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A Parameter Value Optimization Technique for a Reflectionless Transmission-Line Model of the Cochlea

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Abstract— The cochlea is a good candidate for a high-performance Fourier analyzer used in engineering applications. Therefore, electronic circuit implementation of cochlear function is important. However, no high-performance cochlear model is suitable for circuit implementation.

Therefore, Kohda et al. proposed a simple cochlear model that can effectively reproduce the characteristics of the cochlea. The model is based on an ideal distributed constant circuit, and is referred to as a reflectionless transmission line model (cochlear reflectionless transmission line model). It can reproduce the physiological characteristics of the cochlea by adjusting circuit parameter values.

In this paper, we describe an improved method for quantitative reproduction of cochlear characteristics using an optimization technique.

1. Introduction

The cochlea is a peripheral organ in the inner ear. It converts sound from the eardrum into a nerve signal. In addition, the cochlea is a good candidate for a high-performance Fourier analyzer.

Oono and Kohda proposed a passive reflectionless transmission-line model of the cochlea (passive model) based on an ideal distributed constant circuit [1]. The model can reproduce the passive properties of a physiological cochlea [2] by adjusting the values of circuit parameters. However, parameter tunings alone do not enable the passive model to reproduce the active properties of the cochlea. Subsequently, Kohda proposed an active reflectionless transmission-line model with a negative resistance (active model) [3].

In addition, Kohda et al. incorporated the function of the outer hair cell into the model, and modified the model using a hydro-mechanical transducer and a cubic nonlinear element [4]. This model can be transferred back into the passive model through an equivalent circuit conversion. Therefore, it is important to determine the circuit parameter values of the passive model prior to creating the active model, so that the passive model can quantitatively reproduce the passive properties of the cochlea. We proposed a method for determining the parameter values of the passive model to qualitatively reproduce the passive properties of the cochlea [5, 6].

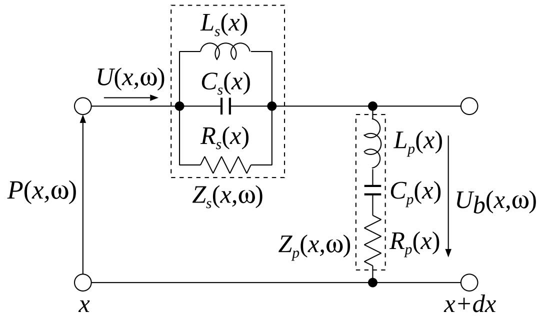

Figure 1: Cochlear passive model [1].

In this paper, we optimize the parameter values in the passive model to quantitatively design the cochlear passive model.

2. Passive model

The passive model of the cochlea is shown in Fig. 1 [1]. A parallel impedance $Z_p(x,\omega)$ in Fig. 1 is given by

$$Z_p(x,\omega) = j\omega L_p(x) + R_p(x) + \frac{1}{j\omega C_p(x)}, \tag{1}$$

where x is a distance from the input of the transmission line, and ω is an angular frequency of the input signal.

If the value of each circuit elements in $Z_p(x,\omega)$ is assumed to change exponentially with distance, circuit elements can be written as

$$R_p(x) = R_0 e^{-ax},\ L_p(x) = L_0 e^{ax},\ \text{and}\ C_p(x) = C_0 e^{ax}, \tag{2}$$

where R_0, L_0, and C_0 are values of circuit elements when $x = 0$, and a is a constant determined by the characteristics of the cochlea.

