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A PSO algorithm with hybrid population structure

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Abstract—A particle swarm optimization (PSO) algorithm with hybrid population topology is proposed in this paper. In contrast to the fully connected network in traditional PSO, the hybrid network is mixed with regular network which has strong exploration ability and scale-free network whose heterogeneous degree distribution diversify the searching ability of each particle. Based on the comparisons with some existing PSO variations, the effectiveness of the proposed PSO is confirmed in terms of solution quality and algorithm robustness.

1. Introduction

Particle swarm optimization (PSO) is an algorithm to simulate the behavior of flocks of birds and schools of fish [1]. Due to its simple concept, easy implementation and quick convergence, PSO has been widely used to deal with many nonlinear and complex practical problems, such as the power loss minimization problem [2].

To improve the performance of PSO, various topological neighborhoods have been considered. In standard PSO, the position of each particle is updated according to the best position found by the whole swarm and by itself, thus it has a fully connected network in which each particle is connected with others. Due to this feature, the standard PSO is easily trapped into local optima in solving the multimodel problems. Thus, the investigation of the suitable networks for PSO has attracted much attention. In [3], various network topologies such as rectangular, hexagonal, cylinder and toroidal networks are studied. In recent years, some concepts of complex networks have been borrowed into the population structure of PSO. For example, a scale-free informed PSO is proposed in [4], the BA model [5] is used as a self-organizing network generation mechanism to adaptively produce a population topology with scale-free property. Based on the small-world network structure [6], the particle is updated according to the distance between it and the one with the best function value among the whole swarm.

Based on the fact that the regular network has good exploration ability [7] and the scale-free network has strong exploration ability [4] respectively, these two types of networks are mixed together to form a hybrid network for the PSO neighborhood design in this paper. As a result, the proposed PSO could converge to better solutions with higher success rate than some existing optimization approaches.

2. Design of Optimization Algorithm

2.1. The standard PSO and variants

In PSO, the population is called a swarm and each individual is defined as a particle which represents a possible solution. The i−th particle has a position vector $x_i = (x_{i1}, x_{i2}, \cdots, x_{iD})$ and a velocity vector $v_i = (v_{i1}, v_{i2}, \cdots, v_{iD})$, where D is the dimension of the searching space. The velocity and position of the i−th particle is adjusted based on the information of the local best position found so far by itself (denoted as x_p) and the global best position discovered by the whole swarm (denoted as x_g) according to the following updating rule:

$$\begin{aligned} v_i &= w_t v_i + c_1 r_1 (x_p - x_i) + c_2 r_2 (x_g - x_i) \\ x_i &= x_i + v_i \end{aligned} \tag{1}$$

where w_t is the inertia weight; c_1 and c_2 are two positive constant acceleration coefficients; r_1 and r_2 are two uniformly distributed random values in (0, 1).

It is noted that w_t can be set as a constant value, it also can be updated as follows [8]:

$$w_t = w_{max} - t\frac{w_{max} - w_{min}}{iter_{max}} \tag{2}$$

where w_{max} and w_{min} are the maximum and minimum inertia weights, respectively; $iter_{max}$ is the maximum iteration.

In Eq. (1), the calculation methods of x_g give rise to two main PSO variants with respect to the neighborhood of a particle. One is the *gbest* PSO in which each particle is guided by the current global best particle. The other one is the *lbest* PSO whose particles learn form neighbors. Simulation results in [3, 4] prove that the proper neighborhood in *lbest* PSOs would be more likely to avoid the premature convergence which is always met in *gbest* PSOs. Thus, an effective *lbest* PSO with hybrid network topology (denoted as HPSO) is proposed in the following.

