



Soft Fault Diagnosis of Nonlinear Analog Circuits Using the Continuation Approach

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Abstract– This paper offers a method for multiple soft fault diagnosis of nonlinear analog circuits using the continuation (homotopy) approach. The considered soft faults include both slight and considerable deviations from the tolerance ranges. The method enables us to locate the faulty elements and evaluate the parameters both the faulty elements and the other elements considered as possible faulty. An important property of the method is its orientation on finding multiple solutions of the nonlinear diagnostic equation. As a result, it is able to find different sets of the parameters which satisfy the diagnostic test, rather than one specific set. The developed method is especially useful at the pre-production stage, where corrections of the technological process are possible and the diagnostic time is not crucial. To illustrate the proposed approach, a numerical example is given.

1. Introduction

Fault diagnosis of analog circuits is an important problem in the design and testing of electronic devices [1]-[7]. If circuit simulations take place after any testing, the diagnosis method is classified as the simulation-after-test (SAT) approach. A fault is called soft when the parameter deviates from its tolerance range, but does not produce a short circuit or an open circuit. Otherwise, when a fault leads to open nodes, shorts between nodes or other topological changes in a circuit, it is called catastrophic. Generally, fault diagnosis includes detecting faulty circuits, locating (identification) faulty elements and determining their parameters.

During the last several years, many methods devoted to soft fault diagnosis of analog circuits have been developed [6]-[13]. Appropriate tools for soft-fault diagnosis are SAT methods, based on measurements at some test points and analyses of the circuit under test. Most of the works address only the case when just one element is faulty. Several papers have been focused on the multiple fault diagnosis, e.g. [9]-[13].

Although many achievements in this field have been made, the problem is still open and no fully automatic method is available for analog circuits.

Many diagnostic methods employ linearized test equations. Consequently, they work correctly only if the parameters are slightly drifted from their tolerance ranges.

The approach proposed in this paper does not utilize such simplification, enabling us to diagnose larger faults and evaluate the parameters of all elements in the set of elements considered as possible faulty. The number of faulty elements in this set does not influence significantly the time consumed by the method.

An important property of the method proposed in this paper is its orientation on finding multiple solutions of the nonlinear diagnostic equation. As a result it can find different sets of parameters which satisfy the test, rather than one specific set.

Location (identification) of the faulty elements and determination of their parameters play a very important role at the pre-production stage, where corrections of the technological process are possible. In this way some defects of the technological process can be eliminated, leading to its improvement. The approach proposed in this paper is especially useful at this stage of the production. In such a case, the time consumed by the diagnostic method is not crucial.

Let us consider a nonlinear resistive circuit driven by DC independent sources, e.g. a diode-transistor one. Suppose that n circuit elements, specified by parameters x_1, \dots, x_n such as resistances, factors of controlled sources, β forward factors of bipolar transistors, are considered as possible faulty. We wish to find the actual values of all the parameters. For this purpose a diagnostic test must be performed leading to a systems of n nonlinear equations in n unknown variables x_1, \dots, x_n . The test equation can be presented in the form

$$\mathbf{f}(\hat{\mathbf{x}}) = \mathbf{u}, \quad (1)$$

where $\hat{\mathbf{x}} = [x_1 \dots x_n]^T \in R^n$ is a vector of the parameters, $\mathbf{u} = [u_1 \dots u_n]^T \in R^n$ is a vector of the measured signals, $\mathbf{f}(\hat{\mathbf{x}}) = [f_1(\hat{\mathbf{x}}) \dots f_n(\hat{\mathbf{x}})]^T$ is a nonlinear function mapping R^n into R^n . Generally it is impossible to formulate the nonlinear functions $f_i(\hat{\mathbf{x}})$, $i = 1, \dots, n$, in explicit analytical form, but the values of $f_i(x_1, \dots, x_n)$ and their derivatives with respect to x_j ($i, j = 1, \dots, n$) can be found numerically for given values of x_1, \dots, x_n . For example, the test can be arranged as follows. Assume that the nonlinear circuit under test has one accessible input

