2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

Event-triggered synchronization control of networked oscillators and its application in smart grid

Qiang Jia†, Mei Sun†

†Faculty of Science, Jiangsu University
301 Xuefu Road, Zhenjiang, Jiangsu Province, P. R. China
Email: qiangjia@ujs.edu.cn, sunm@ujs.edu.cn

Abstract– This work is dedicated to frequency synchronization problem of a group of networked phase oscillators via event-triggered mechanism. An efficient event-triggered control protocol is designed for reaching synchronization, and criterions with coupling strength and certain trigger function are also established. It is also shown that the inter-event times are lower bounded and hence there is no Zeno behavior. To illustrate and verify the effectiveness of the proposed control strategy, a simple example of frequency synchronization of networked oscillators involved in smart grid is also demonstrated. Numerical results further demonstrate the validity of our theoretical results.

1. Introduction

Synchronization phenomena widely exist in natural world and also occur in man-made systems in various fields such as neural networks, sensor networks, biological networks, and smart grids. Frequency synchronization is reported vital for stable operation of electrical power grid [1], and the synchronization control problems of phase oscillators have attracted many attentions. The pioneering work by Ali Jadbabaie et al. analyzed the collective dynamics of networked traditional Kuramoto oscillators with identical natural frequency, and arbitrary topology [2]. Nikhil Chopra and Mark W. Spong studied conditions for synchronization of coupled phase with different natural frequency [3]. Recently, Miroslav Michev et al. studied cooperative behaviors in coupled oscillators with non-identical interactions, and discovered the sufficient condition for exponential synchronization [4]. As event-triggered control [5] shows considerable advantage in reducing the possibility of communication delay, and packet loss in data transferring in communication channels, and outperforms continuous-time control for energy saving since it only measures the system state intermittently. An event-based approach for Kuramoto oscillators was presented very recently, and all oscillators with all-to-all connections are proved to reach the average of all natural frequencies under certain trigger condition and event-triggered control mechanism [6].

In real networks such as smart grid, it is too luxurious to assume all generators bear fully-connected topology and the final synchronous state may deviate from the average of their natural frequencies. Inspired by the work of [2] and [3], a directed weighted topology is introduced to depict the entangled links between all generators. Meanwhile, an event-triggered control mechanism is designed to guarantee frequency synchronization for the purpose of energy-saving and packet-loss attenuation in communication.

2. Problem formulation

2.1. Networked oscillator model

In this work, we consider a Kuramoto-like model, which governs the interactions between coupled oscillators with underlying all-to-all graph, has the following form

$$\dot{\theta}_i = \omega_i + k\sum_{j\neq i}\sin(\theta_j - \theta_i) \qquad (1)$$

where θ_i stands for the phase of i-th oscillator, ω_i is the natural frequency, the parameter $k > 0$ is the coupling strength between each pair of oscillators. For simplicity, it is assumed that all nodes are connected to all other node, indicating an all-to-all topology and the natural frequency ω_i are assumed to be non-identical and randomly selected positive numbers.

In light of the work [2], by defining the incidence matrix of the topology, $B = (b_{ij})$, where $b_{ij} = -1$ denotes the edge j is incoming to oscillator i, and $b_{ij} = 1$ denotes the edge j is out-coming to oscillator i, one can obtain the Laplacian matrix of the underlying graph, $L = BB^T$. The above network model may be rewritten in a concise form

$$\dot{\Theta} = \omega - B\sin(B^T\Theta) \qquad (2)$$

where

$\Theta(t) = [\theta_1(t), \theta_2(t), \cdots \theta_N(t)]^T, \omega = [\omega_1, \omega_2, \cdots, \omega_N]$.

The objective of this work is to establish sufficient conditions for ensuring frequency synchronization under given coupling strength with an event-triggered control mechanism and suitable coupling strengths.

Definition: Frequency synchronization

For any pair of oscillators, if their phase frequencies satisfies

$$\dot{\theta}_i(t) - \dot{\theta}_j(t) \to 0 \qquad (3)$$

as $t \to \infty, \forall i, j = 1,2,\cdots, N.$

It indicates each pair of oscillators reaches a common frequency whereas their phase differences $\theta_i(t) - \theta_j(t)$ become invariant asymptotically.

Although a great deal of works has focused on this topic, continuous-time communication between oscillators is usually assumed. It requires that oscillators collected data in a continuous way, and hence may affected by various factors to a considerable extent, such as communication failure, delay transition, and packet-loss. It is of interest to consider under what conditions the coupled oscillators reach their mean natural frequencies by using an event-triggered control mechanism, and how to establish the relation between control strength and the synchronization for reaching mean natural frequency.

This work will concentrate on the above problems and establish criteria for frequency synchronization with event-triggered mechanism.

