2015 International Symposium on Nonlinear Theory and its Applications
NOLTA2015, Kowloon, Hong Kong, China, December 1-4, 2015

A Performance Study of China UHV Power Grid Based on Oscillator Network

Minqi Du, Wallace K.S. Tang and Zhongyan Fan

Department of Electronic Engineering, City University of Hong Kong
83 Tat Chee Avenue, Kowloon, Hong Kong SAR, P. R. China
Email: minqidu2-c@my.cityu.edu.hk, eekstang@cityu.edu.hk, zyfan1991@gmail.com

Abstract—This paper is to perform analysis on an UHV power grid recently being developing in China. By modeling the power grid as a network of Kuramoto oscillators, its topological design is studied by investigating the synchronizability and resistivity to node failure of the associated model. Better network performance is observed by removing particular edges of the topology, implying that further improvement on the original design may be possible.

1. Introduction

Since 1975, the seminal model proposed by Kuramoto [1] has played an important role for the study of collective dynamics of coupled oscillators. The Kuramoto model is popular due to its simplicity and mathematically tractability. There are also many variants, including the first-order model with additional phase lag, with time delay [2] or with additive noise [3]; the second-order model [4]; and models with high order interaction [5, 6].

Network of Kuramoto oscillators has also been widely used for dynamical analysis of real-world systems, such as chemical reaction, biological neural network, laser array, financial system, etc. In particular, its usage in analysing power grid system has recently received a lot of interests [7, 8, 9, 10].

It is shown in [8] that a mapping between Kuramoto oscillators and the elements in power grid can be established. By defining two types of oscillators, the power delivery from the generators to the loads in an electrical power distribution grid can be described in a Kuramoto model. The importance of the model's topology is explored in [10], by indicating the existence of a Braess's paradox phenomenon based on synchronizability. Recently, significant efforts have been paid to provide the condition of synchronization, for example, an inequality in terms of the topology and oscillator's parameters is derived in [7].

In this paper, we are particularly interested in analyzing the power grid which has been rapidly developing in China. According to the State Grid Corporation of China [11], China is building a large scale of Ultra-high-voltage (UHV) grid, which acts as the backbone grid connecting the power generated in remote western and northern regions to main load cities, mostly located at the eastern and southern coasts. Our objective is to study the proposed grid under the framework of Kuramoto model. We focus on two significant factors, i.e. the synchronizability and the robostness. Synchronization assures the stability and maximizes the power transmission between generators and loads, while robustness provides resistance to disturbances or local failure. It is considered that this study may provide some insights about the current design and the possible improvement from the view point of topology.

2. Kuramoto Model for Power Grid

2.1. China UHV power grid

Figure 1 depicts the corresponding China's UHV power grid construction plan for year 2015 [12]. There are 40 cities ($N = 40$) in the network, where the power generated by the cities at the western and northern of China (as coloured in RED in Fig. 1), can be transferred to main load cities located at the eastern and southern coasts, as marked in GREEN.

2.2. The 2nd-order Kuramoto model

To model the China UHV power grid, the following second-order oscillator model [10] is adopted:

$$\frac{d^2\theta_j}{dt^2} = P_j - \alpha\frac{d\theta_j}{dt} + \sum_{i=1}^{N} K_{ij}\sin(\theta_i - \theta_j) \quad (1)$$

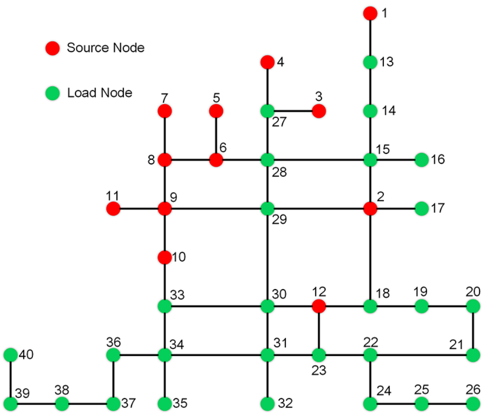
where each node of the power grid is now represented by an oscillator and θ_j denotes its phase, P_j is the power injection with $P_j > 0$ and $P_j < 0$ indicating source and load nodes, respectively, and $\sum P_j = 0$. The last term in (1) indicates the power transmitted between nodes i and j. The coupling gain K_{ij} is assumed to be fixed and identical, thus we have

$$K_{ij} = \begin{cases} \kappa & \text{if nodes } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

2.3. Phase Synchronization

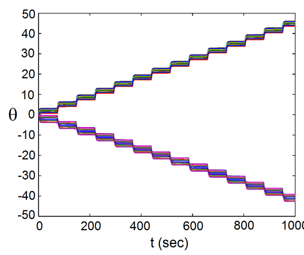
As stated in [7, 8], the stability of a power grid can be reflected by the condition of phase synchronization. Phase synchronization in (1) is reached when κ is above a threshold value, denoted as κ_c. As illustrated in Fig. 2, the nodes in (1) with topology as shown in Fig. 1 (b) are unsynchronized when κ is small, but become synchronized when κ increases to 6.2026. As a result, $\kappa_c = 6.2026$.

