})) I_i(\hat{x}) \quad (8) \end{aligned}$$

where

$$K = \sum_{i=0}^{M} K_i I_i(\hat{x}) \quad (9)$$

Note that $f(\hat{x})$ is nonlinear function so that (8) is of nonlinear observer type.

Consider a special case of $I_i(\hat{x}) = 1$ in which $N = -a_{ij} = b_{ij} \to \infty$ in (3).

Put $X = [x^T, \hat{x}_{[n+1]}]^T$, then Eqs.(4) and (7) yield

$$\dot{X} = \bar{A}_i X + \bar{B} u_i + \bar{\varepsilon}_i$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{\varepsilon}_i = \begin{bmatrix} \varepsilon_i \\ 0 \end{bmatrix},$$

0 is an appropriate dimensional null matrix. Eqs.(7) and (8) by putting $e = x - \hat{x}$ derive

$$\dot{e} = (A_i - K_i H)e + \Delta\varepsilon_i$$

from (2) where $\Delta\varepsilon_i = \varepsilon_i(x) - \varepsilon_i(\hat{x})$. Thus we have

$$\begin{bmatrix} \dot{X} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \bar{A}_i - \bar{B}F_i & \bar{B}F_iE_z \\ 0 & A_i - K_iH \end{bmatrix} \begin{bmatrix} X \\ e \end{bmatrix} + \tilde{\varepsilon}_i \quad (10)$$

if $u_i = -F_i X$, where $\tilde{\varepsilon}_i = [\bar{\varepsilon}_i^T, \Delta\varepsilon_i^T]^T$.

The characteristic equation of a closed-loop system of (10) when considered $\tilde{\varepsilon}_i$ as another input becomes

$$\det[sI - (\bar{A}_i - \bar{B}F_i)]\det[sI - (A_i - K_i H)],$$

in which both $(\bar{A}_i - \bar{B}F_i)$ and $(A_i - K_i H)$ must be stabilized. It means that the separation property of the observer-control design procedure could be used to get F_i and K_i.

In this section we make use of the optimal observer-control approach. To get F_i, we apply the LQ control theory as follows. Consider that the system and cost function

$$\Sigma : \begin{cases} \dot{X} = \bar{A}_i X + \bar{B}u \\ J_i = \frac{1}{2}\int_0^\infty (X^T Q X + u_i^T R u_i) dt \end{cases} \tag{11}$$

is given. Then an application of the linear optimal control theory [2] yields

$$\begin{aligned} u_i(X) &= -F_i X \\ F_i &= R^{-1}\bar{B}^T P_i \end{aligned} \tag{12}$$

where the $(n+1)\times(n+1)$ matrix P_i satisfies with the Riccati equation:

$$P_i\bar{A}_i + \bar{A}_i^T P_i + Q - P_i\bar{B}R^{-1}\bar{B}^T P_i = 0. \tag{13}$$

Here, $Q = Q^T > 0$ and $R = R^T > 0$ which denote positive symmetric matrices. Values of Q and R are properly determined based on engineering experience [8].

To get K_i, we apply the linear identity observer theory for the dual system of Σ. Then we have

$$K_i = S_i H^T V^{-1} \tag{14}$$

where the $n \times n$ matrix S_i satisfies with the Riccati equation:

$$A_i S_i + S_i A_i^T - S_i H^T V^{-1} H S_i + W = 0 \tag{15}$$

Here, $W = W^T > 0$ and $V = V^T > 0$ should be properly selected taking about the loop transfer recovery [8].

As a result, we have the AACCNO formula in case of the sigmoid type as follows.

[AACCNO formula]

$$\dot{\hat{x}} = f(\hat{x}) + Bu + K(y - H\hat{x})$$
$$\dot{\hat{x}}_{[n+1]} = -\sigma\hat{x}_{[n+1]} \quad (\hat{x}_{[n+1]}(0) \simeq 1)$$
$$u = \sum_{i=0}^{M} u_i I_i(\hat{x})$$
$$K = \sum_{i=0}^{M} K_i I_i(\hat{x})$$

where

$$A_i = \partial f(x)/\partial x^T|_{x=\hat{\chi}_i}, \quad w_i = f(\hat{\chi}_i) - A_i\hat{\chi}_i$$