2.2. Event-triggered control strategy

The main idea of event-triggered control strategy is, for each control unit, to update the controllers at specified time instants at which some predefined triggering conditions are satisfied. It shows good performance in energy saving and bandwidth usage attenuation, especially for networked control system with often data transmission. By considering whether all individuals over the network use same triggering functions with all their states, such a control mechanism may be categorized as centralized and distributed approaches.

For simplicity consideration, only the centralized situation is studied in this work. Now we are in a position to propose our event-triggered control law for frequency synchronization. The above model with event-triggered controllers becomes

$$\dot{\theta}_i(t) = \omega_i + k\sum_{j\neq i}\sin(\theta_j(t_k) - \theta_i(t_k)), \forall t \in [t_k, t_{k+1}) \quad (4)$$

where t_k is the k-th event instant of the entire network. The above model implies that the control protocols can only be actuated at specified time instant triggered only when the trigger function meet certain conditions.

In order to achieve frequency synchronization, the knowledge about the updating time instants is also needed together with the event-triggered controllers.

It is assumed that all initial conditions are contained in the following compact set

$$D = \{\theta_i, \theta_j \big| |\theta_i - \theta_j| \le \pi/2 - 2\varepsilon, \forall i, j = 1,2,\cdots\}. \quad (5)$$

Firstly, a lightly modified lemma on the basis of [3] is given for latter use.

Lemma 1[3]: For the network model (1), let all initial phase difference be contained in a bounded set (5). Then, there exists a coupling gain

$$k > \frac{|\omega_{\max} - \omega_{\min}|}{2\cos(2\varepsilon)}, \quad (6)$$

such that $\theta_i - \theta_j \in D$ for any $t > 0$.

It is assumed that each oscillator has only the local information, the actual frequency $\dot{\theta}_i(t)$ and the measured frequency $\dot{\theta}_i(t_k)$. In order to constitute efficient trigger function and criteria for synchronization, define the following measurement errors for each oscillator,

$$e_i(t) = \dot{\theta}_i(t_k) - \dot{\theta}_i(t), k = 0,1,2\cdots, \quad (7)$$

An obvious observation is that $e_i(t_k) = 0.$

2.3. Main results

In this section, our main result will be given. For the above oscillator networks, by concatenating all states of the oscillators, it may be represented by using the following compact form

$$\dot{\Theta}(t) = \omega - kB\sin(B^T(\Theta(t) + e(t))), \forall t \in [t_k, t_{k+1}) \quad (8)$$

with $e(t) = [e_1(t), e_2(t), \cdots, e_N(t)]^T$.

One can readily compute the derivate of the oscillator as

$$\begin{aligned}\ddot{\Theta}(t) &= -kBdiag(\cos(B^T\Theta(t)))B^T\dot{\Theta}(t)\\ &= -k\tilde{L}\left(\dot{\Theta}(t) + e(t)\right)\end{aligned}$$

where $\tilde{L} = Bdiag(\cos(B^T\Theta))B^T$ remains symmetric, positive semi-definite matrix with initial phase difference restriction (5).

Noting that $\tilde{L}\Omega\mathbf{1} = 0$, and defining $\delta(t) = \dot{\Theta} - \Omega\mathbf{1}$, one may use the traditional Lyapunov function method for synchronization study of the above oscillator networks.

Since the objective is to make all oscillators reach a common frequency, the mean natural frequency, one may pick a Lyapunov candidate for the oscillator system as below

$$V = \frac{1}{2}\delta(t)^T\delta(t)$$

After some algebraic operations, it yields that

$$\begin{aligned}\dot{V} &= \delta(t)^T\dot{\delta}(t)\\ &= \delta(t)^T\left(-k\tilde{L}(\dot{\Theta}(t) + e(t))\right) = \delta(t)^T\left(-k\tilde{L}(\delta(t) + e(t))\right)\end{aligned}$$

Recalling the definition of matrix $\tilde{L}$ and the properties of triangular function, restricting K with condition (6), one obtains that

$$\begin{aligned}\dot{V} &\le -\sin(2\varepsilon)k\lambda_2(L)\|\delta(t)\|^2 - k\delta(t)^T\tilde{L}e(t)\\ &\le -\sin(2\varepsilon)k\lambda_2(L)\|\delta(t)\|^2 + \frac{\beta}{2}\|\tilde{L}\delta(t)\|^2 + \frac{k^2}{2\beta}\|e(t)\|^2\end{aligned}$$

By selecting $\beta = \lambda_N$, and using the symmetry of the matrix $L = BB^T$, one can obtain that

$$\dot V \le -\sin(2\varepsilon)k\lambda_2(L)\|\delta(t)\|^2 + \frac{\lambda_N}{2}\|\delta(t)\|^2 + \frac{k^2}{2\lambda_N}\|e(t)\|^2$$

$$= -k\left(\sin(2\varepsilon)\lambda_2(L) - \frac{\lambda_N(L)}{2}\right)\|\delta(t)\|^2 + \frac{k^2\lambda_N(L)}{2}\|e(t)\|^2$$