2.2. The hybrid network topology

To obtain high quality solutions, the swarm population is mixed with regular network and scale-free network, and

Table 1: Four benchmark functions

Function name	Formula	D	Search space	Minimum value	Criterion
Sphere function	$f_1(x_i) = \sum_{i=1}^{D}(x_i)^2$	30	$[-100, 100]^D$	0	0.01
Quartic function	$f_2(x_i) = \sum_{i=1}^{D}\left(x_i^4 + random[0,1)\right)$	30	$[-1.28, 1.28]^D$	0	1
Rastrigin function	$f_3(x_i) = \sum_{i=1}^{D}\left((x_i)^2 - 10\cos(2\pi x_i) + 10\right)$	30	$[-1.28, 1.28]^D$	0	100
Griewank function	$f_4(x_i) = \frac{1}{4000}\sum_{i=1}^{D}(x_i)^2 - \prod_{i=1}^{D}\cos\frac{x_i}{\sqrt{i}} + 1$	30	$[100, 100]^D$	0	0.05

an illustration of this hybrid topology is shown in Fig. 1. In the regular network, each node is connected to its nearest k neighbors, this statistic implies that a long time is required to transfer the information from one particle to others of the graph [2], thus different regions of the search space could be explored at the same time. In the scale-free network, there exists a few nodes with large neighborhood size and most nodes with relatively small neighborhood size, this heterogeneous property means that the hub particles can guide the search direction of the low-degree particles which have effects in small search regions [4], thus the hybrid network can guarantee the exploration and exploitation ability simultaneously.

To generate a network with scale-free property, the BA model [5] is used and its construction procedure is described as: Based on a fully connected network with m nodes, at each step, a new node is added and connected to n existing nodes with the probability $P_i = k_i / \sum_i(k_i)$, where k_i is the degree of node i.

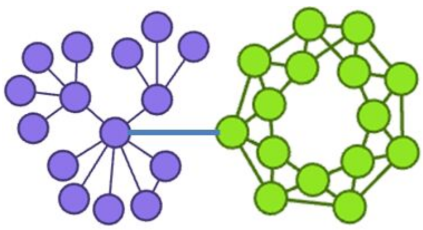

Figure 1: An illustration of the hybrid topology.

2.3. The PSO algorithm with hybrid topology

The operation procedures of the proposed HPSO are described as follows:

Step 1: Set the original iteration as $t = 0$. Randomly initialize the position x_i and velocity v_i of each particle i, the x_p is set as the copy of the current position x_i.

Step 2: Evaluate the fitness function for each particle i. Among its neighbors, find x_g with the best fitness value and update x_p.

Step 3: Calculate the positions and velocities of all particles according to Eq. (1).

Step 4: Start the next generation $t = t + 1$ and go to Step 2. This process is repeated until the maximum iteration is reached.

3. Simulation Results and Discussions

To demonstrate the effectiveness of the proposed HPSO, it is compared with some existing methods:

1. **FPSO**: This is the standard PSO with fully connected network.
2. **LWPSO**: The PSO algorithm with nonlinear decreasing inertia weight w_t as shown in Eq. (2).
3. **RPSO**: The PSO with regular network in which each node is connected to its nearest 2 neighbors.
4. **SFPSO**: The PSO with scale-free network, and parameters are set as $m = 5$, $n = 4$.
5. **IPSO**: This is an independent-minded PSO with dynamically changing network [9], the probability that a particle is influenced by the swarm is set as 0.2.

In the above six algorithms, each swarm has 50 particles, other parameters are set as $c_1 = c_2 = 1.479$. Every algorithm is run 100 times with the maximum iteration 10,000. In HPSO, the regular network and scale-free network have the same size 25, with parameters $k = 2$, $m = 5$, $n = 4$, $w_{max} = 0.9$ and $w_{min} = 0.4$.

For performance comparison, the above six optimization algorithms are applied on four benchmark optimization problems summarized as in Table 1. All the functions are minimum problems and the minimum value is the best value. The criterion is used to evaluate whether the optimization is successful or not. In each trial, if the criterion is not met within 10,000, it is thought this trial is unsuccessful. It is noted that f_1 and f_2 are unimodel functions, while f_3 and f_4 are multimodel functions with numerous local minima.

