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Chaotic Oscillations in Coupled Oscillators Networks with Scale-Free Property

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Abstract– In recent years, various oscillation and synchronization phenomena have been investigated for coupled oscillators. In this paper, we study oscillation in coupled oscillators networks with scale-free network structure. We have confirmed intermittency chaos in coupled network with ten oscillators.

1. Introduction

In recent years, various oscillation and synchronization phenomena have been investigated for coupled oscillators [1]–[5]. We have proposed star-coupled oscillators in which N oscillators are coupled by one resister [2]. Because the current through the coupling resister should be reduced to a minimum, N-phase synchronization phenomena can be observed. It is considered that this coupled oscillator can be used in various fields, because a variety of synchronization phenomena are exhibited. Especially, it is considered that star-coupled oscillators which are arranged in lattice or hexagonal structure can be used as some kinds of cellular neural networks [3].

On the other hand, there are not only regular networks like lattice or hexagonal shape, but also complex networks like small-world or scale-free network [4]–[6]. Among them, a scale-free network is focused in this paper.

Scale-free networks have features that most nodes has very few connections but a small number of particular nodes has many connections. From this feature, even if some parts of most nodes which have very few connections are removed, a global connection in a network is preserved. However, if small number of particular nodes which have many connections are removed, a network is interrupted simply. That is, they are robust to random removal, but they are vulnerable when the most connected nodes are removed. These features of scale-free networks are shown in many networks of various fields, e.g., internet, process diffused of word of mouth, metabolic network, etc. That is, to analyze the features of scale-free networks may be used for analyzing those practical networks.

In this paper, thereby first, we study about oscillation phenomena of a scale-free coupled oscillators network. We compose a scale-free network called Barabási-Albert model by resistively coupled van der Pol oscillators, and confirm the oscillation phenomena in the proposed network by both numerical calculation and circuit experiments. As a result, we show that intermittency chaos is observed in the proposed network.

2. LC oscillators Coupled by One Resistor [1]

If two LC oscillators are coupled by one resister as shown in Fig. 1, anti-phase synchronization phenomena can be stably observed because of the current through the coupling resister is reduced to a minimum [1] (see Fig. 2). In this study, a scale-free network is composed by the coupled oscillators shown in this section.

3. Barabási-Albert Model [7,8]

Barabási-Albert model is scale-free network model proposed by Barabási and Albert [7,8]. This model is constructed by growth and preferential attachment. A new node having the n links is added to initial network step by step with the probability Π_i. Π_i is the probability when the node i is selected to the destination of a new link, and decided by the following equation,

$$\Pi_i = \frac{N_i}{\Sigma_j N_j} \tag{1}$$

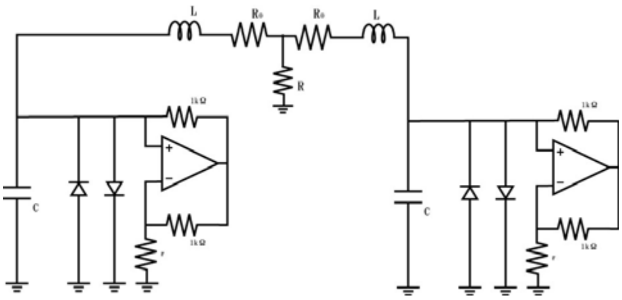

Fig.1: A circuit in which two oscillators are coupled by one resister.

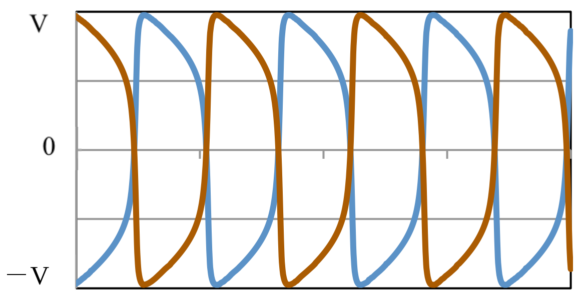

Fig.2: Synchronization in a circuit shown in Fig. 1.

where i and j are node numbers in the network and N_i is a number of links of node i.