Let

$$\eta = \left(k\sin(2\varepsilon)\lambda_2(L) - \frac{\lambda_N}{2}\right),$$

and restrict $e(t)$ to satisfy

$$\|e(t)\| \le \frac{\sqrt{2\lambda_N(L)(\sigma-1+\eta)}}{k}\|\delta(t)\|, 0<\sigma<1, \quad (9)$$

One can obtain that

$$\dot V \le -(1-\sigma)\|\delta(t)\|^2. \quad (10)$$

To design a feasible event-triggered control strategy, and recall condition (6) and $\lambda_2(L) = \lambda_N(L) = N$, one may assume

$$k > \max\{\frac{|\omega_{\max} - \omega_{\min}|}{2\cos(2\varepsilon)}, \frac{1}{2\sin(2\varepsilon)}\frac{\lambda_N(L)}{\lambda_2(L)}\} \quad (11)$$

From the above deduction, one may choose the trigger function as

$$f(e(t),\dot\Theta) = \|e(t)\| - \frac{\sqrt{2\lambda_N(L)(\sigma-1+\eta)}}{k}\|\delta(t)\| = 0. \quad (12)$$

The event times is then defined by the triggering function $f(e(t_k),\delta(t_k)) = 0$, for an increasing sequel $k = 1,2,\cdots$.

At each instant t_k, one may observe that the measurement error $e(t_k) = \Theta(t_k) - \Theta(t_k) = 0$, which guarantees the validity of the triggering condition. For any other time instant, all oscillators evolve by using the frequency data collected at t_k and hence follows a piecewise constant control between the time interval $[t_k, t_{k+1}), \forall k = 1,2,\cdots$, which ensures all oscillators reach the common phase frequency, i.e., the mean natural frequency.

Theorem 1: Consider the coupled oscillator network (1), with all-to-all connections depicted by the incidence matrix *B*. For a specified coupling strength satisfying condition (11), the designed event-triggered control law (4) with triggering function (12) guarantee all oscillators reach the mean natural frequency, and the synchronization manifold is locally exponentially stable with an exponential convergence rate no worse than $1-\sigma$, where parameter $0<\sigma<1$ is defined as above.

Following the similar line as in Ref. [5], one can also obtain the following result, implies the oscillator network system shows no Zeno behaviors, which gives our another result

Theorem 2: For the above studied system with control law (11). For any $0<\sigma<1$, and the specified initial condition (4), the inter-event times $\{t_{k+1} - t_k\}$ given by the triggering condition (12) are lower bounded by

$$\tau = k\|L\|\int_0^\mu (1+y)^2 dy > 0,$$

with $\mu = \frac{\sqrt{2\lambda_N(L)(\sigma-1+\eta)}}{k}$.

Proof.

Inspired by [5], one can calculate the derivative of the quotient $\frac{\|e(t)\|}{\|\delta(t)\|}$.

$$\frac{d}{dt}\frac{\|e(t)\|}{\|\delta(t)\|} = \frac{e(t)^T\dot e(t)}{\|e(t)\|\|\delta(t)\|} - \frac{\delta(t)^T\dot\delta(t)}{\|\delta(t)\|^2}\frac{\|e(t)\|}{\|\delta(t)\|}$$

$$\le \frac{\|\dot e(t)\|}{\|\delta(t)\|} + \frac{\|e(t)\|\|\dot\delta(t)\|}{\|\delta(t)\|^2}$$

$$= \left(1 + \frac{\|e(t)\|}{\|\delta(t)\|}\right)\frac{\|\dot e(t)\|}{\|\delta(t)\|}$$

$$= \left(1 + \frac{\|e(t)\|}{\|\delta(t)\|}\right)\frac{\|\ddot\Theta(t)\|}{\|\delta(t)\|}$$

$$\le \left(1 + \frac{\|e(t)\|}{\|\delta(t)\|}\right)k\|L\|\frac{\|\delta(t)\| + \|e(t)\|}{\|\delta(t)\|}.$$

The last inequality above is allowed due to $\|\tilde L\| \le \|L\|, L\Omega\mathbf{1} = 0.$

By using the notation $y = \frac{\|e(t)\|}{\|\delta(t)\|}$, one obtains that

$$\dot y \le k\|L\|(1+y)^2$$

and y is upper bounded by

$$y(t) \le \varphi(t,\varphi_0),$$

with $\varphi(t,\varphi_0)$ is the solutions of

$$\dot\varphi = k\|L\|(1+y)^2, \varphi(0,\varphi_0) = \varphi_0.$$

The inter-event times are lower bounded by τ subject to

$$\varphi(\tau,0) = \frac{\sqrt{2\lambda_N(L)(\sigma-1+\eta)}}{k},$$

which is implicitly denoted by (12).