In Fig. 1, if we assume that the characteristic impedance $Z_0(x,\omega)$ is independent of distance [7], then the characteristic impedance can be given by

$$Z_0(x,\omega) = r, \tag{3}$$

where r is a positive constant. As a result, the series impedance $Z_s(x,\omega)$ in Fig. 1 is defined as

$$Z_s(x,\omega) = \frac{r^2}{Z_p(x,\omega)}. \tag{4}$$

The circuit elements in $Z_s(x, \omega)$ should satisfy the following conditions.

$$R_s(x) = \frac{r^2}{R_p(x)},$$
$$L_s(x) = r^2 C(x), \tag{5}$$
$$C_s(x) = \frac{L(x)}{r^2}.$$

The propagation constant $\gamma(x, \omega)$ of the transmission line is given by

$$\gamma(x, \omega) = \sqrt{\frac{Z_s(x, \omega)}{Z_p(x, \omega)}} = \frac{r}{Z_p(x, \omega)}. \tag{6}$$

Substituting Eq. (1) into Eq. (6) gives

$$\gamma(x, \omega) = \frac{r}{j\omega L_p(x) + R_p(x) + 1/j\omega C_p(x)}. \tag{7}$$

Resonance angular frequency $\beta(x)$ and sharpness $Q(x)$ are respectively given by

$$\beta(x) = \beta_0 e^{-ax},\ \beta_0 = \frac{1}{\sqrt{L_0 C_0}}, \tag{8}$$

$$Q(x) = Q_0 e^{ax},\ Q_0 = \frac{1}{R_0}\sqrt{\frac{L_0}{C_0}}, \tag{9}$$

where β_0 and Q_0 are the resonance angular frequency and sharpness of the circuit, respectively, when $x = 0$.

Taking the integral of the propagation constant with respect to x gives

$$\Gamma(x, \omega) = \int_0^x \gamma(y, \omega) dy, \tag{10}$$

or

$$\Gamma(x, \omega) = \frac{jr}{2a\sqrt{L_0/C_0 + jR_0 L_0 \omega}} \times \left\{ \ln\left(\frac{\sqrt{1 + jR_0 C_0 \omega} + \sqrt{L_0 C_0} e^{ax} \omega}{\sqrt{1 + jR_0 C_0 \omega} - \sqrt{L_0 C_0} e^{ax} \omega} \right) - \ln\left(\frac{\sqrt{1 + jR_0 C_0 \omega} + \sqrt{L_0 C_0} \omega}{\sqrt{1 + jR_0 C_0 \omega} - \sqrt{L_0 C_0} \omega} \right) \right\}. \tag{11}$$

The transfer function is defined as

$$F(x, \omega) = \frac{U_b(x, \omega)}{P(0, \omega)}, \tag{12}$$

where $U_b(x, \omega)$ is the current flowing in the parallel impedance, and $P(0, \omega)$ is the input voltage of the transmission line.

We can rewrite Eq. (12) using Eqs. (1) and (10) as

$$F(x, \omega) = \frac{1}{Z_p(x, \omega)} \exp(-\Gamma(x, \omega)). \tag{13}$$

The gain and phase characteristics of the passive model are numerically simulated with Eq. (13) and shown in Figs. 2 and 3, respectively, where $a = 0.288$, $L_0 = 2.385 \times 10^{-7}$, $C_0 = 2.132 \times 10^{-7}$, $R_0 = 1.5$, and $r = 5$.

As shown in Fig. 2, the gain characteristic has a peak, i.e., the maximum gain, at a particular frequency. We refer to this as the "peak frequency." Beyond the peak frequency, the gain rapidly decreases towards the resonance frequency. Note that the resonance frequency is different from the peak frequency. In Figs. 2 and 3, a $\circ$ symbol shows the peak frequency, and in Fig. 3, a $\times$ symbol shows the resonance frequency.

3. A method for determining parameter values

We describe below the basic principle to determine the values of parameters a, L_0, C_0, R_0, and r in the passive model [6].

First, we introduce a variable n that represents the square root of the ratio of L_0 and C_0 as

$$n = \sqrt{\frac{L_0}{C_0}}. \tag{14}$$

We can tune the maximum gain at the peak frequency through n.

Substituting Eq. (9) into Eq. (14) gives

$$Q_0 = \frac{n}{R_0}. \tag{15}$$

Therefore, using R_0, we can determine the sharpness without changing the resonance frequency.