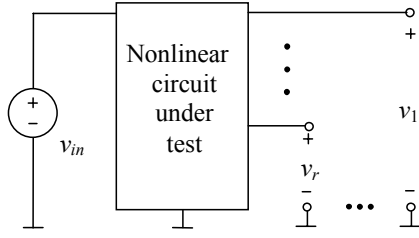


Fig. 1. Arrangement of a diagnostic test

node and r accessible output nodes. We apply a DC voltage source to the input node (see Fig. 1) and for K different values of this source measure the corresponding output voltages v_1, \dots, v_r . As a result we obtain rK values of the output voltages: $v_1^{(1)} = u_1, \dots, v_r^{(1)} = u_r$, \dots , $v_1^{(K)} = u_{(K-1)r+1}, \dots, v_r^{(K)} = u_{Kr}$. This test enables us to perform the diagnosis of a circuit with $n \leq rK$ elements considered as possible faulty. Some other appropriate variants of the test can be arranged.

In this paper we propose the continuation (homotopy) method [14]-[15] for finding multiple solutions of the diagnostic equation (1). The idea of the method is as follows. To solve an equation $\mathbf{g}(\mathbf{x}) = \mathbf{0}$ a parameter λ is embedded into this equation to obtain a new one

$$\mathbf{h}(\mathbf{x}, \lambda) = \mathbf{0}, \quad (2)$$

called a homotopy equation, such that for $\lambda = 0$, $\mathbf{h}(\mathbf{x}, \lambda) = \mathbf{0}$ can be easily solved and for $\lambda = 1$ it reduces to the original equation $\mathbf{g}(\mathbf{x}) = \mathbf{0}$. The augmented equation (2) is deformed as the parameter λ varies. At each stage of the deformation the corresponding solution is calculated, taking into account the previous solution. The Newton-Raphson method can be used for this purpose. As a result a solution (homotopy) path is traced and each intersection of the path with $\lambda = 1$ plane is a solution of the diagnostic equation.

2. Fault Diagnosis

Let us consider a circuit belonging to the class defined in Section 1 and assume that the parameters x_1, \dots, x_n have their nominal values $\hat{\mathbf{x}} = \hat{\mathbf{x}}^{(0)}$. We analyse this circuit and find the output signals leading to $\mathbf{u} = \mathbf{u}^{(0)}$. Thus, the equation

$$\mathbf{f}(\hat{\mathbf{x}}^{(0)}) = \mathbf{u}^{(0)} \quad (3)$$

holds. For the diagnostic equation (1) repeated below

$$\mathbf{f}(\hat{\mathbf{x}}) = \mathbf{u}, \quad (4)$$

we form the Newton homotopy [14]

$$\mathbf{h}(\hat{\mathbf{x}}, \lambda) = \mathbf{0}, \quad (5)$$

where

$$\mathbf{h}(\hat{\mathbf{x}}, \lambda) = \mathbf{f}(\hat{\mathbf{x}}) - \mathbf{u}^{(0)} - \lambda(\mathbf{u} - \mathbf{u}^{(0)}). \quad (6)$$

Note that for $\lambda = 0$ the equation specified by (5) and (6) reduces to the equation $\mathbf{f}(\hat{\mathbf{x}}) = \mathbf{u}^{(0)}$ having the known solution $\hat{\mathbf{x}}^{(0)}$, whereas for $\lambda = 1$ it becomes the original diagnostic equation. Denote $x_{n+1} = \lambda$ and form an augmented vector $\mathbf{x} = [x_1 \dots x_n x_{n+1}]^T$. Then the homotopy equation can be written as

$$\hat{\mathbf{h}}(\mathbf{x}) = \mathbf{0}. \quad (7)$$

Equation (7) represents a system of n nonlinear scalar equations in $n+1$ variables. As $x_{n+1} = \lambda$ varies, starting from $x_{n+1} = 0$, the solution of (7) traces a homotopy path.