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$u_i = -R^{-1}\bar{B}^T P_i \hat{X}, \quad \hat{X} = [\hat{x}^T, \hat{x}_{[n+1]}]^T$$

$$P_i\bar{A}_i + \bar{A}_i^T P_i + Q - P_i\bar{B}R^{-1}\bar{B}^T P_i = 0$$

$$K_i = S_i H^T V^{-1}$$

$$A_i S_i + S_i A_i^T - S_i H^T V^{-1} H S_i + W = 0$$

$$\begin{aligned} I_i(\hat{x}) \;=\; \prod_{j=1}^{L}\Bigg\{ 1 &- \frac{1}{1+\exp\left(2N\left(C_j(\hat{x}) - a_{ij}\right)\right)} \\ &- \frac{1}{1+\exp\left(-2N\left(C_j(\hat{x}) - b_{ij}\right)\right)} \Bigg\} \end{aligned}$$

Since this formula is of a structure-specified type, each parameter included in the above equations must be properly selected so that the feedback control system (1) by AACCNO could stabilize globally.

4. Numerical Example

Consider a field excitation control problem of power system. Put $x=[x_{[1]}, x_{[2]}, x_{[3]}]^T=[E_I - \hat{E}_I, \delta - \hat{\delta}_0, \dot{\delta}]^T$ and $u = E_{fd} - \hat{E}_{fd}$.
Here δ: phase angle, $\dot{\delta}$: rotor speed, E_I: open circuit voltage, E_{fd}: field excitation voltage.
Then this system is described by Eqs.(1)and(2), where $n = 3, r = 1, m = 2$,

$$\begin{aligned} f_1(x) &= -\frac{1}{kT'_{d0}}\left(x_{[1]} + \hat{E}_I - \hat{E}_{fd}\right) \\ &\quad + \frac{(X_d - X'_d)\widetilde{V}Y_{12}}{k}x_{[3]}\cos\left(\theta_{12} - x_{[2]} - \hat{\delta}_0\right) \\ f_2(x) &= x_{[3]} \\ f_3(x) &= -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}\left(x_{[1]} + \hat{E}_I\right)\cos\left(\theta_{12} - x_{[2]} - \hat{\delta}_0\right) \\ &\quad - \frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}\left(x_{[1]} + \hat{E}_I\right)^2 - \frac{\widetilde{D}(x)}{\widetilde{M}}x_{[3]} + \frac{P_{in}}{\widetilde{M}} \\ \widetilde{D}(x) &= \widetilde{V}^2\Bigg\{\frac{T''_{d0}(X'_d - X''_d)}{(X'_d + X_e)^2}\sin^2\left(x_{[2]} + \hat{\delta}_0\right) \\ &\quad + \frac{T''_{q0}(X_q - X''_q)}{(X_q + X_e)^2}\cos^2\left(x_{[2]} + \hat{\delta}_0\right)\Bigg\} \\ b_1 &= \frac{1}{kT'_{d0}}, \quad k = 1 + (X_d - X'_d)\,Y_{11}\sin\theta_{11} \\ H &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = [b_1, 0, 0]^T. \end{aligned}$$

Parameters are

$\widetilde{M} =$	$0.016095[pu]$	$T'_{d0} =$	$5.09907[sec]$
$\widetilde{V} =$	$1.0[pu]$	$P_{in} =$	$1.2[pu]$
$X_d =$	$0.875[pu]$	$X'_d =$	$0.422[pu]$
$Y_{11} =$	$1.04276[pu]$	$Y_{12} =$	$1.03084[pu]$
$\theta_{11} =$	$-1.56495[pu]$	$\theta_{12} =$	$1.56189[pu]$
$X_e =$	$1.15[pu]$	$X''_d =$	$0.238[pu]$
$X_q =$	$0.6[pu]$	$X''_q =$	$0.3[pu]$
$T''_{d0} =$	$0.0299[pu]$	$T''_{q0} =$	$0.02616[pu]$.