Next, we adjust the phase value at the resonance frequency by r. Through this tuning, the resonance frequency itself is not affected.

We now show the design procedure according to the above.

Step 1: We determine the value of a, such that the characteristics of the resonance frequencies according to the distance match those of the peak frequencies obtained from physiological experiments.

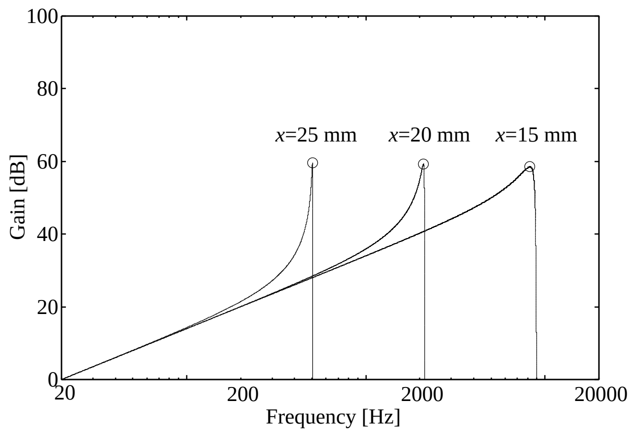

Figure 2: Frequency vs. gain characteristics of the passive model.

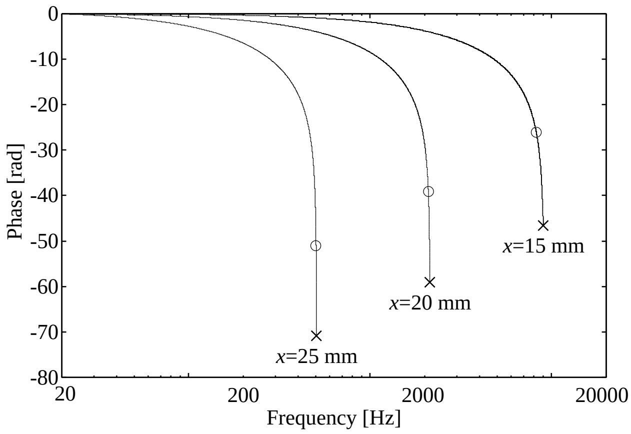

Figure 3: Frequency vs. phase characteristics of the passive model.

Step 2: We set the value of L_0 and C_0 as

$$L_0 = C_0 = \frac{1}{\beta_0}; \tag{16}$$

the initial value of $n = 1$.

Step 3: By fixing the value of L_0 as given in Eq. (16), we tune the peak frequency through n so that it matches its physiological counterpart. In this tuning process, we use the empirical values $R_0 = 1$ and $r = 5$.
The numerical simulation results of the gain and phase characteristics are labeled (a) in Figs. 4 and 5, respectively, as they were at Step 2 (before the value of n was adjusted). The circuit element values used in the simulations are: $a = 0.281$, $n = 1$, $R_0 = 1$, and $r = 5$. After we tune the value of n in Step 3, we obtain the gain and phase characteristics labeled (b) in Figs. 4 and 5, respectively. After tuning, the circuit element values are: $a = 0.281$, $n = 1.2$, $R_0 = 1$, and $r = 5$.

Step 4: We adjust the value of R_0 to match the maximum gain and sharpness at the peak frequency to those obtained from physiological experiments.
The numerical simulation results after the tuning of R_0 are labeled as (c) in Figs. 4 and 5. After tuning, the circuit element values are: $a = 0.281$, $n = 1.2$, $R_0 = 100$, and $r = 5$.

Step 5: We adjust the phase at the resonance frequency by changing r, so that it produces a value similar to that obtained from the physiological experiments.
The numerical simulation results after the adjustment of r are labeled as (d) in Figs. 4 and 5. After tuning, the circuit element values are: $a = 0.281$, $n = 1.2$, $R_0 = 100$, and $r = 3$.

