2010 International Symposium on Nonlinear Theory and its Applications
NOLTA2010, Krakow, Poland, September 5-8, 2010

Single-Tone Moments Based Adjoint Sensitivity Analysis of Nonlinear Intermodulation Distortion in RF Circuits

Dani Tannir and Roni Khazaka

Department of Electrical and Computer Engineering, McGill University
Montreal, QC, Canada, H3A 2A7
Email: dani.tannir@mail.mcgill.ca, roni.khazaka@mcgill.ca

Abstract— Intermodulation distortion analysis is one of the main computational bottlenecks in the simulation of Radio Frequency circuits. Recently, an efficient moments based approach for computing the third order intercept point (IP3) using moments analysis with only single-tone inputs was presented. This approach did not, however, provide any sensitivity information which is critical for design centering, optimization and yield analysis applications among others. In this paper, we propose an efficient method for computing the sensitivity of IP3 using single-tone adjoint moments analysis. The adjoint method finds the sensitivity of one output with respect to all variables with only minimal additional computation cost to the original algorithm, thereby making it very efficient.

1. Introduction

One of the main specifications that engineers must account for in the design of Radio Frequency (RF) front ends is the linearity of the core blocks such as low noise amplifiers (LNAs) and mixers. Measuring the amount of the third order nonlinear distortion is of particular importance since its effects will appear in the passband of the system and are thus very difficult to filter out. This distortion in turn could lead to many undesirable effects such as gain compression and adjacent channel interference [1]. The key figure of merit for quantifying the third order nonlinear distortion is the third order intercept point (IP3) [1]. Determining the value of IP3 through simulation has, however, been one of the main bottlenecks in the design automation process due to the multi-tone input requirement which considerably slows down steady-state simulators based on techniques such as the Harmonic Balance method [2].

In [3], [4], an efficient method for computing the value of IP3 was presented based on the computation of the Harmonic Balance moments [5] from the general Harmonic Balance equations with only a single-tone input and without the need to find the solution of the equations. This reduced the computation cost to that of solving a system of sparse, linear equations, and also resulted in a significantly smaller system than that with multi-tone inputs. This is especially the case with mixer circuits where only two tones (1 RF tone in addition to that of the Local Oscillator) are required instead of the typical three tones. However, this approach did not provide any insight into the sensitivity of IP3 with respect to various circuit parameters. For any sensitivity analysis to be performed, only brute-force perturbation could be employed, which is very inefficient. In [6], a new approach for computing the sensitivity of IP3, based on the adjoint sensitivity method [7], was presented. The method in [6], however, is limited in its application to the multi-tone moments method presented in [8].

In this paper, we extend the method in [6] to the computation of IP3 sensitivity using the single-tone moments method. The adjoint method has been a classical tool for the sensitivity analysis of both linear and nonlinear circuits, including those operating under large signal periodic and almost-periodic conditions as is the case with the Harmonic Balance method [9]. The approach proposed in this paper benefits from the same computational cost advantage as [3], [4], while also providing the sensitivity of IP3 with respect to all the circuit parameters. This would provide a key advantage for circuit optimization, design space exploration and design centering analysis. It is to be noted that similarly to the approach in [3], [4] this method is general and easily automated for any arbitrary circuit topology. Finally, since the adjoint moments are computed using the same set of linear equations that are used to determine the Harmonic Balance moments, the CPU cost of the operation is reduced to that of finding three additional moments over the CPU cost of the method in [3], which is very cheap.

2. System Formulation

In this section, we present the general formulation of the nonlinear Harmonic Balance equations followed by overviews of the moments computation algorithm and of the method for computing IP3 using single-tone moments analysis. This will provide the necessary background information for the new single-tone sensitivity analysis method presented in Section 3.

2.1. Harmonic Balance Formulation

The Harmonic Balance equations for a nonlinear system are of the form [10]

$$\boldsymbol{A}\boldsymbol{X} + \boldsymbol{F}(\boldsymbol{X}) = \boldsymbol{B}_{DC} + \alpha\boldsymbol{B}_{RF} + \beta\boldsymbol{B}_{LO}, \tag{1}$$

where

- $\boldsymbol{X} \in \mathbb{R}^{N_h}$ is a vector of unknown cosine and sine coefficients for each of the variables in $\boldsymbol{x}(t)$.

- $\boldsymbol{B}_{DC} \in \mathbb{R}^{N_h}$ contains the contribution of the DC independent sources while $\boldsymbol{B}_{RF} \in \mathbb{R}^{N_h}$ and $\boldsymbol{B}_{LO} \in \mathbb{R}^{N_h}$ show the location of the RF and LO input frequency tones, respectively.