To check whether the algorithm could reach the predefined criterion as shown in Table 1, the successful rate S, defined as the percentage of successful runs, is illustrated in Table 2. Obviously, the proposed HPSO reaches $S = 100\%$ in most cases, thus the HPSO has strongest robustness among all PSOs. Moreover, compared with FPSO and LWPSO having fully connected network, RPSO, SFPSO and HPSO have better success rate. The reason is in these algorithms, when a particle discovers a solution with good quality, its information needs long time to reach others of the swarm. When a particle traps into

Table 2: The average values of the successful rate S and the convergence speed T of different algorithms

Functions	f_1		f_2		f_3		f_4	
	S	T	S	T	S	T	S	T
FPSO	100%	623	28%	1392	56%	276	43%	2828
LWPSO	100%	**599**	34%	**70**	65%	636	45%	**358**
SFPSO	100%	777	78%	1448	87%	**74**	81%	724
RPSO	100%	1017	87%	1171	92%	89	82%	801
IPSO	100%	910	**100%**	271	97%	459	89%	2458
HPSO	**100%**	881	**100%**	109	**100%**	129	**98%**	581

Table 3: Result of mean (Mean) and standard (Std) values of different algorithms on four benchmark functions

Functions	f_1		f_2		f_3		f_4	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
FPSO	1.74×10^{-18}	0.75×10^{-18}	3.91×10^{-1}	0.31×10^{-1}	8.03×10^{1}	1.03×10^{1}	3.45×10^{-2}	4.45×10^{-3}
LWPSO	$\mathbf{2.94\times10^{-19}}$	$\mathbf{0.32\times10^{-19}}$	7.01×10^{-3}	$\mathbf{0.65\times10^{-3}}$	8.12×10^{1}	0.97×10^{1}	1.18×10^{-2}	2.32×10^{-3}
SFPSO	3.67×10^{-18}	0.72×10^{-18}	5.34×10^{-1}	1.43×10^{-1}	6.68×10^{1}	$\mathbf{0.92\times10^{1}}$	1.09×10^{-2}	2.09×10^{-3}
RPSO	6.58×10^{-18}	0.24×10^{-18}	4.61×10^{-1}	1.03×10^{-1}	2.91×10^{1}	1.04×10^{1}	1.02×10^{-2}	$\mathbf{1.78\times10^{-3}}$
IPSO	5.01×10^{-18}	0.21×10^{-18}	1.74×10^{-1}	0.78×10^{-1}	4.68×10^{1}	0.97×10^{1}	8.92×10^{-3}	3.12×10^{-3}
HPSO	1.87×10^{-18}	0.13×10^{-18}	$\mathbf{5.51\times10^{-3}}$	1.98×10^{-3}	$\mathbf{2.32\times10^{1}}$	0.96×10^{1}	$\mathbf{5.92\times10^{-3}}$	2.15×10^{-3}

a local optima, its local information will be also slowly spread in the whole swarm. In a word, the slow spreading of local information among the swarm results in a high success rate. While in IPSO, the success rate is guaranteed by the dynamical changing network topology which helps avoid the local optima.

The evolving process of different algorithms are shown in Fig. 2, the mean and standard deviation values of the best particle at the maximum iteration over the successful runs are shown in Table 3. Obviously, except the unimodel function f_1 whose best value is obtained by LWPSO, the proposed HPSO achieves higher solution accuracy than all the other algorithms. From Fig. 2, it is observed that FPSO and LWPSO converge faster than the other PSOs in the beginning, then the population stay on a poor optima especially in multimodel functions f_3 and f_4, thus the mean values of FPSO and LWPSO in Table 3 are the worst. Compared with RPSO and SFPSO, the better performance of HPSO rely on two reasons: Firstly, in HPSO, the homogeneous degree distribution of regular networks implies that each particle has strong exploitation capability and weak exploration ability, while the heterogeneous degree distribution of scale-free network means the hub nodes have strong exploration ability and the low degree nodes have good exploitation ability, thus the hybrid network can balance the exploration-exploitation relation. Secondly, according to Eq. (2), in the initial stage, the current position of each particle is greatly influenced by its last position. When iteration is increased, the inertia weight is reduced, then the particle could learn more from the whole swarm to escape from the local optima.