Therefore,

$$\tau = k\|L\|\int_0^\mu (1+y)^2 dy,$$

with $\mu = \frac{\sqrt{2\lambda_N(L)(\sigma - 1 + \eta)}}{k}$.

It completes the proof.

3. Numerical Simulations

For demonstration purpose, a simple example with numerical simulations will be carried out. In smart grids, it is of importance to make the rotators of all generators keep synchronous. It is assumed that there are 6 generators are involved, with all-to-to connections.

With the selection of parameters $k = 0.4, \sigma = 0.6$, one may verify that the conditions proposed are fulfilled. The designed event-triggered control (4) and the triggering function (12) can drive all oscillators reach the mean natural frequency. Fig. 1 and Fig. 2 shows the numerical results, and one can observe that all frequencies converge to the mean natural frequency and the final phase differences remains constants, which verifies the validity and efficiency of proposed control strategy.

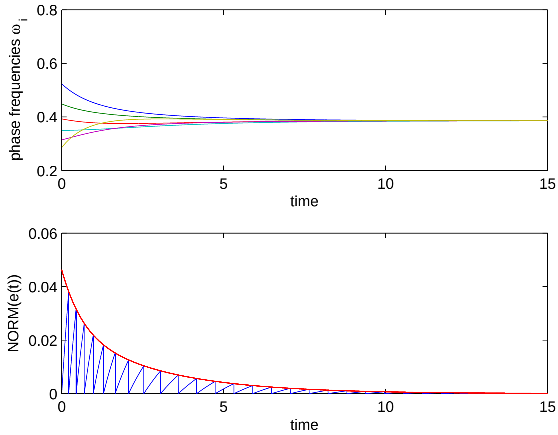

Fig.1: Synchronization errors of frequencies and the time history of the measure errors.

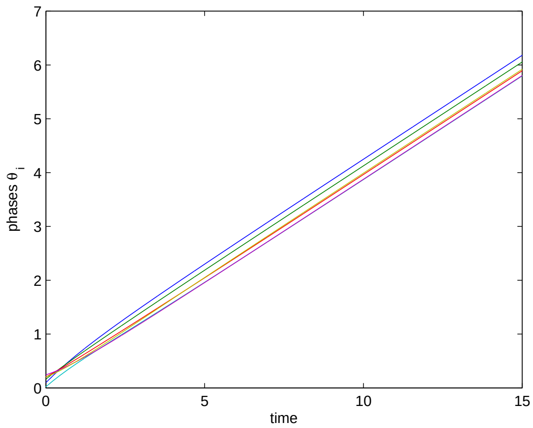

Fig.2: Time history of all oscillators

4. Conclusions

In this work, we consider the frequency synchronization of a group of networked oscillators with non-identical frequency. With Lyapunov stability method, an event-triggered control strategy together with specified triggering condition is established. The triggering condition proposed hereby relies on the measurement error and the all oscillators' deviations from the mean natural frequency. It is also demonstrated that the inter-event times involved is lower bounded and hence there is no Zeno behaviors.

Numerical simulations are also carried out for the verification purpose via a network of phase oscillators which typically models the operations of coupled generators in power grids. It shows that the designed control mechanism can guarantee the frequency synchronization of networked oscillators, and the frequency deviations vanish rapidly and the phase difference remains constant as time elapses.

References

[1] 1. Martin, Rohden, Andreas Sorge, Marc Timme et al., Self-organized synchronization in decentralized power grids, *Physical Review Letters*, vol. 109, 064101, 2012.

[2] Ali Jadbabaie, Nader Motee, Mauricio Barahona, On the stability of the Kuramoto model of coupled nonlinear oscillators, *Proceedings of the 2004 American Control conference*, Vol. 5, pp. 4296-4301, 2004.

[3] Nikhil Chopra, Mark W. Spong, On synchronization of Kuramoto oscillators, *Proceedings of the 44th IEEE Conference on Desicion and Control*, pp. 3916-3922, 2005.

[4] Miroslav Mirchov, Lasko Basnarkov, Fernando Corinto, Ljupco Kocarev, *IEEE Transactions on Circuits and Systems-I: Regular Papers*, vol. 61, no. 3, pp. 811-819, 2014.

[5] Dimos V. Dimarogonas, Karl H. Jahansson, Event-triggered control for multi-agent systems, *Proceeding of the 48th IEEE Conference on Decision and Control*, pp. 7131-7136, 2009.

[6] S. Golfinger, Event-triggered control for synchronization, KTH thesis, 2012.