(a)

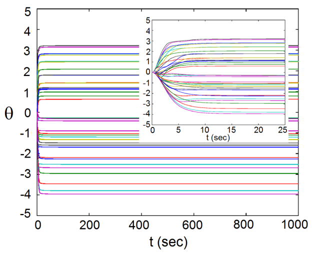


(b)

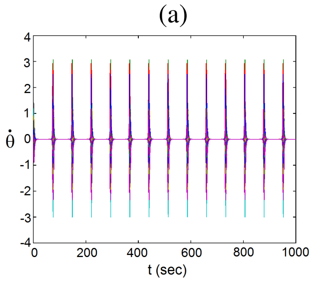
Figure 1: (a) China UHV power grid in Year 2015 (b) Topological representation of the China UHV power grid.



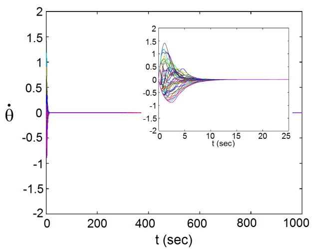
(a)



(b)



(c)



(d)

Figure 2: Unsynchronized state when $\kappa = 6.20$ (a) θ versus time (c) $\dot{\theta}$ versus time. Synchronized state when $\kappa = 6.2026$ (b) θ versus time (d) $\dot{\theta}$ versus time.

For better illustration and comparison, [10] suggests a phase order parameter which is defined by

$$Re^{j\psi} \equiv \frac{1}{N}\sum_{j=1}^{N} e^{i\phi_j}. \tag{3}$$

It is remarked that $R = 0$ if the system cannot reach an equilibrium state. As observed in Fig. 3, R increases after κ reaches κ_c, and saturates when κ further increases.

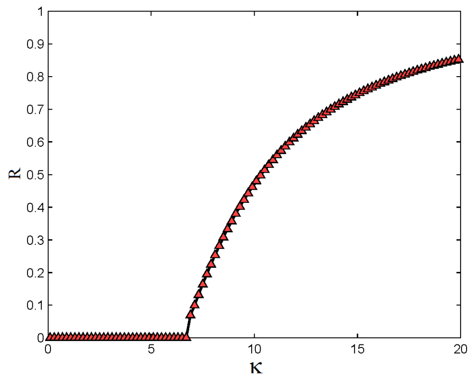


Figure 3: Relationship between R and κ.

3. Simulation Results and Analyses

A bisection search is designed as given in Fig. 4 to determine κ_c of a topology. As noticed in (1), if $\sum P_j = 0$, $\dot{\theta}$ becomes zero when synchronization is reached (also see Fig. 2 (d)). Consider the time window $\Sigma \equiv [(T_{max} - \tau), T_{max}]$, synchronization is thus assumed when:

$$\begin{cases} \left|\dot{\theta}_j(t)\right| < \epsilon, & \text{with } t \in \Sigma \\ \dot{\theta}_j(t_1) > 0 \text{ and } \dot{\theta}_j(t_2) < 0 & \text{for some } t_1, t_2 \in \Sigma \end{cases} \tag{4}$$

for $j = 1, 2, \cdots, N$. In our simulation, we let $\epsilon = 10^{-5}$, $\tau = 2s$ and $T_{max} = 1000s$ such that transient can be eliminated.

```
Preconditions:
(i) network is synchronized with κ=κh
(ii) network is not synchronized with κ=κl

while (κh-κl) ≥ δ
{
  κm = 0.5(κh + κl);
  if (network(κ=κm) is synchronized)
      κh = κm;
  else
      κl = κm;
}
return(κh);
```

Figure 4: Bisection algorithm for searching κ_c of a topology.

To study the goodness of the topology in Fig. 1 (b), we firstly remove a possible edge once at a time and obtain the new values of κ_c by the Algorithm in Fig. 4. The results are shown in Fig. 5 and tabulated in Table 1. Those gray edges in Fig. 5 are not considered as their removal will generate isolated node(s).