Now, the convergence rate T, defined as the number of iterations required to accomplish the goal in Table 1, are show in Table 2. Obviously, the HPSO doesn't have the fast convergence rate, while it could find the best solution as shown in Table 3, thus a little heavy computation complexity is still worthy.

4. Conclusion

In the neighborhood design of PSO, the regular network whose homogeneous degree distribution implies strong exploitation capability and scale-free network with diversity exploration ability are mixed to form a hybrid network topology. Simulation results show that the proposed PSO finds solutions with better quality and higher success rate in multimodel problems than the traditional PSOs with fully connected networks.

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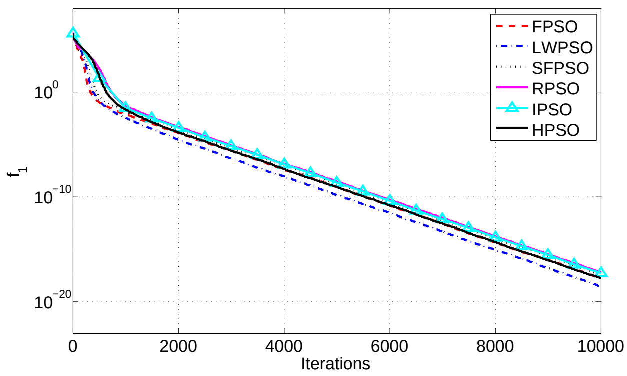
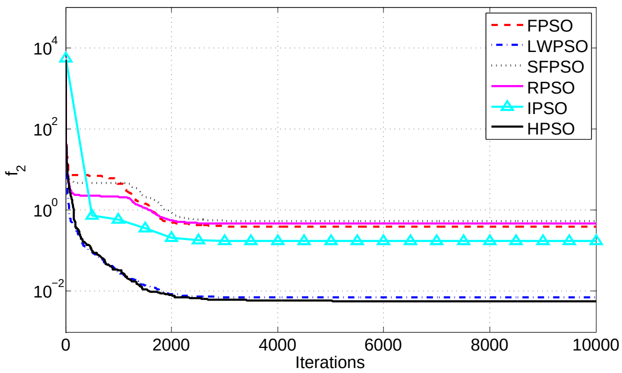
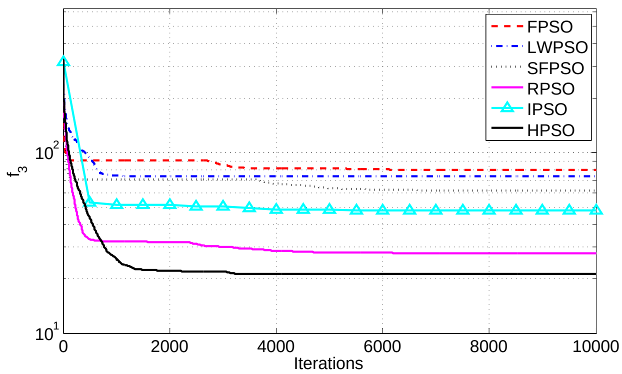


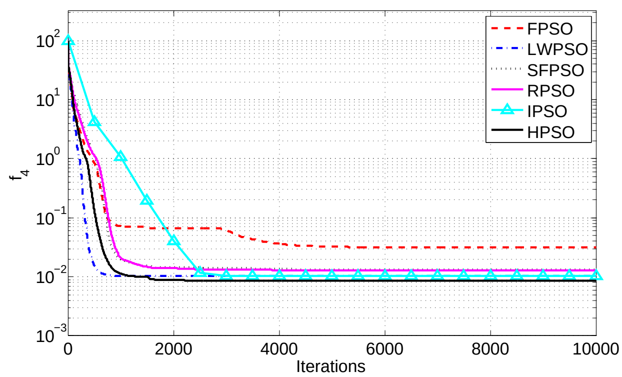
(a) Sphere function



(b) Quartic function



(c) Rastrigin function



(d) Griewank function

Figure 2: The algorithm evolving process versus iterations on different benchmark functions

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