4. Circuit Model

For $n = 2$, we construct a scale-free network by Barabási-Albert model based on the coupled oscillators shown in Fig. 1. Figure 3 shows the network model for this paper. In this network, adjoined two oscillators are coupled by one resister like the circuit shown in Fig. 1. An oscillator subcircuit shown in Fig. 4 composes a node of the network, and the links are composed by inductors and resistors. Table 1 shows the number of links connected to each node decided by Eq. (1).

Circuit equation of this network is described as

$$C\frac{dv_k}{dt} = -\sum_l i_{kl} - i_r(v_k)$$
$$L\frac{di_{kl}}{dt} = v_k - R_0 i_{kl} - R(i_{kl} + i_{lk}) \quad (2)$$

By changing the variables,

$$t = \sqrt{LC}\tau,$$
$$v_k = \sqrt{\frac{g_1}{3g_3}}x_k,\ i_k = \sqrt{\frac{Cg_1}{3Lg_3}}y_k,$$
$$\alpha = R\sqrt{\frac{C}{L}},\quad \beta = R_0\sqrt{\frac{C}{L}},\quad \varepsilon = g_1\sqrt{\frac{L}{C}}$$

equation (2) can be normalized as follows,

$$\dot{x}_k = -\sum_l y_{kl} + \varepsilon(x_k - \frac{x_k^3}{3})$$

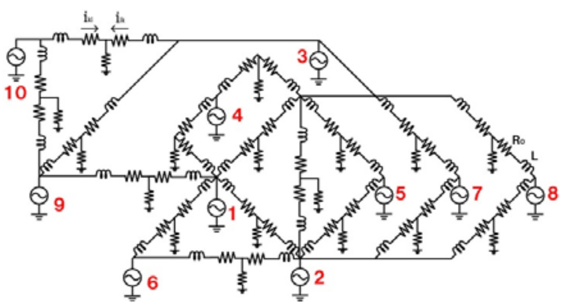

Fig. 3: Network model.

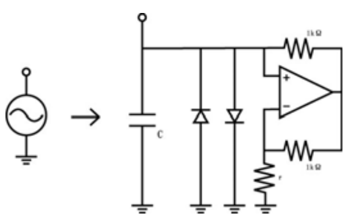

Fig. 4: Oscillator subcircuit model.

Table 1: Number of links for each node.

Oscillator	1	2	3	4	5	6	7	8	9	10
Links	5	6	8	2	2	2	2	2	3	2

$$\dot{y}_{kl} = x_k - \beta y_{kl} - \alpha(y_{kl} + y_{lk}) \quad (3)$$

where $\dot{x} = dx/d\tau$. α is the coupling factor and ε is the strength of nonlinearirity.

5. Numerical and Experimental Results

In this section, numerical calculations of Eq. (3) using fourth order Runge-Kutta method are carried out. The circuit parameter values are $L = 10$mH, $C = 0.068\mu$F, $R = 300\,\Omega$, $R_0 = 10\,\Omega$, $\varepsilon = 5$.

The waveforms of x_k $(k = 1,2,\cdots,10)$ obtained by numerical calculation are shown in Fig. 5. The discrete Fourier transforms and the attractors for oscillator 9 and 10 for $\tau = 9180.8 \sim 10000$ are shown in Figs. 6 and 7, respectively.

From Fig. 5, it is confirmed that the oscillators with large number of links show the higher frequency oscillations, because the number of inductors connected in parallel are larger. No.5, No.7 and No.8 oscillators have two links to both No.2 and No.3 oscillators. As shown in Fig. 3, the graph of these oscillators shows similar form. No.4, No.6 and No.10 oscillators have two links, too. But those exhibit different forms.

Moreover, we cannot see synchronous waveforms but intermittency chaos can be observed in the network, nevertheless the network is based on the coupled oscillators with synchronous state as shown in Sect. 2. to focus on No.9 and No.10, oscillation vary irregularly especially around $x_k = -1, 1$. Also, in attractors shown in Fig.7, we see loci turn round with small radius.

Intermittency chaos is a phenomenon which repeats regular and quiet state aperiodically, and disorderly and intense state for short time. Because an intermittency chaos is not observed in regular networks with coupled identical van der Pol oscillators, this is very interesting result.