- α and β are the amplitudes of the RF and the LO voltages, respectively.

- $\boldsymbol{A} \in \mathbb{R}^{N_h \times N_h}$ is a block matrix representing the contribution of the linear elements.

- $\boldsymbol{F}(\boldsymbol{X}) \in \mathbb{R}^{N_h}$ is the vector of nonlinear equations.

2.2. Harmonic Balance Moments

The Harmonic Balance moments are the coefficients of the Taylor series expansion of the Harmonic Balance vector of unknowns, $\boldsymbol{X}$, with respect to the signal amplitude voltage α, as given by

$$\boldsymbol{X} = \boldsymbol{M}_0 + \boldsymbol{M}_1\alpha + \boldsymbol{M}_2\alpha^2 + \boldsymbol{M}_3\alpha^3 + \ldots = \sum_{i=0}^{\infty} \boldsymbol{M}_i\alpha^i \qquad (2)$$

where $\boldsymbol{M}_k$ is the k^{th} moment vector. The zeroth moment $\boldsymbol{M}_0$, is obtained by finding the solution of the system described by (1) with $\alpha = 0$. The remaining moment vectors $\boldsymbol{M}_n$ can then be found by solving the system of equations given by [5]

$$\boldsymbol{\Phi}\boldsymbol{M}_1 = \boldsymbol{B}_{RF} \qquad (3)$$

$$\boldsymbol{\Phi}\boldsymbol{M}_n = -\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\boldsymbol{T}_j\boldsymbol{M}_{n-j}, \quad n \geq 2 \qquad (4)$$

where $\boldsymbol{\Phi}$ is the moments computation matrix given by

$$\boldsymbol{\Phi} = \boldsymbol{A} + \left.\frac{\partial \boldsymbol{F}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right|_{(\alpha=0)}, \qquad (5)$$

and $\boldsymbol{T}_j$ are the coefficients of the Taylor series expansion with respect to α of the nonlinear Jacobian given by

$$\frac{\partial \boldsymbol{F}(\boldsymbol{X})}{\partial \boldsymbol{X}} = \sum_{i=0}^{\infty} \boldsymbol{T}_i\alpha^i, \qquad (6)$$

It is important to note that the matrix $\boldsymbol{\Phi}$ has the same structure as a Jacobian matrix but is evaluated with only the DC and LO tones present which makes it very sparse. As can be seen from (3) and (4), the computation of the moment vectors is a solution of a set of sparse linear algebraic equations where the left-hand-side matrix is the same throughout and is therefore very efficient.

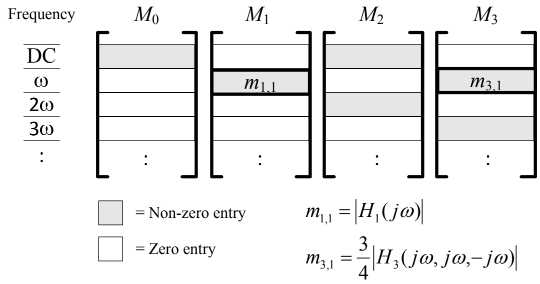

Figure 1: Location of distortion terms in moments for amplifier circuits

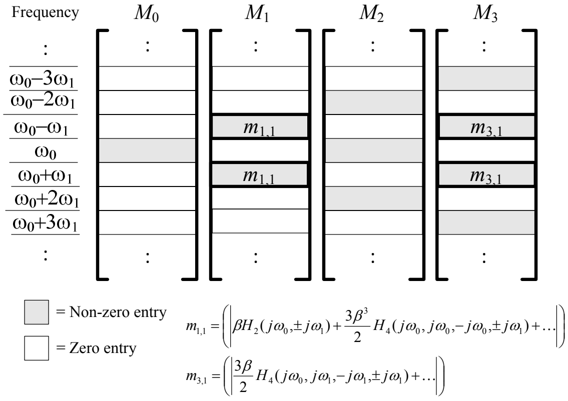

Figure 2: Location of distortion terms in moments for mixer circuits

2.3. Computation of IP3 from the Moments

The value of the input referred IP3 (IIP3) for a circuit excited with a single frequency tone RF signal of $V_{in}(t) = \alpha\cos(\omega t)$, can be determined from the moments using the following relation [3],

$$IIP3 = \sqrt{\frac{m_{1,1}}{m_{1,3}}} \qquad (7)$$

In this equation, $m_{1,1}$ represents the entry in the first moment vector at the fundamental frequency of ω, while the term $m_{1,3}$ represents the entry in the third moment vector at the same fundamental frequency as illustrated in Fig. 1 for amplifiers and in Fig. 2 for mixers.