Each intersection of this path with the $x_{n+1} = 1$ plane is a solution of the diagnostic equation (1). If there are several intersection points, then the test equation has several different solutions. Let us parameterize the path with respect to arc length [15]-[16]. This is the standard treatment of representing a path in parametric coordinates $\mathbf{x} = \mathbf{x}(s)$. If

$$\sqrt{\left(\frac{dx_1}{ds}\right)^2 + \dots + \left(\frac{dx_n}{ds}\right)^2 + \left(\frac{dx_{n+1}}{ds}\right)^2} = 1,$$

then s is the arc length of the path. Using the parameterization we form the set of equations

$$\hat{\mathbf{h}}(\mathbf{x}) = \mathbf{0}, \quad (8)$$

$$\sum_{i=1}^{n+1} \left(\frac{dx_i}{ds}\right)^2 = 1. \quad (9)$$

As $s = s_j$, then $\mathbf{x} = \mathbf{x}^{(j)}$. The derivative $\frac{dx_i}{ds}$ at $s = s_{j+1}$,

labeled $\left(\frac{dx_i}{ds}\right)^{(j+1)}$, can be expressed in terms of $x_i^{(j+1)}$

and $x_i^{(j)}$ using the approximate formula

$$\left(\frac{dx_i}{ds}\right)^{(j+1)} = \frac{1}{h} (x_i^{(j+1)} - x_i^{(j)}), \quad i = 1, \dots, n, \quad (10)$$

where $h = s_{j+1} - s_j$. Using (10) we formulate the set of equations (8)-(9) at $s = s_{j+1}$ as follows

$$\mathbf{w}(\mathbf{x}^{(j+1)}) = \mathbf{0}, \quad (11)$$

where

$$\mathbf{w}(\mathbf{x}^{(j+1)}) = \begin{bmatrix} \hat{h}_1(x_1^{(j+1)}, \dots, x_{n+1}^{(j+1)}) \\ \dots \\ \hat{h}_n(x_1^{(j+1)}, \dots, x_{n+1}^{(j+1)}) \\ \sum_{i=1}^{n+1} (x_i^{(j+1)} - x_i^{(j)})^2 - h^2 \end{bmatrix}. \quad (12)$$

To simplify the notation we denote $[x_1^{(j+1)} \dots x_{n+1}^{(j+1)}]^T = \mathbf{z} = [z_1 \dots z_{n+1}]^T$ and $x_i^{(j)} = X_i$. As a result the equation (11) becomes

$$\mathbf{w}(\mathbf{z}) = \begin{bmatrix} \hat{h}_1(z_1, \dots, z_{n+1}) \\ \dots \\ \hat{h}_n(z_1, \dots, z_{n+1}) \\ \sum_{i=1}^{n+1} (z_i - X_i)^2 - h^2 \end{bmatrix} = \mathbf{0} . \quad (13)$$

To solve the equation (13) we apply the Newton-Raphson method

$$\mathbf{z}^{(m+1)} = \mathbf{z}^{(m)} - \mathbf{J}^{-1}(\mathbf{z}^{(m)}) \mathbf{w}(\mathbf{z}^{(m)}) , \quad (14)$$

where m is the index of iteration. The Jacoby matrix is

$$\mathbf{J}(\mathbf{z}^{(m)}) = \begin{bmatrix} \frac{\partial \hat{h}_1}{\partial z_1} & \dots & \frac{\partial \hat{h}_1}{\partial z_n} & \frac{\partial \hat{h}_1}{\partial z_{n+1}} \\ \dots & \dots & \dots & \dots \\ \frac{\partial \hat{h}_n}{\partial z_1} & \dots & \frac{\partial \hat{h}_n}{\partial z_n} & \frac{\partial \hat{h}_n}{\partial z_{n+1}} \\ 2(z_1 - X_1) & \dots & 2(z_n - X_n) & 2(z_{n+1} - X_{n+1}) \end{bmatrix} \quad (15)$$

where $\mathbf{z} = \mathbf{z}^{(m)}$. Using (6) and (7) we find

$$\frac{\partial \hat{h}_1}{\partial z_{n+1}} = u_1^{(0)} - u_1 , \dots , \frac{\partial \hat{h}_n}{\partial z_{n+1}} = u_n^{(0)} - u_n . \quad (16)$$

Moreover, for $k, l = 1, \dots, n$ it holds

$$\left. \frac{\partial \hat{h}_k}{\partial z_l} \right|_{\mathbf{z}^{(m)}} = \left. \frac{\partial f_k}{\partial x_l} \right|_{\mathbf{z}^{(m)}} . \quad (17)$$