In order to confirm adequacy of the numerical result, we also carried out circuit experiments. Figure 8 shows a result of the measurements by oscilloscope. In real circuit, we can see similar results to numerical results.

7. Conclusion

In this paper, we have studied oscillation phenomena in scale-free coupled oscillators network using Barabási-Albert model. In the proposed coupled oscillators network, an intermittency chaos could be observed. To study the intermittency chaos will become an issue in the future.

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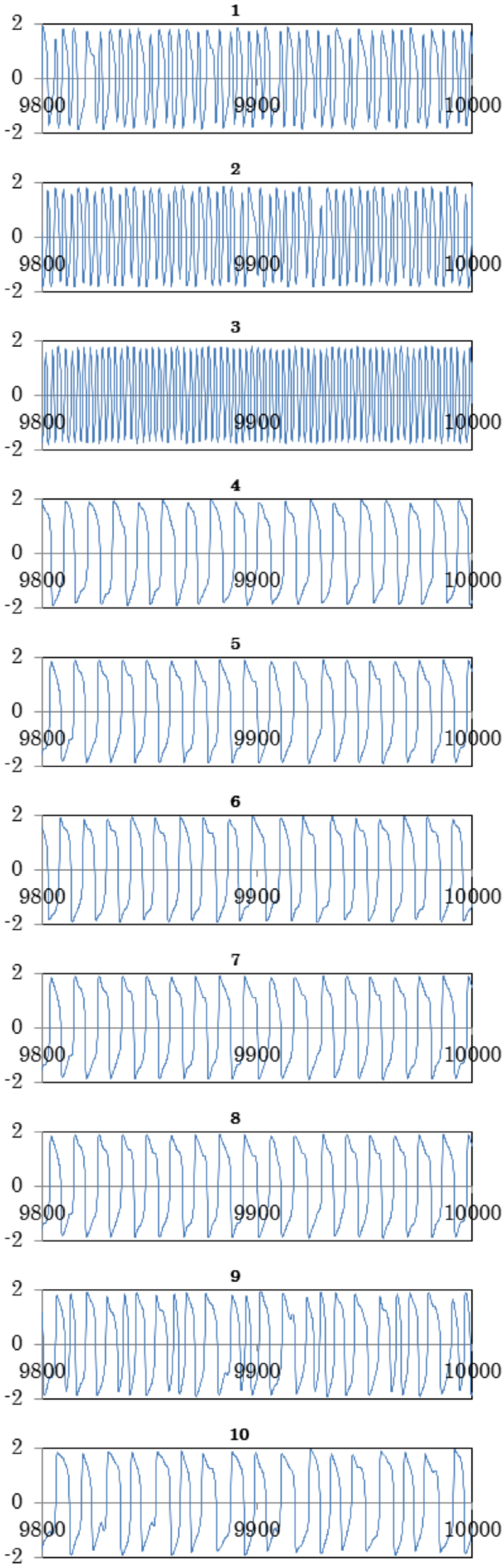

Fig. 5: Waveforms by the numerical calculation.

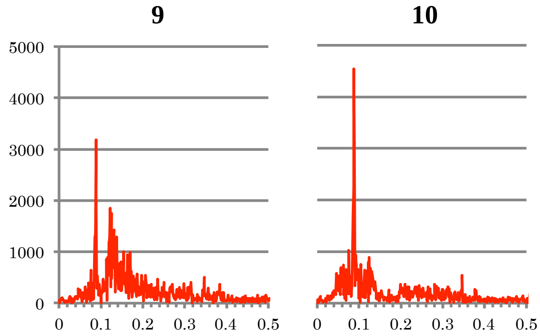

Fig. 6 DFT of oscillator No.9, 10

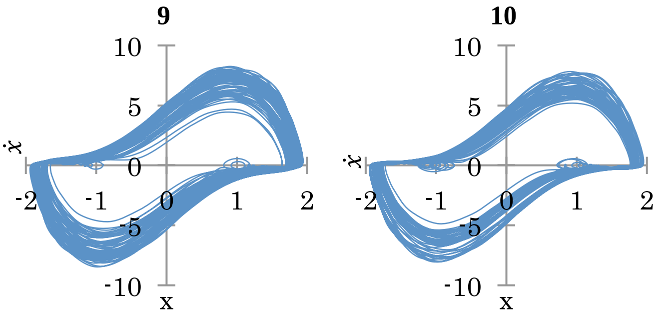

Fig. 7: Attractors of oscillator No.9 and 10.