3. Moments Based Sensitivity

The relative sensitivity of IIP3 with respect to a general parameter '$\gamma = \lambda_0 + \lambda$', is defined as follows

$$S_{\gamma}^{IIP3} = \lambda_0\frac{\partial(IIP3)}{\partial\gamma} = \lambda_0\frac{\partial(IIP3)}{\partial\lambda} \qquad (8)$$

where λ_0 is the nominal value of the parameter and λ is the change in its value. Essentially, To find the sensitivity of

IP3 with respect to the parameter λ, from (7) we note that we need to obtain

$$\frac{\partial}{\partial\lambda}(IIP3) = \frac{1}{2}\left(\frac{m_{1,1}}{m_{1,3}}\right)^{-\frac{1}{2}} \frac{m_{1,3}\frac{\partial m_{1,1}}{\partial\lambda} - m_{1,1}\frac{\partial m_{1,3}}{\partial\lambda}}{(m_{1,3})^2} \quad (9)$$

From (9), we observe that determining the sensitivity of IP3 now essentially comes down to determining the value of $\frac{\partial m_{1,1}}{\partial\lambda}$ and $\frac{\partial m_{1,3}}{\partial\lambda}$, since the terms $m_{1,1}$ and $m_{1,3}$ are already available from the original computation of IP3.

3.1. Adjoint Moments Sensitivity Analysis

The computation of the sensitivity of IP3 has been reduced to finding the derivatives of $m_{1,1}$ and $m_{1,3}$ with respect to λ. In this section, we derive an efficient adjoint-based approach for computing these derivatives. As illustrated in Fig. 1 and Fig. 2, the terms $m_{1,1}$ and $m_{1,3}$ can be written as

$$m_{1,1} = \boldsymbol{d}^T \boldsymbol{M}_1 \quad (10)$$

$$m_{1,3} = \boldsymbol{d}^T \boldsymbol{M}_3 \quad (11)$$

where $\boldsymbol{d}$ is a selection vector. Note that $m_{1,1}$ and $m_{1,3}$ appear in the Taylor expansion of X_{out} defined as

$$X_{out} = \boldsymbol{d}^T \boldsymbol{X} = m_{1,0} + m_{1,1}\alpha + m_{1,2}\alpha^2 + m_{1,3}\alpha^3 + \ldots \quad (12)$$

The derivative of $\boldsymbol{X}_{out}$ with respect to λ can now be written as

$$\frac{\partial X_{out}}{\partial\lambda} = \frac{\partial m_{1,0}}{\partial\lambda} + \frac{\boldsymbol{\partial m_{1,1}}}{\boldsymbol{\partial\lambda}}\alpha + \frac{\partial m_{1,2}}{\partial\lambda}\alpha^2 + \frac{\boldsymbol{\partial m_{1,3}}}{\boldsymbol{\partial\lambda}}\alpha^3 + \ldots \quad (13)$$

From this equation, we can deduce that the terms $\frac{\partial m_{1,1}}{\partial\lambda}$ and $\frac{\partial m_{1,3}}{\partial\lambda}$, required in (9), are the first and third moments of the expansion of $\frac{\partial X_{out}}{\partial\lambda}$. In order to compute these moments, we start by using the general Harmonic Balance adjoint sensitivity expression to write [9]

$$\frac{\partial X_{out}}{\partial\lambda} = \boldsymbol{X}_a^T \frac{\partial \boldsymbol{A}}{\partial\lambda}\boldsymbol{X} \quad (14)$$

where $\boldsymbol{X}_a$ is the solution of the Adjoint equations

$$\boldsymbol{J}^T \boldsymbol{X}_a = -\boldsymbol{d} \quad (15)$$

From (14), it can be seen that the moments of $\frac{\partial X_{out}}{\partial\lambda}$ can be expressed in terms of the moments of $\boldsymbol{X}$ and $\boldsymbol{X}_a$.