The derivatives df_k/dx_l , ($k, l = 1, \dots, n$) cannot be computed directly, because the function f is not given in explicit analytical form. To find the derivatives (17) we set $x_i = z_i^{(m)}$, ($i = 1, \dots, n$) and perform the DC and sensitivity analyses of the tested circuit at K values of the input voltages. In this way we find $f(\mathbf{z}^{(m)})$ and the sensitivities of the output voltages v_1, \dots, v_r with respect to the parameters x_1, \dots, x_n

$$\left. \frac{\partial v_d}{\partial x_l} \right|_{\mathbf{z}^{(m)}} = \left. \frac{\partial f_k}{\partial x_l} \right|_{\mathbf{z}^{(m)}} , \quad (18)$$

where v_d is an appropriate output voltage. The vector $\mathbf{w}(\mathbf{z}^{(m)})$ that appears in (14) is given by

$$\mathbf{w}(\mathbf{z}^{(m)}) = \begin{bmatrix} f_1(z_1^{(m)}, \dots, z_n^{(m)}) - u_1^{(0)} - z_{n+1}^{(m)}(u_1 - u_1^{(0)}) \\ \dots \\ f_n(z_1^{(m)}, \dots, z_n^{(m)}) - u_n^{(0)} - z_{n+1}^{(m)}(u_n - u_n^{(0)}) \\ \sum_{i=1}^{n+1} (z_i^{(m)} - X_i)^2 - h^2 \end{bmatrix} . \quad (19)$$

To perform the computation process efficiently some strategy of changing the step h has been worked out and applied.

The proposed method was implemented on a computer using Delphi and tested on several circuits. The

computations were executed using PC Pentium Core 2 Duo E6400.

3. Numerical Example

Let us consider the transistor circuit shown in Fig. 2. The nominal parameters, having the tolerances 5%, are indicated on the circuit diagram. To test the proposed method two series of diagnoses were carried out. In the first series all resistors $R_1 - R_7$ of the circuit were considered as possible faulty. Under the assumption that the nodes A and B are accessible for measurement we performed the diagnostic test, as described in Section 1, for different values of the input voltages: $v_{in}^{(1)} = 15V$, $v_{in}^{(2)} = 15V$; $v_{in}^{(1)} = 5V$, $v_{in}^{(2)} = 15V$; $v_{in}^{(1)} = 15V$, $v_{in}^{(2)} = 7V$; $v_{in}^{(1)} = 7V$, $v_{in}^{(2)} = 4V$. For these sets of input voltages we found (measured) 7 values of the output voltages. The measurement accuracy was assumed to be $1\mu V$.

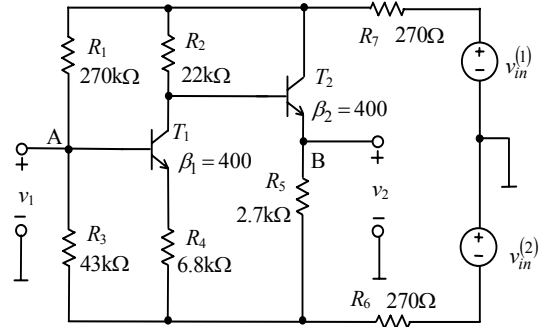


Fig. 2. A circuit for example

We considered 20 different sets of the circuit parameters. In most of the cases, the method gave more than one set of the parameters which satisfied the test. Some of them can be discarded due to negative values of certain resistances. In each case, the correct set of parameters appeared in the provided results.

One out of 20 considered cases is described below.

The parameters are as follows: $\{R_1 = 108k\Omega$ (-60%), $R_2 = 5k\Omega$ (-77%), $R_3 = 21k\Omega$ (-51%), $R_4 = 3k\Omega$ (-56%), $R_5 = 1.89k\Omega$ (-30%), $R_6 = 127\Omega$ (-53%), $R_7 = 125\Omega$ (-54%) $\}$.