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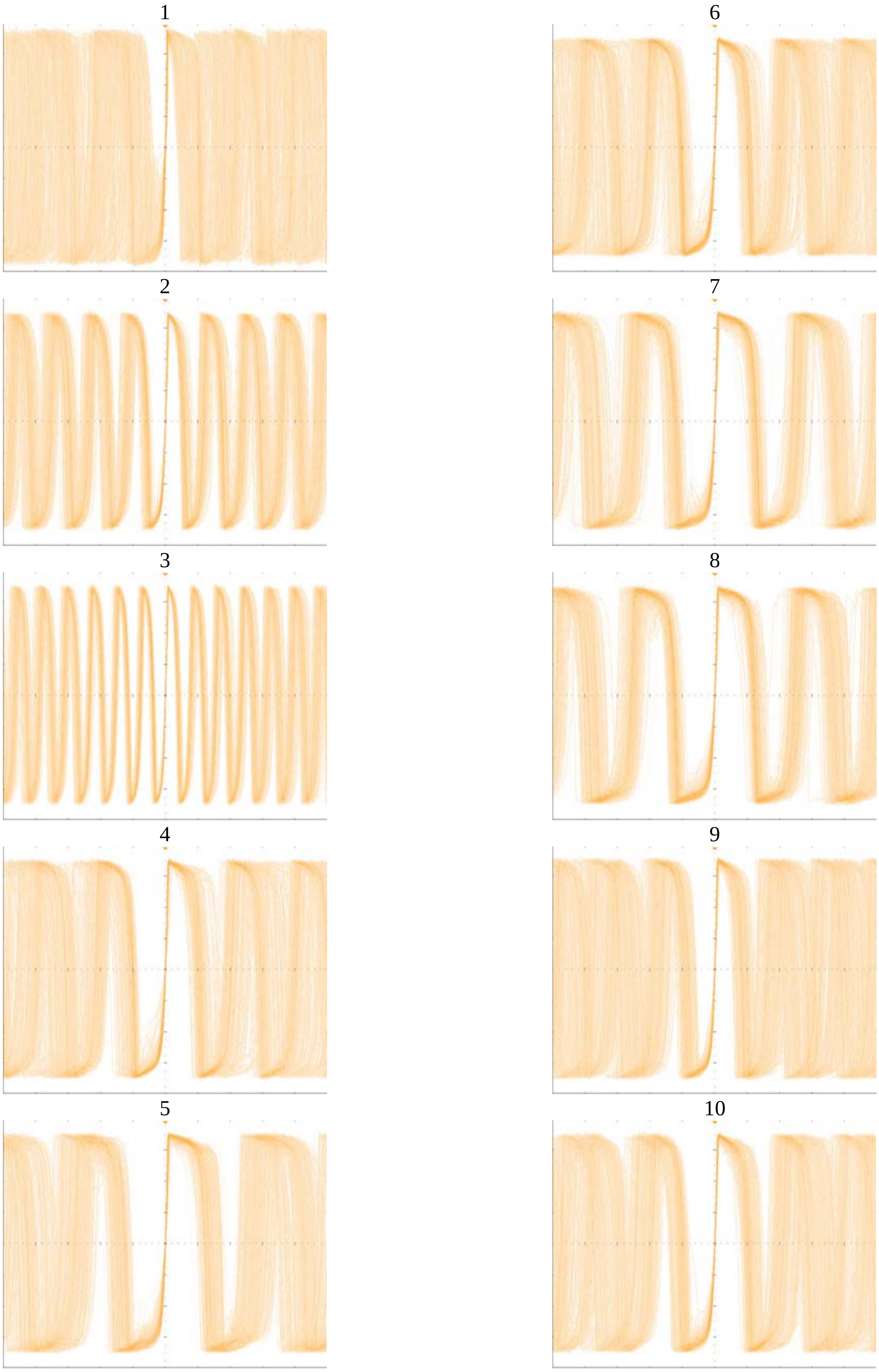

Fig.8: Experimental results.