The adjoint moment vectors are defined as the Taylor series coefficients of the expansion of the adjoint solution vector $\boldsymbol{X}_a$, defined in (15), with respect to the signal amplitude voltage α. The expansion of $\boldsymbol{X}_a$ can therefore be expressed as

$$\boldsymbol{X}_a = \boldsymbol{M}_{a0} + \boldsymbol{M}_{a1}\alpha + \boldsymbol{M}_{a2}\alpha^2 + \boldsymbol{M}_{a3}\alpha^3 + \ldots = \sum_{i=0}^{\infty} \boldsymbol{M}_{ai}\alpha^i \quad (16)$$

where $\boldsymbol{M}_{ak}$ is the k^{th} adjoint moment. By substituting (2), (13) and (16) in (14), we get the final expressions in terms of the moments. By equating the powers of α and α^3 on both sides of the resulting expressions we can write

$$\frac{\partial m_{1,1}}{\partial\lambda} = \boldsymbol{M}_{a0}^T\left(\frac{\partial \boldsymbol{A}}{\partial\lambda}\right)\boldsymbol{M}_1 + \boldsymbol{M}_{a1}^T\left(\frac{\partial \boldsymbol{A}}{\partial\lambda}\right)\boldsymbol{M}_0 \quad (17)$$

$$\frac{\partial m_{1,3}}{\partial\lambda} = \boldsymbol{M}_{a0}^T\left(\frac{\partial \boldsymbol{A}}{\partial\lambda}\right)\boldsymbol{M}_3 + \boldsymbol{M}_{a1}^T\left(\frac{\partial \boldsymbol{A}}{\partial\lambda}\right)\boldsymbol{M}_2 + \boldsymbol{M}_{a2}^T\left(\frac{\partial \boldsymbol{A}}{\partial\lambda}\right)\boldsymbol{M}_1 + \boldsymbol{M}_{a3}^T\left(\frac{\partial \boldsymbol{A}}{\partial\lambda}\right)\boldsymbol{M}_0 \quad (18)$$

It is important to note that the matrix $\frac{\partial A}{\partial\lambda}$ contains only the harmonic balance 'stamp' of the derivative of the element that λ is a parameter of. It is, therefore, an extremely sparse matrix with at most four non-zero block entries.

3.2. Computation of the Adjoint Moments

The computation of the adjoint moment vectors (defined in (16)) is a very CPU efficient algorithm. This is achieved by first substituting (6) and (16) into (15), which gives the following general relation

$$\left(\boldsymbol{A} + \sum_{i=0}^{\infty} \boldsymbol{T}_i\alpha^i\right)^T \left(\sum_{i=0}^{\infty} \boldsymbol{M}_{ia}\alpha^i\right) = -\boldsymbol{d} \quad (19)$$

To determine the expressions for computing each individual adjoint moment vector, we equate powers of α on both sides of (19). This results in the following set of equations that can be solved sequentially:

$$\boldsymbol{\Phi}^T \boldsymbol{M}_{0a} = -\boldsymbol{d} \quad (20)$$

$$\boldsymbol{\Phi}^T \boldsymbol{M}_{1a} = -\boldsymbol{T}_1^T \boldsymbol{M}_{0a} \quad (21)$$

$$\boldsymbol{\Phi}^T \boldsymbol{M}_{2a} = -\boldsymbol{T}_1^T \boldsymbol{M}_{1a} - \boldsymbol{T}_2^T \boldsymbol{M}_{0a} \quad (22)$$

$$\boldsymbol{\Phi}^T \boldsymbol{M}_{3a} = -\boldsymbol{T}_1^T \boldsymbol{M}_{2a} - \boldsymbol{T}_2^T \boldsymbol{M}_{1a} - \boldsymbol{T}_3^T \boldsymbol{M}_{0a} \quad (23)$$

Note that $\boldsymbol{\Phi}$ is the same sparse moments computation matrix that was used to determine the Harmonic Balance moments in (3) and (4). This means that no additional LU decompositions are required to find the adjoint moments.

4. Numerical Examples

In this section, we compute the value of IP3 and its sensitivity for two example circuits using the single-tone moments method and then compare the results to those obtained using Harmonic Balance to demonstrate the accuracy and speed-up of the new approach. The two circuits considered are a differential Low Noise Amplifier (LNA) with an IIP3 of −7.24dBm, in addition to a singly-balanced mixer circuit with an IIP3 of −3.4dBm. Both circuits are implemented using Bipolar Junction Transistors and the sensitivity computed is with respect to a collector resistor R_C. The computation cost of the sensitivity with respect to additional parameters is negligible in both methods.

First, we use the Harmonic Balance method to determine the steady-state solution of both circuits. The Differential

Table 1: CPU Cost Comparison of Finding both IP3 and its Adjoint Sensitivity for the Differential Amplifier Circuit.