It is interesting to point out that, although κ_c will generally be increased when edges are removed, topology without edge (12–30) or (23–31) can further reduce κ_c, and the topology without edge (12–30) is the best. For the sake of clarification, we called the original topology as Topology-A and the one with edge (12–30) removed as Topology-B. The below comparisons will be focused on these two topologies.

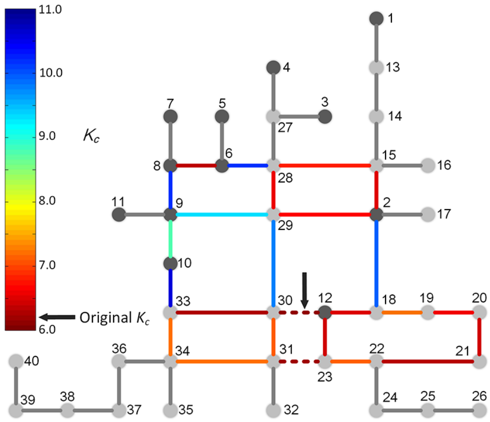


Figure 5: Critical value κ_c when the corresponding edge is removed.

Table 1: The values of κ_c for the topologies with the corresponding edge being removed.

Edge	κ_c	Edge	κ_c	Edge	κ_c
2–15	6.4237	12–18	6.3878	**23–31**	**6.1631**
2–18	9.161 8	12–23	6.3664	28–29	6.5039
2–29	6.5170	**12–30**	**6.1220**	29–30	9.0087
6–8	6.2888	15–28	6.6611	30–31	6.8729
6–28	9.4645	18–19	6.9972	30–33	6.2388
8–9	9.3422	19–20	6.4543	31–34	6.9972
9–10	8.2414	20–21	6.3479	33–34	6.9974
9–29	8.6667	21–22	6.2537		
10–33	9.6860	22–23	6.9971		

The second performance study is on the robustness of the topology. A topology is considered to be more robust if synchronization can be retained after a node failure. Here, three different power redistribution schemes are assumed. When node j is failed, its power injection P_j will be

1. uniformly redistributed to its connected neighbors;
2. redistributed to neighbors in a portion inversely proportional to the Eulerian distance; or
3. redistributed to neighbour with the nearest Eulerian distance.

Assuming that node 20 is failed at $t = 500s$ and its load is redistributed to neighbors according to the second redistribution scheme, Topology A will lose its synchronization while Topology B will not (see Fig. 6).

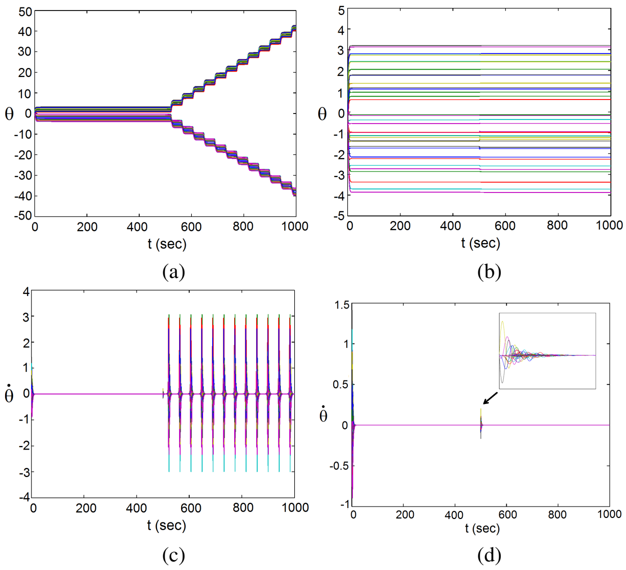


Figure 6: Node failure occurs in node 20. Synchronization is lost in Topology A: (a) θ versus time (c) $\dot{\theta}$ versus time. Synchronization can be retained in Topology B: (b) θ versus time (d) $\dot{\theta}$ versus time.

The robustness can be measured in several ways. Firstly, it is assumed that the same κ is used in both Topologies A and B. A node is critical if its failure will cause the desynchronization of the network. Figure 7 shows the number of critical nodes of the two topologies under the three redistribution schemes, where $\kappa = 6.2026$ is used. It can be concluded that Topology B is more robust, as there are less critical nodes at all the schemes. Moreover, it can be observed that less critical nodes are resulted if the third redistribution scheme is applied.

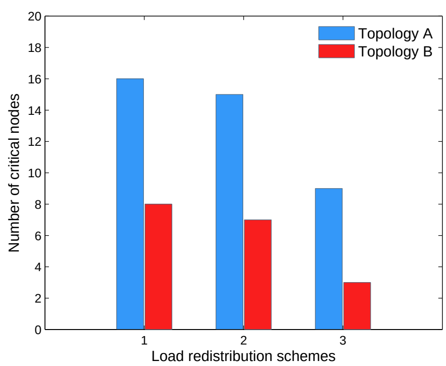


Figure 7: Numbers of critical nodes in Topologies A and B under different load redistribution schemes.