Thus, all 7 elements of the set are faulty. The proposed method gives two sets of parameters which satisfy the test: $\{R_1^{(1)} = 107.7k\Omega$, $R_2^{(1)} = 4.98k\Omega$, $R_3^{(1)} = 20.9k\Omega$, $R_4^{(1)} = 2.99k\Omega$, $R_5^{(1)} = 1.89k\Omega$, $R_6^{(1)} = 127\Omega$, $R_7^{(1)} = 125\Omega\}$ and $\{R_1^{(2)} = 58.8k\Omega$, $R_2^{(2)} = 789\Omega$, $R_3^{(2)} = 11.8k\Omega$, $R_4^{(2)} = 495\Omega$, $R_5^{(2)} = 792\Omega$, $R_6^{(2)} = 51\Omega$, $R_7^{(2)} = 64\Omega\}$.

Thus, the circuit is detected as faulty. The time consumed by the method is 14s. The homotopy path is a closed curve, its projection on $\lambda - R_1$ plane is shown in Fig. 3.

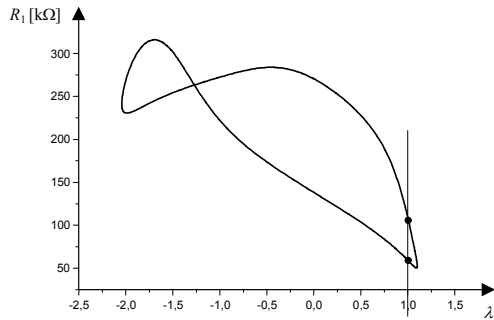


Fig. 3. Projection of the homotopy path on $\lambda - R_1$ plane

The points of intersection of the curve with the vertical line $\lambda = 1$ correspond to $R_1^{(1)}$ and $R_1^{(2)}$.

The second series of the diagnoses is discussed below. We consider all the resistors $R_1 - R_7$ and β forward factors of the transistors (β_1 and β_2) as possible faulty. The tolerances of all the parameters are 5%. We extend the previous test by adding a set of the input voltages: $v_{in}^{(1)} = 10\text{V}$, $v_{in}^{(2)} = 0\text{V}$ and find (measure) 9 values of the output voltages. The measurement accuracy is assumed to be $1\mu\text{V}$. We considered 10 different sets of the parameters. In most of the cases the method gave one set of the parameters which satisfied the test. In each case the correct set appeared in the provided results. One of the cases is described below. The parameters are as follows: $\{R_1 = 340\text{k}\Omega$ (26%), $R_2 = 12\text{k}\Omega$ (-45%), $R_3 = 44\text{k}\Omega$ (2%), $R_4 = 7.1\text{k}\Omega$ (4%), $R_5 = 2.5\text{k}\Omega$ (-7%), $R_6 = 265\Omega$ (-2%), $R_7 = 262\Omega$ (-3%), $\beta_1 = 340$ (-15%), $\beta_2 = 340$ (-15%)}. Thus, 5 elements are faulty. The proposed method gives just one (correct) set of the parameters. The time consumed by the method is 27s. It is satisfactory at the pre-production stage. For illustration, the projection of the obtained homotopy path on $\lambda - \beta_2$ plane is shown in Fig. 4. There is one point of intersection of the curve with the vertical line $\lambda = 1$, corresponding to $\beta_2 = 340$.

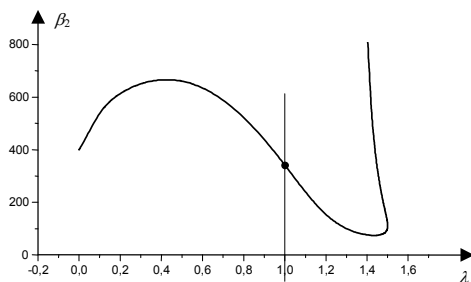


Fig. 4. Projection of the homotopy path on $\lambda - \beta_2$ plane

4. Conclusion

The method enables us to locate multiple soft faults of nonlinear analog circuits and evaluate the parameters of all elements considered as possible faulty. The new aspect of the proposed approach is its orientation on finding

different sets of parameters, which satisfy the diagnostic test, rather than one specific set. The soft faults accepted by the method may be caused both by slight and considerable deviations of the circuit parameters.

Acknowledgments

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