Step 6: Repeating the procedures from Step 3 to Step 5, we further tune the values of n, R_0, and r, so that the gain and phase responses sufficiently match those obtained from the physiological experiments.

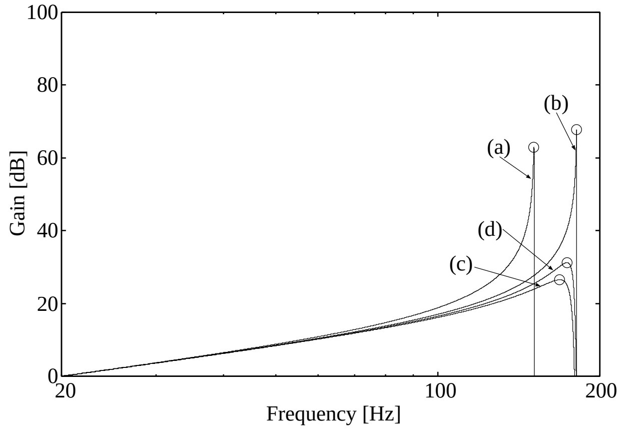

Figure 4: The gain characteristics of the passive model at $x = 30$ mm, when we tune the parameters. (a) Step 2, (b) Step 3, (c) Step 4, and (d) Step 5.

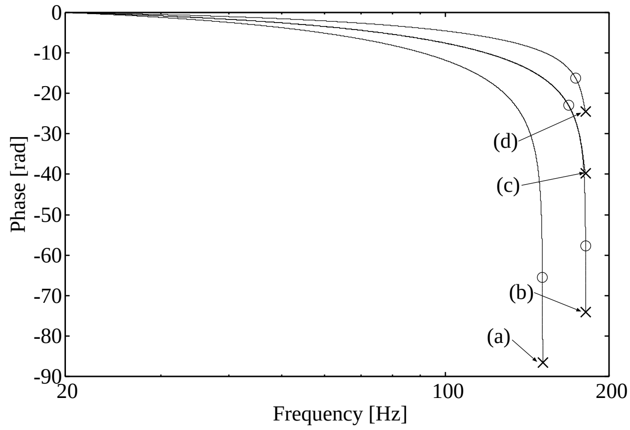

Figure 5: The phase characteristics of the passive model at $x = 30$ mm, when we tune the parameters. (a) Step 2, (b) Step 3, (c) Step 4, and (d) Step 5.

4. Optimization of the parameter values

To obtain improved parameter values that provide a better match between the gain and phase responses of the passive model and those of the physiological data, we use an optimization technique.

First, we introduce a constraint on parameter a as

$$\frac{\ln f_{max} - \ln f_{min}}{x_{max}} \leq a, \tag{17}$$

where f_{max} and f_{min} are the maximum and minimum frequencies of the human audible range, respectively, and x_{max} is the length of a human cochlea. In this paper, we use $f_{max} = 10$ kHz, $f_{min} = 100$ Hz, and $x_{max} = 35$ mm.

Next, the object function $E(a, R_0, n, r)$ can be defined as

$$\begin{aligned} E(a, R_0, n, r) &= w_Q \cdot EQ(a, R_0, n, r) \\ &+ w_f \cdot EF(a, R_0, n, r) + w_g \cdot EG(a, R_0, n, r) \\ &+ w_p \cdot EP(a, R_0, n, r), \end{aligned} \tag{18}$$

where w_Q, w_f, w_g, and w_p are weights, and $EQ(a, R_0, n, r)$ (Eq. (19)), $EF(a, R_0, n, r)$ (Eq.