Type of Computation	Harmonic Balance (s)	Proposed Method (s)	Speed-Up
IP3	44.67	0.34	129 x
Sensitivity	6.03	0.03	201 x
Total	50.7	0.37	**137 x**

Table 2: CPU Cost Comparison of Finding both IP3 and its Adjoint Sensitivity for the Mixer Circuit.

Type of Computation	Harmonic Balance (s)	Proposed Method (s)	Speed-Up
IP3	118.43	0.43	275 x
Sensitivity	21.63	0.21	103 x
Total	140.06	0.64	**218 x**

LNA circuit was run with two input tones at $f_1 = 1000$ MHz and $f_2 = 1001$ MHz. The sensitivity of IP3 with respect to a change in R_C was found to be $-2.354 \times 10^{-4}V$. For the singly-balanced mixer, the two input RF tones were at $f_1 = 100$MHz and $f_2 = 100.1$MHz, while the LO frequency was set to $f_{LO} = 1$GHz. The sensitivity of IP3 with respect to R_C was found to be $3.104 \times 10^{-2}V$ for this particular circuit.

Next, using the single-tone moments method, we first compute the adjoint moments using the relations given in (20)–(23) and only a single tone at $f = 1000$MHz for the LNA, and at $f = 100$MHz for the mixer. The sensitivity expressions are then determined using (17) and (18) with the matrix $\frac{\partial A}{\partial \lambda}$ being the Harmonic Balance stamp of the resistor. The sensitivity of IP3 is then computed by evaluating (9). The IP3 sensitivity obtained showed a difference of 0.08% for the LNA, and a difference of 0.63% for the mixer when compared to perturbation. As can be seen, the results are very accurate.

Tables 1 and 2 show a comparison of the CPU times between traditional Harmonic Balance and the proposed method for determining IP3 and its adjoint sensitivity obtained using a prototype MATLAB simulator on a local workstation. As can be seen, the proposed method presents significant computational speedup in the CPU time needed to find both IP3 and its sensitivity.

5. Conclusion

In this paper, a method for the efficient sensitivity analysis of third order nonlinear distortion based on single-tone adjoint moments analysis was presented. This approach adds insight to the results of the single-tone moments based method for computing IP3 presented in [3], while still remaining significantly more efficient than traditional multi-tone simulation approaches based on Harmonic Balance.

References

[1] J. Rogers and C. Plett, *Radio Frequency Integrated Circuit Design*. Norwood, MA: Artech House, 2003.

[2] K. S. Kundert, J. K. White, and A. Sangiovanni-Vincentelli, *Steady-State Methods for Simulating Analog and Microwave Circuits*. Boston: Kluwer Academic, 1990.

[3] D. Tannir and R. Khazaka, "Computation of intermodulation distortion in RF circuits using single-tone moments analysis," *IEEE Trans. on Computer Aided Design of Integrated Circuits and Systems*, 2010.

[4] D. Tannir and R. Khazaka, "Computation of IP3 using single-tone moments analysis," in *Proc. Design Automation and Test in Europe Conference*, pp. 718–723, 2009.

[5] E. Gad, R. Khazaka, M. Nakhla, and R. Griffith, "A circuit reduction technique for finding the steady-state solution of nonlinear circuits," *IEEE Trans. on Microwave Theory and Techniques*, vol. 48, pp. 2389–2396, Dec. 2000.

[6] D. Tannir and R. Khazaka, "Adjoint sensitivity analysis of nonlinear distortion in RF circuits," in *Proceedings IEEE Custom Integrated Circuits Conference*, pp. 633–636, 2009.

[7] S. W. Director and R. A. Rohrer, "The generalized adjoint network and network sensitivities," *IEEE Trans. on Circuit Theory*, vol. CT–16, pp. 318–323, Dec. 1969.

[8] D. Tannir and R. Khazaka, "Moments based computation of intermodulation distortion in RF circuits," *IEEE Trans. on Microwave Theory and Techniques*, vol. 55, pp. 2135–2146, Oct. 2007.

[9] J. W. Bandler, Q. J. Zhang, and R. M. Biernacki, "A unified theory for frequency-domain simulation and sensitivity analysis of linear and nonlinear circuits," *IEEE Trans. on Microwave Theory and Techniques*, vol. 36, pp. 1661–1669, Dec. 1988.

[10] C. W. Ho, A. E. Ruehli, and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Transactions on Circuits and Systems*, vol. 22, pp. 504–509, June 1975.