Secondly, a numerical metric, called S-value, can be proposed to reflect the robustness. Let $\sigma_\theta(j)$ be the standard deviation of nodes' phase after the transient state when node j is failed, the S-value can be defined as:

$$S = \sum_{j=1}^{N} \sigma_\theta(j) \tag{5}$$

where a smaller S means better.

The results are tabulated in Table 2, and as shown, the S-value of Topology A is smaller than that of Topology B.

Table 2: The S-values of Topologies A and B for having single node failure.

Redistribution Scheme	1	2	3
Topology A	22.9176	21.0350	11.7774
Topology B	16.9566	15.4216	5.8865

Lastly, comparison can also be made by computing the new κ_c under node failure such that no critical node exists in the topology. The results are shown in Table 3, and again, κ_c of Topology B under all three load redistribution schemes are smaller, implying that Topology B is better.

Table 3: New κ_C of Topologies A and B such that synchronisation can be retained under any single node failure.

Redistribution Scheme	1	2	3
Topology A	6.9156	6.8635	6.7100
Topology B	6.7692	6.7411	6.6770

4. Conclusions

In this paper, the topological design of the Year 2015 China UHV Power Grid is studied based on the framework of Kuramoto model. By modeling the cities as nodes and their connections as edges, a network of Kuramoto oscillators is constructed. The synchronizability and robustness of this network are then investigated. It is found that, although edge removal will reduce the network performance for most of the cases, two subnets with particular edge being removed can indeed perform better. Based on the simulation results, it is also concluded that, when node failure occurs, it will be better to redistribute the load to one of the neighbours instead of to all of them. Finally, this piece of work also furnishes oru future work on network optimization, for which the critical value κ_c and S-value can be considered as the objective values.

Acknowledgments

This work was supported in part by the Alexander von Humboldt Research Group Linkage 3.4-IP-DEU/1009882.

References

[1] Y. Kuramoto, "Self-entrainment of a population of coupled nonlinear oscillators," *Int. Symp. Mathematical Problems in Theoretical Physics, Lecture Notes in Physics*, vol. 39. Springer-Verlag, New York. pp. 420, 1975.

[2] A Nordenfelt, A. Wagemakers and M.A.F. Sanjuan, "Frequency dispersion in the time-delayed Kuramoto model," *Phys. Rev. E*, vol. 89, article 032905, 2014.

[3] S. Gupta, A. Campa and S. Ruffo, "Nonequilibrium first-order phase transition in coupled oscillator systems with inertia and noise," *Phys. Rev. E*, vol. 89, article 022123, 2014.

[4] Y.P. Choi, Z. Li, S.Y. Ha, X. Xue and S.B. Yun, "Complete entrainment of Kuramoto oscillators with inertia on networks via gradient-like flow," *J. Differential Equations,* vol. 257, pp. 2591-2621, 2014.

[5] J.R. Engelbrecht and R. Mirollo, "Structure of long-term average frequencies for Kuramoto oscillator systems," *Phys. Rev. Lett.* vol. 109, article 034103, 2012.

[6] K. Li, S. Ma, H. Li and J. Yang, "Transition to synchronization in a Kuramoto model with the first- and second-order interaction terms," *Phys. Rev. E,* vol. 89, article 032917, 2014.

[7] F. Dorfler, M. Chertkov and F. Bullo, "Synchronization in complex oscillator networks and smart grids," *Proc. Natl. Acad. Sci.,* vol. 110, no. 6, pp. 2005-2010, 2013.

[8] G. Filatrella, A.H. Nielsen and N.F. Pedersen, "Analysis of a power grid using a Kuramoto-like model," *Eur. Phys. J. B.*, vol. 61, pp. 485-491, 2008.

[9] M. Rohden, A.Sorge, M. Timme and D. Witthaut, "Self-organized synchronization in decentralized power grids," *Phy. Rev. Lett.* vol. 109, article 064101, 2012.

[10] D. Witthaut and M. Timme, "Braess's paradox in oscillator networks, desynchronization and power outage," *New J. Phys.,* vol. 14, article 083036, 2012.

[11] State Grid Corporation of China, Ultra high voltage AC grid, http://www.sgcc.com.cn/ztzl/tgyzl/default.shtml

[12] Picture of Year 2015 China UHV Power Grid Plan, http://image1.caing.com/2011-04-24/100251799.jpg