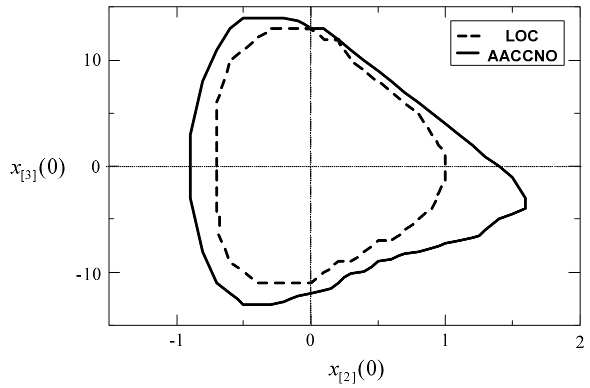

Figure 3: Stable region

Steady state values are

$$\begin{aligned} \hat{E}_I &= 1.52243[pu] & \hat{\delta}_0 &= 48.57^\circ \\ \hat{\dot{\delta}}_0 &= 0.0[deg/sec] & \hat{E}_{fd} &= 1.52243[pu]. \end{aligned}$$

Set $X = [x^T, \hat{x}_{[4]}]^T = [x_{[1]}, x_{[2]}, x_{[3]}, \hat{x}_{[4]}]^T$, $\hat{\delta}_0 = 48.57^\circ$, $C(x){=}x_{[2]}$, $L = 1$, $M = 1$, $a_1 = 61.37^\circ$, $\hat{\chi}_0 = 0$, $\hat{\chi}_1 = [0, 80^\circ - \hat{\delta}_0, 0]^T$, $\sigma = 0.3262$, $R = 1$, $Q = diag(1,1,1,1)$, $V = diag(1,1)$, $W = diag(139.1,1,1)$, $\hat{X}(0) = [0,0,0,1]^T$. These values are selected by trial-and-error referring to Ref.[7]. Experiments are carried out for the new control(AACCNO) and the ordinary linear optimal control (LOC)[1, 2]. Figure 3 depicts the cross section of the stable region for AACCNO and LOC, where $x_{[1]}(0) = 0$. Figure 4 shows the time responses of $x_{[1]} \sim x_{[3]}$ when $X(0) = [0, 1.4, 0, 1]^T$. Experimental results indicate that the stable region and trajectories by the new AACCNO are much better than those by the LOC.

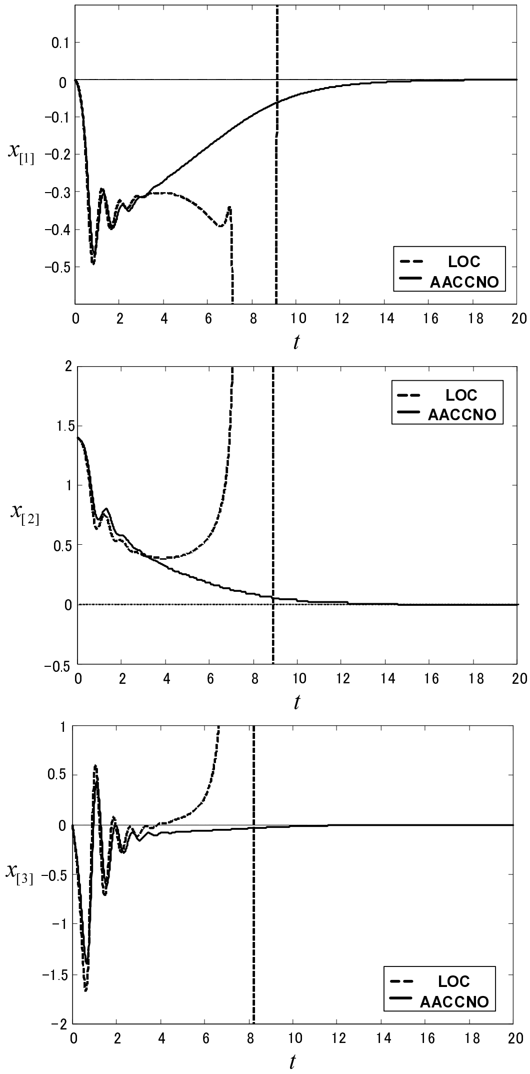

Figure 4: Time responses

5. Conclusions

We have studied an augmented automatic choosing control of nonlinear observer type (AACCNO) for nonlinear systems with linear measurement. This controller has been applied to a field excitation control problem of power system. Simulation results have shown that the new controller is able to improve the transient stability considerably well.

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