(20)), $EG(a,\ R_0,\ n,\ r)$ (Eq. (21)), and $EP(a,\ R_0,\ n,\ r)$ (Eq. (22)) are the errors from the target values in the sharpness, peak frequency, maximum gain, and phase at resonance frequency of the designed circuit, respectively.

$$EQ(a,\ R_0,\ n,\ r) = \left|\frac{Q_x - Q(x;\ a,\ R_0,\ n,\ r)}{Q_x}\right|, \qquad (19)$$

$$EF(a,\ R_0,\ n,\ r) = \left|\frac{pf_x - f_p(x;\ a,\ R_0,\ n,\ r)}{pf_x}\right|, \qquad (20)$$

$$EG(a,\ R_0,\ n,\ r) = \left|\frac{g_x - g(x;\ a,\ R_0,\ n,\ r)}{g_x}\right|, \qquad (21)$$

$$EP(a,\ R_0,\ n,\ r) = \left|\frac{p_x - p(x;\ a,\ R_0,\ n,\ r)}{p_x}\right|, \qquad (22)$$

where Q_x, pf_x, g_x and p_x are the target values for the sharpness, peak frequency, maximum gain, and phase at resonance frequency at a distance of x, respectively, and $Q(x;\ a,\ R_0,\ n,\ r)$, $f_p(x;\ a,\ R_0,\ n,\ r)$, $g(x;\ a,\ R_0,\ n,\ r)$ and $p(x;\ a,\ R_0,\ n,\ r)$ are the current values for the sharpness, peak frequency, maximum gain, and phase at resonance frequency given by Eq. (13) at a distance of x, respectively.

The constraint given by Eq. (17) is continuous, which allows us to classify the above optimization problem as a continuous optimization problem. In addition, this constraints are primary inequalities; however the objective function in Eq. (18) is not a linear function.

Consequently, the target optimization problem is a nonlinear optimization problem. Therefore, we use the downhill simplex method to determine the parameter values.

4.1. Design example

We demonstrate the optimization of the parameters through a design example. The target values for Q_x, pf_x, g_x, and p_x at $x = 30$ mm used in the design example are summarized in Table 1.

Before the optimization, we first determined the initial values of the parameters a, R_0, n, and r according to the procedure described in Sec. 3. In addition, small random numbers were added to those parameter values.

We then performed the optimization proposed above. The total number of the trials were 10 with 200 iterations in each trial.

Through the optimization, we obtained the values of $a = 0.156385$, $R_0 = 0.200919$, $n = 1.057865$, and $r = 2.133622$. Figure 6 shows the gain and phase responses with these optimized parameters. The relative errors from the target values defined by Eqs. (19) to (22) are summarized in Table 2, that confirms good matches.

5. Conclusion

We have proposed a technique to determine the parameter values in a passive model of the cochlea. The proposed technique was formulated as a nonlinear optimization problem. We use the downhill simplex method to solve the optimization problem. Through the design example, we confirmed the effectiveness of the proposed method.

In the future, we will try other solving technique rather than the downhill simplex method. In addition, we will propose a design technique including a hydro-mechanical transducer and a cubic nonlinear element.

Table 1: The target values.

x [mm]	Q_x	pf_x [Hz]	g_x [dB]	p_x [rad]
30	200	200	50	-50

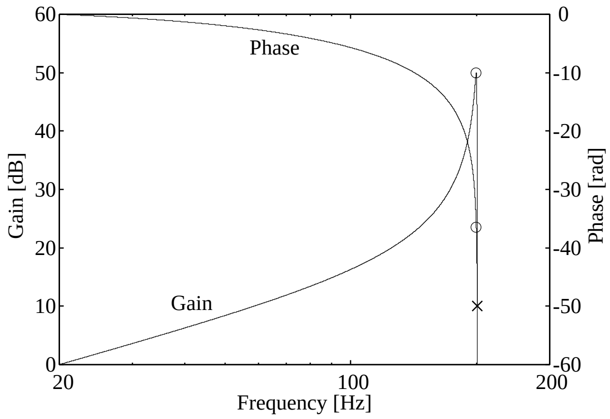

Figure 6: Frequency vs. gain and phase characteristics as a result of the parameter optimization.

Table 2: The relative errors from the target values.

	Relative Error [%]
EQ	0.01
EF	0.01
EG	0.028
EP	0.028

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